DETECTION OF INELASTIC EXCURSIONS IN HYSTERETIC SYSTEMS FOR CUMULATIVE DAMAGE ESTIMATION USING WAVELET TRANSFORMOF RESPONSE TIME HISTORIES

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SUMMARY

It is very important to estimate the cumulative damage of buildings by a strong ground motion quantitatively, because it is necessary to judge whether they are able to be used or not in future. Even after they are judged to be able to be used, it is also necessary to judge the amount or method of reinforcing them. In this paper, the estimation of cumulative damage of building with hysteretic restoring force by using wavelet analysis to strong response motion is proposed. Firstly, it was assumed that the abnormal signals might be contained in the observed acceleration records, which should be produced at the point changing from the elastic region to the plastic region and might be detected by wavelet analysis. Through the numerical evaluation using the model, it is proven that this assumption is correct and it is confirmed that the wavelet analysis will be used to estimate the cumulative damage of buildings by earthquake, effectively.

INTRODUCTION

In January of 1995, a strong earthquake occurred in the southern part of Hyogo prefecture (the Great Hanshin-Awaji Earthquake). Many buildings were damaged by this earthquake. It is very important to estimate the damage rate of buildings by a strong ground motion quantitatively, because it is necessary to judge if the damage is within repairable limits. It is also necessary to judge the method of reinforcing them. At present, however, the damage rate of buildings is estimated by visual inspection by experts. Therefore, in order to recognize the damage of building, it is strongly required to introduce the quantitative and systematic method.

The cumulative damage of building in a strong ground motion is the low cycle fatigue with large amplitude and small number of cycle, which occurs during time interval of 20 or 30 second. The authors have proposed a health monitoring system by using wavelet transform of response as a method for evaluating the safety of structures quantitatively [4]. This system can detect the abnormal signal, which occurred in progress of the low cycle fatigue of structure under noisy condition.

In this paper, especially, we consider a single degree of freedom system with bilinear restoring force subjected to white noise ground excitation. The discontinuity that should be produced at the point changing from the elastic region to the plastic region in restoring force is detected by wavelet analysis with Meyer's wavelet and Daubechies' wavelet. Moreover, the relationship between the ratio of preyield stiffness to postyield stiffness in restoring force and the detection results of discontinuity is discussed.

DISCRETE WAVELET TRANSFORM

In the wavelet transform, the singularity of function or signal is analyzed by the basis compactly supported in both time domain and frequency domain. However, this basis is generally the oblique system and over-complete

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system. Therefore, for the actual computation of wavelet transform, the dilation parameter a and translation parameter b should be discretized. This wavelet transform using this discrete wavelet is called the discrete wavelet transform. For some very special choices of y , the discretized wavelets $\{y_{j,k}\}$ constitute an orthonormal basis. By using the orthonormal bases, the wavelet expansion of a function x(t) and the coefficients of wavelet expansion are defined as follows[1],[2],[5]:

$$x(t) = \sum_{j} \sum_{k} a_{j,k} y_{j,k}(t) \tag{1}$$

$$a_{j,k} = \int_{-\infty}^{\infty} x(t) \overline{y_{j,k}(t)}\, dt = \left\langle x(t), y_{j,k}(t) \right\rangle \tag{2}$$

where $a_{j,k}$ is the coefficients of wavelet expansion and $y_{j,k}$ is the discrete basis generated by dilating and translating an analyzing wavelet y . The symbol of < , > stands for the inner product. Integers j and k are the dilation parameter and the translation parameter, respectively. Generally, by choosing the dilation parameter a to be $a_j = 2^j$,$y_{j,k}$ is expressed by

$$y_{j,k}(t) = 2^{j/2} y\,(2^j t - k) \tag{3}$$

In this study, as analyzing wavelets, Meyer's wavelet and Daubechies' wavelet are introduced. These wavelets are an orthonormal wavelet basis with compact support in frequency domain. The generation of this wavelet is also based on the multi-resolution analysis [3], [5]. Figure 1 and Fig.2 show the Meyer's analyzing wavelet and Daubeshies' analyzing wavelet, respectively.

