CUMULATIVE DAMAGE ESTIMATION USING WAVELET TRANSFORM OF STRUCTURAL RESPONSE

Ryutaro SEGAWA[1], Shizuo YAMAMOTO[2], Akira SONE[3] And Arata MASUDA[4]

SUMMARY

During a strong earthquake, the response of a structure with bilinear restoring force has discontinuity in jerk. In this paper, the method of estimate the ductility factor by the wavelet analysis of absolute acceleration response is proposed. It is found that the wavelet coefficient obtained by wavelet analysis of acceleration response is proportional to the velocity at yield point and the ductility factor is evaluated from the velocity at yield point by the energy conservation law. Using the proposed method, the ductility factor is estimated from wavelet coefficient with good accuracy and its applicability is verified through some simulations.

INTRODUCTION

The cumulative damage of structure subjected to a strong seismic motion is the low cycle fatigue with large amplitude and small number of cycle, which occurs during time interval of 20 or 30 second. The ductility factor; that is defined as the ratio of peak relative displacement and the yield relative displacement of structure is utilized for estimating the cumulative damage of low cycle fatigue. For the estimation of cumulative damage, it is necessary to know the following two factors.

(1) Peak displacement in each cycle oscillation.

(2) Number of these oscillations.

When the structures with hysteretic restoring force behave from the elastic region to plastic region, the acceleration response changes rapidly and its time derivative (jerk) has discontinuity at the yield point. Therefore, for estimating second item, the wavelet analysis seems to be a suitable method.

The authors [Sone, et al., 1995] have been proposing the estimation method of cumulative damage due to low cycle fatigue of structures subjected to a strong earthquake. It was assumed that the discontinuity might be contained in the observed acceleration records, which should be produced at the yield point changing from the elastic region to the plastic region and might be detected by wavelet analysis. Through the numerical simulations using a single degree of freedom model, it was proven that this assumption was correct and it was confirmed that the wavelet analysis was used to estimate the cumulative damage of structures by earthquake, effectively. However, we do not apply this analysis to the estimation of above first item.

Considering the low cycle fatigue of structures, it is important to estimate not only the number of discontinuity but also damage rate due to the low cycle fatigue. In this paper, we consider single degree of freedom model with bilinear restoring force subjected to filtered white noise excitation with dominant frequency. This paper aims at identifying the ductility factor, using the magnitude of wavelet coefficient obtained by the wavelet analysis of acceleration response. Especially, the relation between the velocity at the yield point and the ductility factor is derived based on the energy balance technique. The relation between the velocity at the yield point and the magnitude of wavelet coefficient for acceleration response is also discussed.

[1] *Dept of Mechanical and System Engineering, Kyoto Institute of Technology, Japan.Email: segawa@viblab.mech.kit.ac.jp*
[2] *Dept of Mechanical and System Engineering, Kyoto Institute of Technology, Japan.Email: segawa@viblab.mech.kit.ac.jp*
[3] *Dept of Mechanical and System Engineering, Kyoto Institute of Technology, Japan.Email: sone@ipc.kit.ac.jp*
[4] *Dept of Mechanical and System Engineering, Kyoto Institute of Technology, Japan.Email: masuda@ipc.kit.ac.jp*

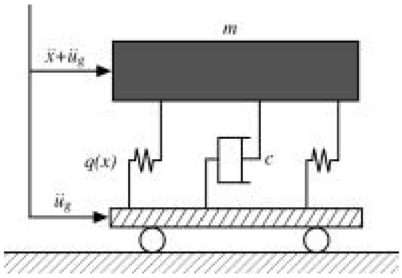


Figure 1: Model of structure with bilinear restoring force

BILINEAR RESTORING FORCE AND DISCONTINUITY IN JERK AT YIELD DISPLACEMENT

Bilinear Restoring Force

The equation of motion of structure shown in Fig.1 is given as follows:

$$m\ddot{x}+c\dot{x}+q(x)=-m\ddot{u}_g \tag{1}$$

Where *m*, *c* and *q(x)* are mass, damping coefficient, restoring force, respectively. $\ddot{u}_g$ and x are seismic input acceleration and relative displacement. Dividing the both sides of Eq.(1) by mx_y (x_y: yield relative displacement), the following nondimensional equation is given by

$$\ddot{X}+2\zeta\omega_n\dot{X}+Q(X)=-\ddot{U}_g \tag{2}$$

Where, the restoring force *Q(X)* is shown as the linear combination of a linear component and a hysteretic component [Suzuki and Minai, 1994].