WAVELET ANALYSIS OF RESPONSE OF BILINEAR SYSTEMS

As the analytical model of building, a single degree of freedom system with bilinear restoring force as shown in Fig.3 is considered. m and c are mass and damping coefficient of system , respectively. $f(x)$ is the hysteretic restoring force. Input acceleration $\ddot{u}_g$ is assumed to be white noise. Figure 4 shows the hysteresis curve of bilinear restoring force. x, x_y and x_m are the relative displacement response, yield relative displacement response and maximum relative displacement response, respectively. f_y is the yield shear force. k_1 and k_2 are the preyield stiffness and postyield stiffness, respectively.

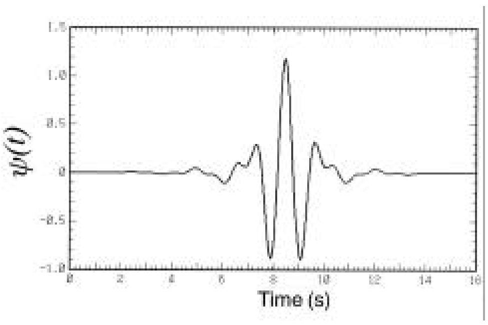


Figure 1: Meyer's analyzing wavelet

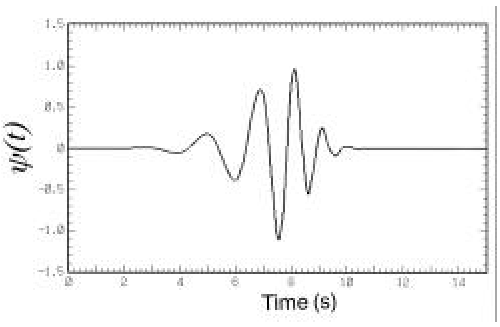
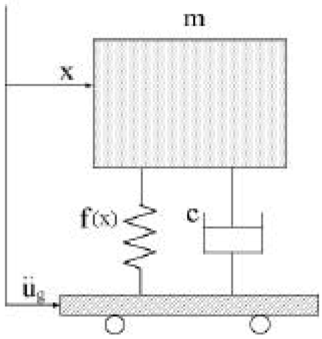


Figure 2: Daubechies' analyzing wavelet



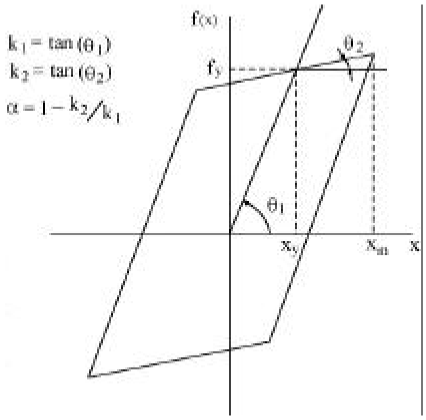
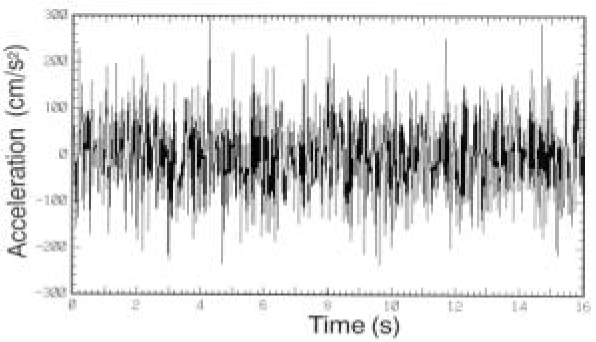