$$Q(X)=\gamma\omega_n^2X+(1-\gamma)\omega_n^2Z \tag{3}$$

Where γ is the ratio of postyield stiffness and preyield stiffness. X and Z are nondimensional parameters as follows.

$$X=\frac{x}{x_y},Z=\frac{z}{x_y},\zeta=\frac{c}{2\sqrt{mk}},\omega_n=\sqrt{\frac{k}{m}},\ddot{U}_g=\frac{\ddot{u}_g}{x_y} \tag{4}$$

ζ and ω_n are the damping ratio and natural circular frequency of structure. $\ddot{U}_g$ is the normalized acceleration. Time derivative $\dot{Z}$ of nondimensional restoring force Z is given by using the step function $U(\cdot)$.

$$\dot{Z}=\dot{X}\left[1-U(\dot{X})U(Z-1)-U(-\dot{X})U(-Z-1)\right] \tag{5}$$

The physical meaning for Eq.(3) is explained by taking the examples $\gamma=1$ and $\gamma=0$ as shown in Fig.2. That is, when $\gamma=1$, the second term in Eq.(3) becomes zero and the first term shows the elastic restoring force as shown in Fig.2(a). On the other hand, when $\gamma=0$, the first term in Eq.(3) becomes zero and the second term shows the restoring force for $\gamma=0$ as shown in Fig.2(b). When $\gamma=0.5$, the value of vertical axis in Fig.2(a) and (b) reduces by half and the combination of two figures is shown in Fig.2(c).

Discontinuity in Jerk Response at Yield Point

Rearranging Eq.(2), the absolute acceleration $\ddot{X}+\ddot{U}_g$ is given by the following equation.

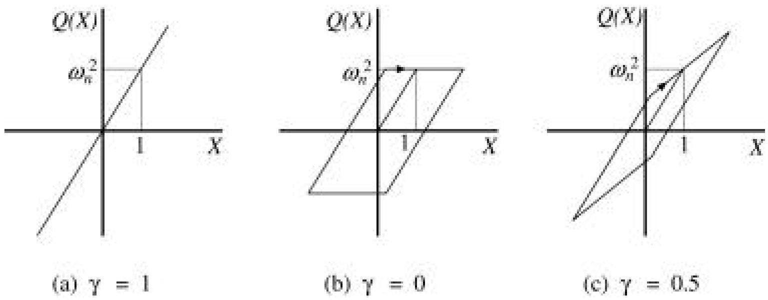


Figure 2: Explanation of bilinear restoring force for example of $\gamma=0.5$

$$\ddot{X}+\ddot{U}_g = -2\zeta\omega_n\dot{X} - \gamma\omega_n^2 X - (1-\gamma)\omega_n^2 Z \tag{6}$$

In the right hand of this equation, the displacement X, the velocity $\dot{X}$ and the restoring force Z are continuous because the equation of motion is second order. Differentiating the both sides of Eq.(6) with respective to time, the following equation is given by

$$\dddot{X}+\dddot{U}_g = -2\zeta\omega_n\ddot{X} - \gamma\omega_n^2 \dot{X} - (1-\gamma)\omega_n^2 \dot{Z} \tag{7}$$

In this equation, the acceleration $\ddot{X}$ is still continuous, but the time derivative of restoring force Z is not continuous. The time derivative of restoring force is shown in Fig.3 and it is found that $\dot{Z}$ is equal to $\dot{X}$ on the inclined lines and is equal to zero on the flat lines. Then, the value of Z at the yield point changing from the inclined line to the flat line has discontinuity. Consequently, the jerk; that is the time derivative of acceleration ($\dddot{X}+\dddot{U}_g$), as shown in Eq.(7) has discontinuity at the yield points and the magnitude of the discontinuity is related to $(1-\gamma)\omega_n^2\dot{X}$.