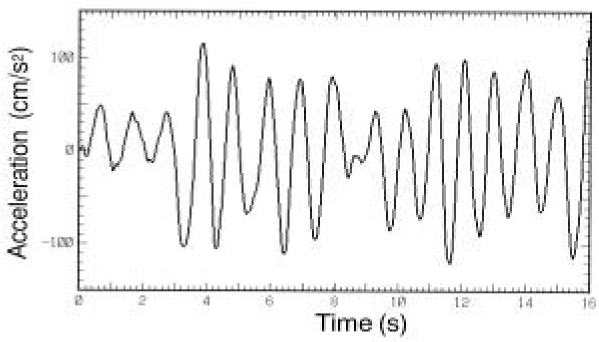
Figure 3: Analytical model **Figure 4: Bilinear restoring force**

In the numerical simulations, the parameters of SDOF system for the elastic deflection range are assumed to be the natural period $T_0 = 1.0\text{s}$, damping ratio $z = 0.01$ and yield relative displacement response $x_y = 2.7\text{cm}$. Also, five values are chosen for maximum input acceleration $\ddot{u}_{gmax}$ among the values of $250\,\text{cm/s}^2$ through $350\,\text{cm/s}^2$. In order to determine the characteristic of bilinear restoring force, the parameter of $a = 1 - k_2/k_1$ is introduced. Four values are chosen for this parameter a among the values of 0.25 through 1.0. $a = 0$ and $a = 1.0$ correspond to the elastic system and perfectly elastoplastic system, respectively.

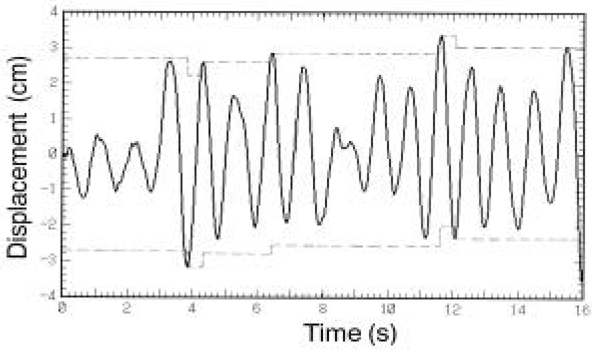
First, the input acceleration, absolute acceleration response and relative displacement response of system for case of $a = 0.5$ and maximum input acceleration $\ddot{u}_{gmax} = 300\ \text{cm/s}^2$ are shown in Fig. 6(a), (b) and (c). The dashed line denotes the yield relative displacement response. When the relative displacement response of system is more than $x_y = 2.7\text{cm}$, the plastic deformation occurs. From this figure, it is shown that small plastic deformations occur at time of about t = 4s and 6.5s, while large plastic deformations occur at time of about t = 11.5s and 16s.This example shows the analytical results of relative displacement response of a SDOF system. Generally, the relative displacement response is not recorded and the absolute acceleration response is recorded. On the other hand, we can not identify the occurrence time of plastic deformation by inspecting only this acceleration response. However, the singularities in the acceleration response may occur as shown in the displacement response.