ESTIMATION OF DUCTILITY FACTOR

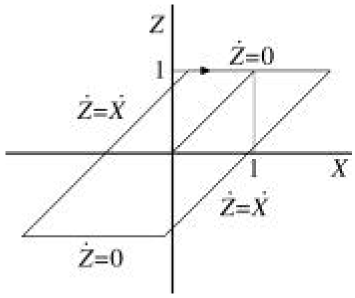


Figure 3: Time derivative of restoring force

Wavelet Analysis

The absolute acceleration shown in Eq.(6) is expanded by the wavelet series $\psi_{j,k}$ as follows [Sone, et al, 1995 and Ashino and Yamamoto, 1997].

$$\ddot{X}+\ddot{U}_g = \sum_j \sum_k \alpha_{j,k}\psi_{j,k}(t) \tag{8}$$

Where $\psi_{j,k}(t)$ is an analysing wavelet. j, k are the dilation and translation parameters, respectively. It is known that wavelet coefficients $\alpha_{j,k}$ are changed dramatically when the absolute acceleration response is analyzed by the wavelet transform [Ashino and Yamamoto, 1997]. Then, the wavelet coefficient $\alpha_{j,k}$ obtained by Eq.(8) can detect discontinuity contained in the jerk at the yield point as shown in Fig.3. Besides, because of the relationship between the magnitude of discontinuity and $(1-\gamma)\omega_n^2\dot{X}$, the wavelet coefficient $\alpha_{j,k}$ is assumed by the following equation.

$$|\alpha_{j,k}| = (1-\gamma)\omega_n^2|v_y|\cdot a \tag{9}$$

Where a is the constant value which is obtained when the kind of wavelet and the level j used in the wavelet analysis are fixed. v_y is the relative velocity at the yield point.

The physical meaning for Eq.(9) is that the sudden discontinuity at the yield point causes a jerk, which registers as high frequency and high amplitude accelerations, and the jerk depends on the velocity. So, as the velocity at the yield point becomes larger, the change in acceleration becomes remarkable. Then, we assumed that the discontinuity at the yield point could be detected by the wavelet coefficient $\alpha_{j,k}$ for the acceleration response. Moreover, it is expected that as the velocity at the yield point increases, the peak displacement (ductility factor) becomes large.

Energy Balance

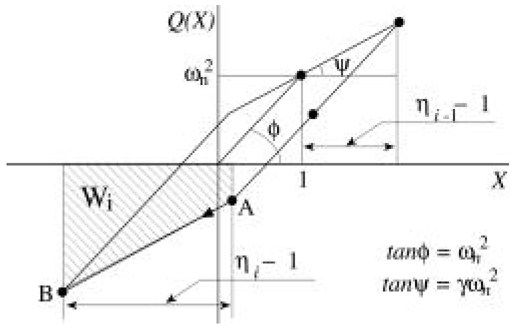


Figure 4: Energy balance

The energy balance of *i*-th step is shown in Fig.4. If the effect of seismic excitation is not considered, the kinetic energy at the point A is equal to the potential energy shown by the shaded area. This potential energy W_i is given by

$$W_i = \frac{1}{2}\omega_n^2\left[2(1-\gamma)(\eta_{i-1}-1)+\gamma(\eta_i-1)\right](\eta_i-1) \qquad (10)$$

Where, η_i is the ductility factor at *i*-th step ($= x_{mi}/x_y$). x_{mi} and x_y are maximum relative displacement of *i*-th step and yield relative displacement. The kinetic energy T_i at the point A is given by

$$T_i = \frac{1}{2}v_{yi}^2 \qquad (11)$$

Then, after equating Eqs.(10) with (11), the relation between velocities v_{yi} at the yield point and ductility factors η_i is obtained as follows.

$$v_{yi} = \sqrt{\omega_n^2\left[2(1-\gamma)(\eta_{i-1}-1)+\gamma(\eta_i-1)\right](\eta_i-1)} \qquad (12)$$

Equation(12) means that the velocity v_y at the yield point depends on the current ductility factor η_i and the prior ductility factor η_{i-1}. Then, if an initial ductility factor is zero, it will be able to evaluate ductility factors from the velocities at the yield point successively.

Therefore, using Eq.(9) and Eq.(12), it is possible to estimate the ductility factor from the wavelet coefficient, which is obtained by wavelet analysis of absolute acceleration response.

VERIFICATION OF PROPOSED METHOD BY SIMULATION

Response Analysis

Since the horizontal seismic motion is ideally assumed to be a motion with a dominant frequency and is stochastic in nature, stochastic modeling of ground motion seems appropriate [Tajimi, 1960]. Therefore, in this paper, a seismic acceleration $\ddot{u}_g$ to a structural model is the filtered white noise which is passed through the second order system ($\omega_f = 4\pi$, $\zeta_f = 0.5$). It stimulates the response of structure in condition of resonance. The mean-square amplitude of input acceleration and its time duration are set on referring to post earthquake motion. The mean-square amplitude of $\ddot{u}_g$ is 1549(cm^2/s^4). The structural parameter of the model as shown in Eq.(2) is taken as

$$\omega_n = 2\pi, \zeta = 0.01, x_y = 5(\text{cm}), \gamma = \text{variable}(0\text{ -}1) \qquad (13)$$

In order to obtain the relationship between the wavelet coefficient and the ductility factor, response analyasis is performed from 0 to 1000 seconds.

In this section, the case of system parameter $\gamma = 0.05$ is examined The sampling frequency is 100Hz. The input acceleration and absolute acceleration response from 0 to 25 seconds are shown in Fig.5 and Fig.6, the time history of restoring force is shown in Fig.7 and the absolute jerk response evaluated by numerical differentiation of absolute acceleration is shown in Fig.8. Though it can be seen from Fig.7 that the displacement response gets into the plastic region five times, the discontinuity at the yield point can not be found only by observing the acceleration response in Fig.6. While, the absolute jerk response ought to contain some discontinuous informations. But unless knowledge of the time entering to plastic region and observation of the signal near the

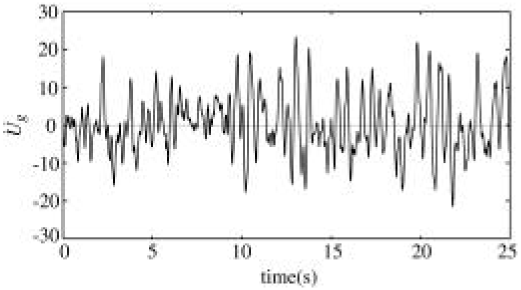


Figure 5: Input acceleration

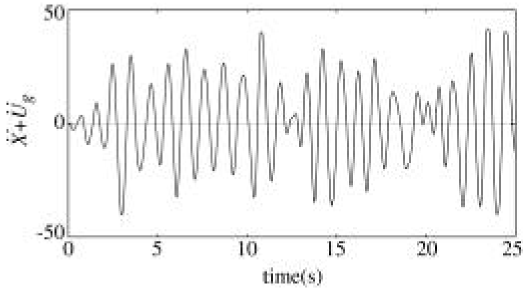


Figure 6: Response of system (g =0.05)

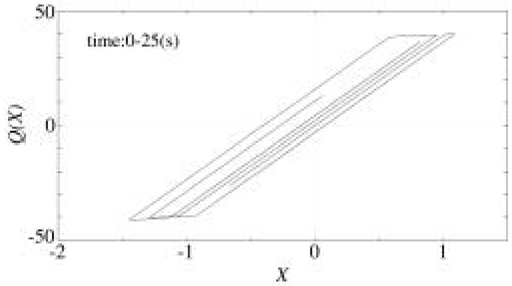


Figure 7: History of restoring force (g =0.05)