(a) Input acceleration



(b) Absolute acceleration response



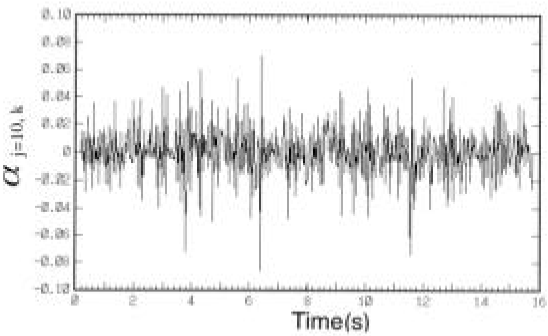
Relative displacement response

Figure 5: Input acceleration and responses

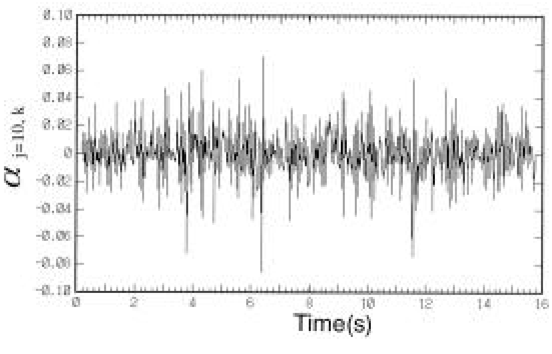
($\ddot{u}_{g\,max} = 300\ \mathrm{cm/s^2}$ and $a = 0.5$)

Next, applying the wavelet transform to this absolute response, its singularities, which can not be observed by visual inspection, are detected and number entering into the plastic region and cumulative plastic deformation are estimated. To apply wavelet transform for this purpose means that we choose the most appropriate filter based on the multi-resolution analysis. However, there are many choice of wavelet. Therefore, we intend to choose the most appropriate wavelet among various types of wavelets for our analysis. In this study, two types of wavelets; Daubechies' wavelet and Meyer's wavelet are used for the wavelet transform and are investigated.

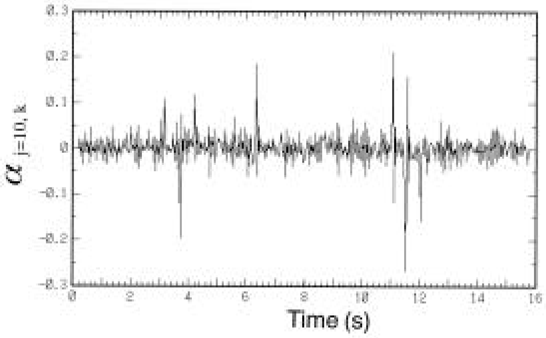
Using Daubechies' wavelet and Meyer's wavelet, the coefficients of wavelet expansion of acceleration response are shown in Figs. 6 and 7, respectively. These coefficients $a_{j=10,k}$ are obtained from the wavelet with the highest scale for detecting the singularities in acceleration response. Because the singularities have high frequency component. These examples are for cases of $a = 0.5$, and maximum input acceleration $\ddot{u}_{g\,max} = 275\mathrm{cm/s^2}$, $300\,\mathrm{cm/s^2}$ and $350\,\mathrm{cm/s^2}$. From these figures, it is clear that the pulsive peaks are observed in coefficients of wavelet expansion. The occurrence time of peaks corresponds to that of singularities in acceleration response, which occurs at the point changing from the elastic region to the plastic region; that is change of stiffness of restoring force.



(a) $275\mathrm{cm/s^2}$

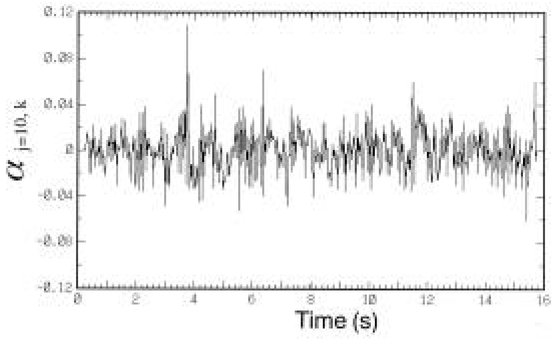


(b) $300\,\mathrm{cm/s^2}$

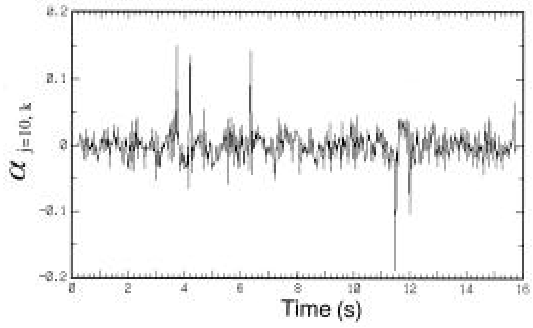


(c) $350\,\mathrm{cm/s^2}$

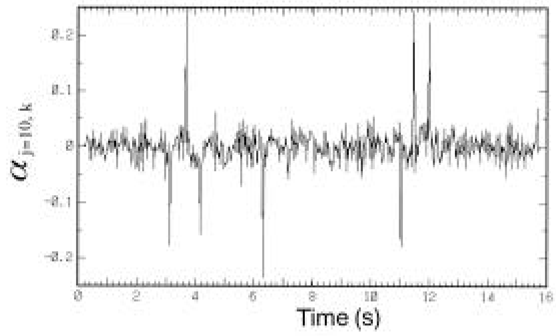
Figure 6: Coefficient of wavelet expansion of acceleration (Daubechies' wavelet with *N*=8 and $a = 0.5$)



(a) $275\mathrm{cm/s^2}$



(b) $300\,\mathrm{cm/s^2}$



(c) $350\,\mathrm{cm/s^2}$

Figure 7: Coefficient of wavelet expansion of acceleration (Meyer's wavelet and $a = 0.5$)

In other words, the occurrence time of peaks in coefficients of wavelet expansion shows the point passing the corner of bilinear restoring force. It is natural from these figures that number of peaks, namely number entering into plastic region, increases as the maximum input acceleration becomes large. The results obtained by two types of analyzing wavelets show a similar tendency. Moreover, comparing these results in detail, in the case of large input acceleration, the pulsive peaks are clearly observed. However, in the case of small input acceleration such as $\ddot{u}_{g\,max} = 275\mathrm{cm/s^2}$, the ability of detection for singularity in acceleration response by Meyer's wavelet is greater than that by Daubechies' wavelet. This is related to the regularities of wavelets and their vanishing moments. Therefore, it is known that the peaks of coefficient of wavelet expansion occurs remarkably when the wavelets having higher regularity and vanishing moment are used. In this study, as Meyer's wavelets have the highest order of vanishing moment and highest regularity among three types of analyzing wavelets, above results seem to be reasonable.

Using Meyer's wavelets, which can clearly detect the singularities in acceleration response, the relation between detected number entering into the plastic region and maximum input acceleration is obtained and discussed. The ratio of detected number entering into the plastic region by wavelet analysis to its real value is shown in Fig. 8. In this figure, x-axis denotes the maximum input acceleration. It is clear from this figure that the detected number entering into the plastic region almost approximates its real value as a becomes large, namely as the restoring force of system is close to a perfectly elastoplastic restoring force