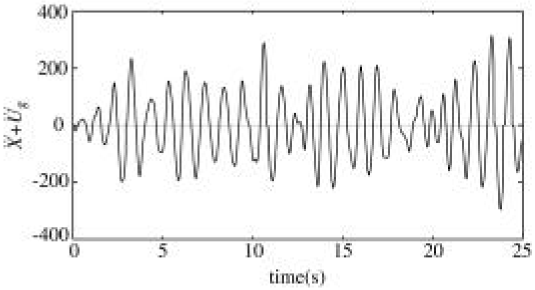
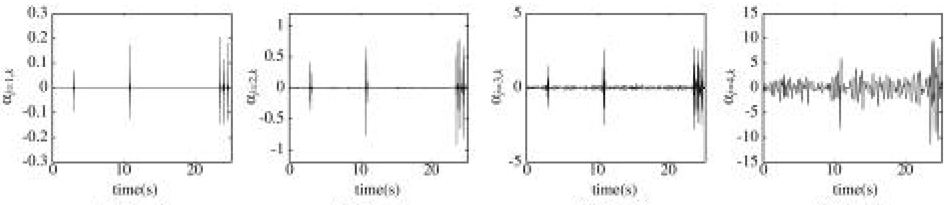


Figure 8: Absolute jerk response (g =0.05)

time carefully, it is impossible to detect the discontinuity. In proportion as the rigidity ratio increases, the detection of discontiniuity becomes more difficult.

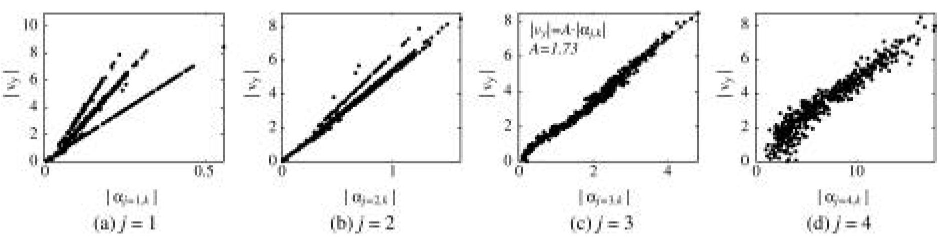
Detection of Discontinuity by Wavelet Analysis

The absolute acceleration response simulated in previous section is analyzed by the wavelet analysis and the wavelet coefficient $a_{j,k}$ is obtained as shown in Fig.9. The center frequency of each analyzing wavelet (level j =1-4) are 29Hz, 14.5Hz, 7.25Hz and 3.625Hz respectively. The kind of wavelet is Daubechies' wavelets (N=6) in this study. In the case of level j = 4, the signal detecting the discontinuity is buried in noise, while in the other cases (level j =1,2,3), the wavelet coefficient detects discontinuity clearly as a pulse at the yield point. As mentioned above, this phenomenon means that sudden discontinuity at the yield point causes the change in acceleration; that is the jerk. Thus, rearranging Eq.(9), the proportional relationship between wavelet coefficient and velocity at yield point is given by following equation.



(a) $j = 1$ (b) $j = 2$ (c) $j = 3$ (d) $j = 4$

Figure 9: Wavelet coefficient $a_{j,k}$ of absolute acceleration response (sampling: 100Hz)



(a) $j = 1$ (b) $j = 2$ (c) $j = 3$ (d) $j = 4$

Figure 10: Relationship between wavelet coefficient and velocity at yield point (sampling: 100Hz)

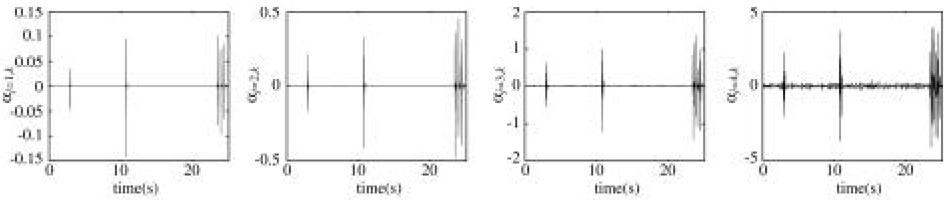
$$|v_y| = A \cdot |\alpha_{j,k}| \tag{14}$$

Where,

$$A = \frac{1}{(1-\gamma)\omega_n^{2} \cdot \alpha} \tag{15}$$

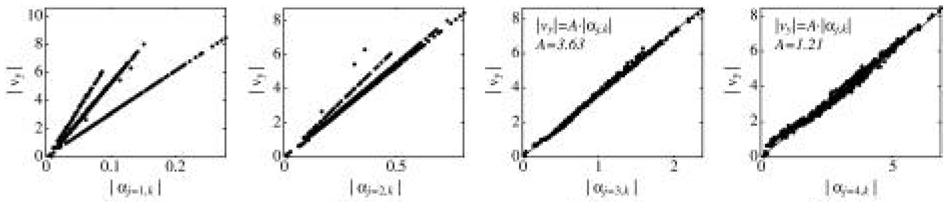
It is natural to consider that the magnitude of wavelet coefficient detecting discontinuity is proportional to the velocity at the yield point. Because, as the velocity at the yield point increases, the discontinuity in acceleration becomes large. In order to investigate this proportional relationship, the relation between the wavelet coefficient and the velocity at the yield points is obtained from the response analysis and it is shown in Fig.10. In the case of level j =3, it is clear that $\alpha_{j,k}$ correlates with v_y. And applying this relationship to Eq.(14), the value of constant α is obtained as 0.0154.

Next, the wavelet analysis that is similar to above is performed in the case of changing sampling frequency (200Hz and 400Hz). The results of these analyses are showed in Figs.11-14. As sampling frequency becomes double, the resolution of an analyzing wavelet becomes double, too. So, in the case of 200Hz sampling, the center frequency of each analyzing wavelet (level j =1-4) are 58Hz, 29Hz, 14.5Hz and 7.25Hz respectively. Also, in the case of 400Hz sampling, the center frequency of each analyzing wavelet (level j =1-4) are 116Hz, 58Hz, 14.5Hz and 7.25Hz respectively. Thus, the clearest case of the pulse detecting discontinuity is reasonably that the sampling frequency is 400Hz and level $j = 1$, but the proportional relationship between wavelet coefficient and velocity at yield point is approved only the case that the level $j \geq 3$.



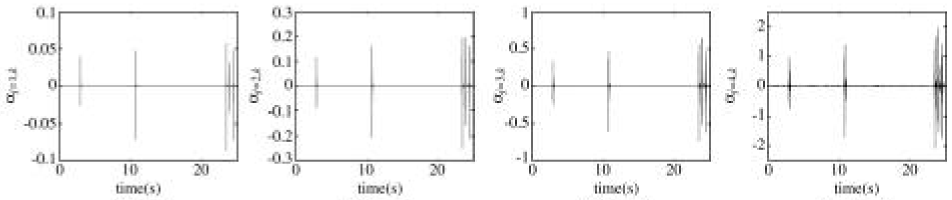
(a) j = 1 (b) j = 2 (c) j = 3 (d) j = 4

Figure 11: Wavelet coefficient $\alpha_{j,k}$ of absolute acceleration response (sampling: 200Hz)



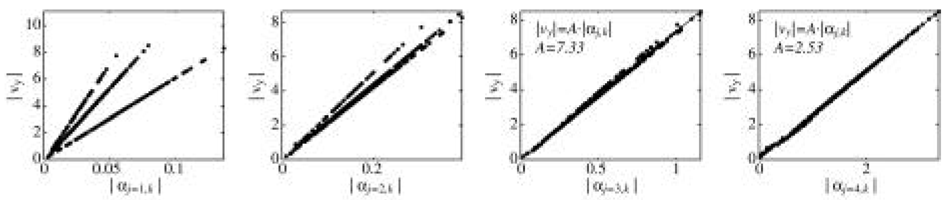
(a) j = 1 (b) j = 2 (c) j = 3 (d) j = 4

Figure 12: Relationship between wavelet coefficient and velocity at yield point (sampling: 200Hz)