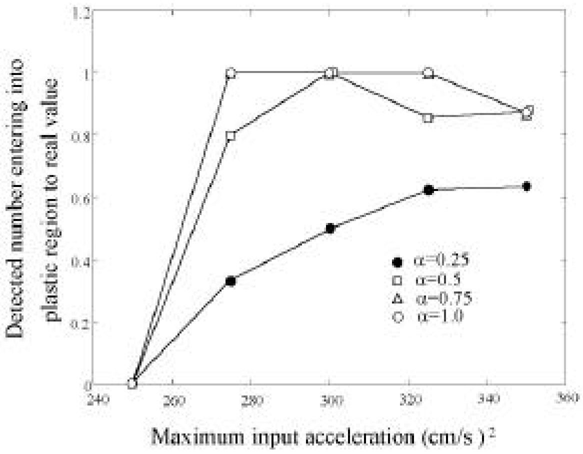


Figure 8: Ratio of detected number entering into plastic region to real value

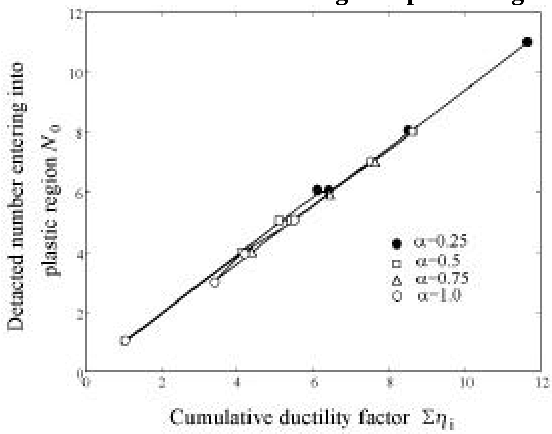


Figure 9: Relation between real number entering into plastic region and cumulative ductility factor

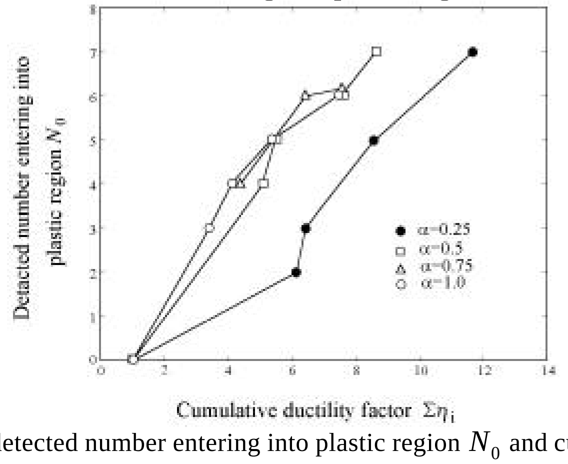


Figure 10: Relation between detected number entering into plastic region N_0 and cumulative ductility factor

On the other hand, the error in the case of small input acceleration becomes large. These results agree with our physical intuition. Applying the wavelet transform to the acceleration response, in the case of the hyteretic system with strong nonlinearity such as and large input acceleration, number entering into the plastic region can be estimated.

Next, in order to predict the cumulative damage of system, the estimation of plastic deformation is important. Therefore, in this study, the relation between detected number entering into the plastic region by wavelet analysis and cumulative ductility factor is obtained and discussed. Figure 9 shows the relation between real number entering into the plastic region obtained from the displacement response and cumulative ductility factor for four values of a . It is clear from this figure that the relationship of real numbers entering into plastic region N_0 and cumulative ductility factor almost linear irrespective of a . Figure 10 shows the relation between detected number entering into the plastic region N_0 and cumulative ductility factor. From this figure, the linear relation between detected number entering into plastic region N_0 and cumulative ductility factor is almost obtained. Accordingly, if number entering into the plastic region N_0 is obtained by the wavelet transform of acceleration response, the cumulative ductility factor can be estimated according to the linear relation as shown in Fig. 10. Based on these results, we can also estimate the cumulative damage of system.

Finally, using the real ground motion record of the Great Hanshin-Awaji Earthquake, we apply the wavelet transform to the acceleration response of the SDOF system. Figure 11 shows the measured earthquake record of JMA Kobe NS, 1995. The absolute acceleration response of SDOF system for the case of $a = 0.5$ is shown in Fig. 12. Using this acceleration, the coefficient of wavelet expansion of acceleration response for Daubechies' wavelet N=8 and $a = 0.5$ is shown in Fig. 13. This example is for $x_y = 15\text{cm}$. It is clear from this figure that the pulsive peaks, namely singularities in acceleration response are remarkably observed. Also, Fig. 14 shows the relation between detected number entering into the plastic region N_0 and cumulative ductility factor for four values of a . From this figure, the linear relation between detected number entering into the plastic region N_0 and cumulative ductility factor is almost obtained by using the real earthquake acceleration record. Therefore, the proposed estimation method is confirmed by the wavelet analysis using real earthquake motion.