(a) j = 1 (b) j = 2 (c) j = 3 (d) j = 4

Figure 13: Wavelet coefficient $\alpha_{j,k}$ of absolute acceleration response (sampling: 400Hz)



(a) j = 1 (b) j = 2 (c) j = 3 (d) j = 4

Figure 14: Relationship between wavelet coefficient and velocity at yield point (sampling: 400Hz)

Table 1 shows the result of some numerical simulations in which structural parameter is changed (rigidity ratio $g = 0.1, 0.3, 0.5, 0.7, 0.9$). The condition of this simulations is that the sampling frequency is 400Hz and level j =4. It is found that the value of constant *a* is almost the same value as 0.01056. Thus, keeping the kind of wavelet and its level *j*, the value of *a* is determined as a constant value.

Velocity at Yield Point and Ductility Factor

The relation between the velocity v_{yi} at the yield point and the ductility factor h_i is given by Eq.(12). This equation is derived by the assumption that the effect of seismic motion is not considered from the yield displacement to the maximum displacement. Also, it is assumed that the kinetic energy at the yield point is equal

Table 1: Constant *a* evaluated from some simulations

Rigidity ratio ã	Constant *a*
0.1	0.01055
0.3	0.01055
0.5	0.01056
0.7	0.01056
0.9	0.01058

to the strain energy from the yield point to the maximum displacement. These assumptions may be justified when the structure is resonant. In this condition, the structure is dramatically stimulated by the small seismic motion, because the damping ratio of structure is quite small. The estimated value by Eq.(12) is shown in Fig.15 as the area enclosing by two solid line obtained by $h_{i-1} = 1$ and $h_{i-1} = 2$, and the simulated values are shown by dots in the same figure. Though the simulated values are effected by the seismic motion applied from the yield point to the maximum displacement, these values are closely distributed around the two solid lines given by Eq.(12).

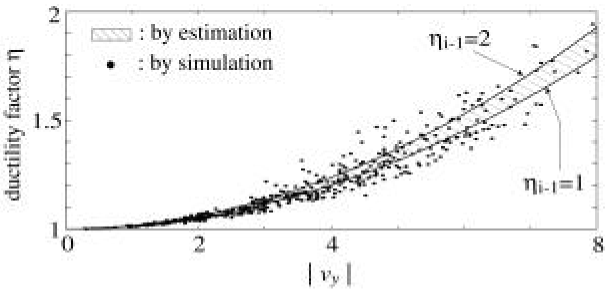


Figure 15: Relation between velocity v_y and ductility factor h

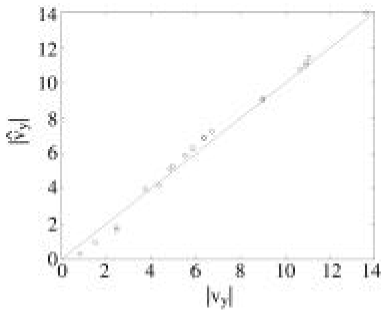
Estimation of ductility factor

In this section, it is tried to estimate the ductility factor from the wavelet coefficient for the acceleration response of a structural model stimulated by the filtered white noise which is passed through the second order system ($w_f = 4p$, $z_f = 0.5$). The structural parameter of the system shown in Eq.(2) is taken as

$$w_n = 4p, z = 0.01, g = 0.5, x_y = 5(\text{cm}) \tag{16}$$

The velocity at the yield point is estimated from wavelet coefficient in Eq.(9) with constant *a*=0.01056 and the relation between the estimated velocities $\hat{v}_y$ at the yield point and true values v_y is shown in Fig.16. It is shown from this figure that the relationship is almost linear and the velocity at the yield point is estimated with good accuracy.

Next, the ductility factor is estimated from estimated velocity at the yield point in Eq.(12) and the relation between the estimated values $\hat{h}$ and the true values h is shown in Fig.17. It can be seen from this figure that the relationship is almost linear and the estimation of the ductility factor is successful with good accuracy. Where, the maximum value of error in this estimation is 2.9%.



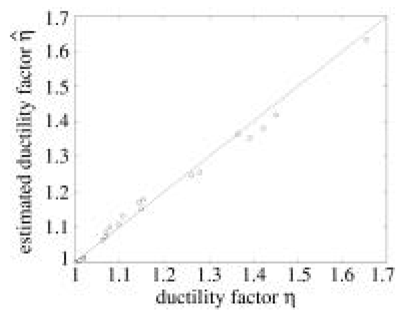


Figure 16: Estimation of velocity at the yield point **Figure 17: Estimation of ductility factor**

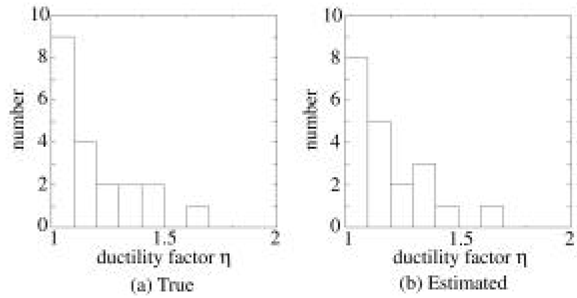


Figure 18: Histogram of ductility factor

As the result of these estimations, from the wavelet coefficient for acceleration response, the ductility factor, namely damage rate can be easily estimated and it can be possible to obtain the histogram of ductility factor as shown in Fig.18.

CONCLUSIONS

It is necessary to know the ductility factor and the number entering the plastic region in order to estimate the cumulative damage of structure. In this paper, the method is proposed to estimate the ductility factor by the wavelet analysis of absolute acceleration response. Wavelet has the ability to detect discontinuity in an acceleration response record, furthermore it is found in this study that the wavelet coefficient is proportional to the velocity at the yield point. Besides, the ductility factor is estimated from the velocity at the yield point by using the energy balance between kinetic energy at the yield point and strain energy from the yield point to the maximum displacement. Thus, using these relationships, the ductility factor is estimated from the wavelet coefficient obtained by the wavelet analysis of acceleration response with good accuracy. The applicability of this method is demonstrated through some simulations.

REFERENCES

1. Ashino R. and Yamamoto S. (1997), "*Wavelet Analysis* (in Japanese)", Kyoritsu Syuppan, Tokyo
2. Sone A., Yamamoto S., Nakaoka A. and Masuda A. (1994), "Health Monitoring System of High-rise Structures by using Wavelet Transformation of Monitoring Signal in Service". *Proceedings of First World Conference of Structural Control*, Vol.2, pp.TA3, 43-52.
3. Sone A., Yamamoto S. and Masuda A., Nakaoka A. and Ashino R. (1995) "Estimation of Cumulative Damage of Building with Hysteretic Restoring Force by Using Wavelet Analysis of Strong Response Records (in Japanese)". *J. Struct. Constr. Eng.*, AIJ, No.476: pp.67-74.
4. Suzuki Y., and Minai R. (1988) "Application of Stochastic Differential Equations to Seismic Reliability Analysis of Hysteretic Structures", *Probabilistic Engineering Mechanics*, 3-1: pp.43-52.
5. Tajimi, H. (1962), "A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake", *Proceedings of Second WCEE*, Vol.2, pp.781-786.
6. Tamai H., Kondoh K. and Hanai M. (1994), "On Low-cycle Fatigue Characteristics of Hysteretic Damper and Its Fatigue Life Prediction Under Sever Earthquake (in Japanese)", *J. Struct. Constr. Eng.*, AIJ, No.462: pp.141-150.