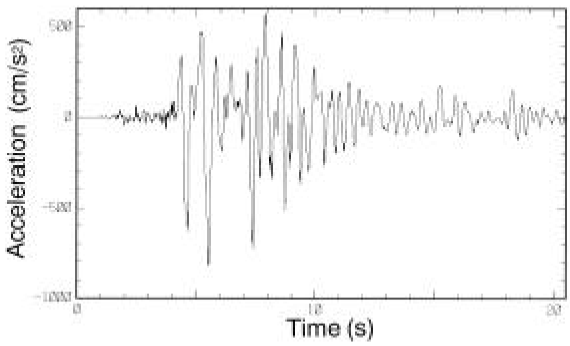


Figure 11: Measured earthquake record of JMA Kobe NS

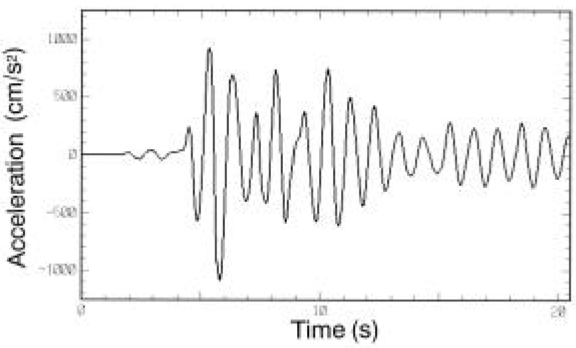


Figure 12: Absolute acceleration response of SDOF system ($a = 0.5$)

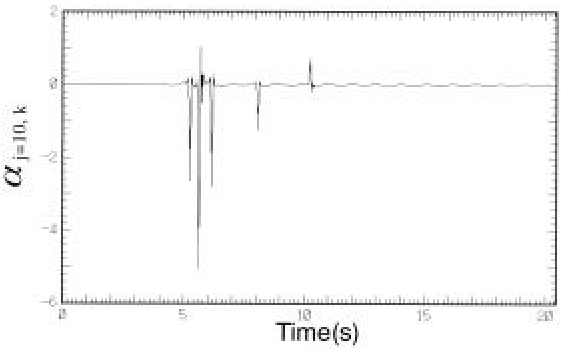


Figure 13: Coefficient of wavelet expansion of acceleration (Daubechies' wavelet *N*=8 and $a = 0.5$)

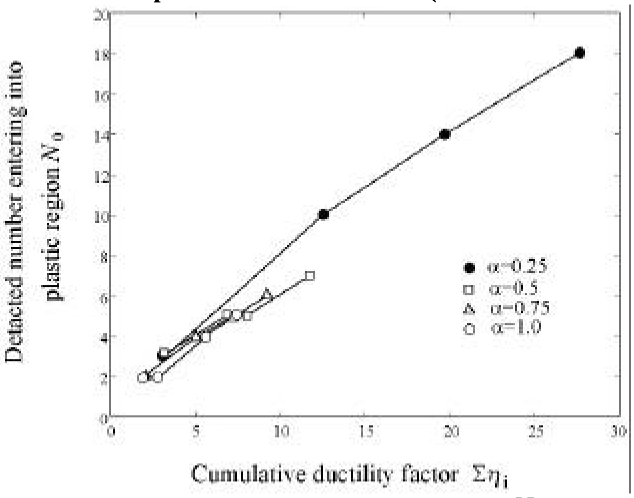


Figure14: Relation between real number entering into plastic region N_0 and cumulative ductility factor

CONCLUSIONS

In this study, the fundamental investigation to estimate cumulative damage of building with hysteretic restoring force by using wavelet analysis is carried out. The results obtained herein are as follows.

In the case of estimating number entering into plastic region by wavelet analysis, Meyer's wavelets are more reasonable than Daubechies' wavelets.

As the change of hysteretic restoring force of system becomes large, number entering into plastic region can be estimated remarkably.

The proposed estimation method for cumulative damage is confirmed by the wavelet analysis using real earthquake motion such as the Great Hanshin-Awaji earthquake.

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