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FRAME DESIGN METHOD BASED ON REDUCED MODEL-FRAME INVERSE TRANSFORMATION

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SUMMARY

A method is proposed for designing member sizes of a frame which would satisfy the design constraints under Level-2 design earthquakes. This method consists of two steps. In the first step, a shear building model is used and story stiffnesses and story strengths of the shear model are determined so that the designed shear model would exhibit a specified distribution of interstory ductility factors under the Level-2 design earthquakes. In the second step, member sizes of the frame are transformed from the story stiffnesses and story strengths of the shear building model based on the proposed shear model-frame inverse transformation. In this transformation, the total member weight is minimized as well as the equivalent transformation of the restoring force characteristics in the story level.

INTRODUCTION

Many useful structural design methods have been developed based mainly on structural optimization or inverse problem approaches [for example, Gallagher and Zienkiewicz, 1973; Bertero and Kamil, 1975; Nakamura, 1980; Morris, 1982; Atrek et. al, 1984; Nakamura and Yamane, 1986; Akiyama, 1987; Chan et. al, 1994; Olhoff and Rozvany, 1995]. However it appears that most of these methods are not used in the practical structural design environments due to the intrinsic problems, e.g. computational efficiency, complicated design conditions in the real world.

The purpose of this paper is to develop a new performance-based seismic resistant design method for steel building structures. The design method is aimed at designing member sizes of a frame which would satisfy the design constraints (including elastic-plastic response constraints) under Level-2 design major earthquakes. This method consists of two steps. In the first step, a shear building model is used and story stiffnesses and story strengths of the shear building model are determined via an original inverse problem formulation [Nagaoka and Tsuji, 1995] so that the designed shear building model would exhibit a specified distribution of interstory drift angles (or ductility factors) under the Level-2 earthquakes. In the second step, the member sizes of the frame are transformed from the story stiffnesses and story strengths of the shear building model based on the proposed "shear building model-frame inverse transformation" which takes advantage of the total quantity expression-based design sensitivity analysis [Uetani and Takewaki, 1998; Takewaki et. al, 1998; Uetani et. al, 1999]. This transformation is based on the fact [Nakamura et. al, 1990] that if the restoring force characteristics in the storylevel are equivalent between the frame model and its reduced shear building model, the inelastic dynamic

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responses of both models exhibit almost equivalent responses under ground motions. In the proposed inverse transformation, the total member weight is minimized as well as the equivalent transformation of the restoring force characteristics in the story level.

Building frames are three-dimensional structures of multi degrees of freedom and it seems quite difficult to use frame models in the preliminary design stage of tall buildings where elastic-plastic dynamic response analysis are performed to check the design constraints. Via the present method, structural designers can determine the member sizes of the building frame based on its structural performance level without laborious elastic-plastic dynamic response analysis for frame models. Remarkable rationalization can be achieved in the structural design process of tall buildings under performance-based design environments. Examples for a 22-story building frame are presented to demonstrate the usefulness of this design method. Member cross-sections are determined from a table of available discrete cross-sections. The accuracy of this design method is investigated through time-history response analysis for shear building models.

2. FRAME MODEL

Consider a multi-plane steel building structure as shown in Fig.1. When this building structure is subjected to a horizontal proportional loading, the story shear force - interstory drift relation may be expressed as an ensemble of finite lines with bending points. Each bending point corresponds to formation of plastic hinge. The P-D effect, shear deformation in members, beam-to-column connection flexibility and interaction with perpendicular beams are not taken into account here for simple presentation of the method. It is possible, if desired, to take into account these effects. The composite beam effect and intermediate loading on beams are taken into consideration. Beams are assumed to exhibit an elastic-perfectly plastic behavior and columns are assumed to remain elastic. An example of the story shear force - interstory drift relation is shown in Fig.2. To simplify the story shear force - interstory drift relation, several bending points are selected and called 'representative bending points'.

3. DEFINITION OF REDUCED MODELS

A shear building model is adopted as a reduced model of the frame model. The shear building model is assumed to possess a normal trilinear hysteretic property in the story shear force - interstory drift relation (Q-D relation). The shear building model may have a multi-linear Q-D relation so far as each element has a positive slope.

4. INELASTIC FRAME DESIGN METHOD VIA REDUCED MODEL-FRAME INVERSE TRANSFORMATION

4.1 Problem of Reduced Model-Frame Inverse Transformation:

In the present reduced model-frame inverse transformation, the criterion is the equivalence of the Q-D relation between the frame model and the shear building model. There exist possibilities that unique transformation is not possible mathematically. In this case another criterion, i.e. the member weight minimization, is introduced.

Consider an N-story frame. The design variable grouping is employed for a specific group of members. Let f_B and f_C denote the number of design variables for beams and that for columns, respectively. The sum of f_B and f_C is denoted by f. Let L_{Bi} denote the total member length of the beams with the cross-sectional area A_{Bi} and let L_{Ci} denote the total member length of the columns with the cross-sectional area A_{Ci}. Here a case is considered where there exist both criteria of equivalence of the Q-D relation and member weight minimization. Consider the following objective function.

$$C = C_W + C_D \tag{1}$$

where C_W is the total member weight and C_D is an index, i.e. a weighted sum of the distances of the Q-D relations between two models, representing equivalence of the Q-D relations between two models. If only C_D is treated, it suffices to put $C_W = 0$. C_W and C_D may be expressed as

$$C_W = \sum_{j=1}^{f_B} A_{Bj} L_{Bj} + \sum_{j=1}^{f_C} A_{Cj} L_{Cj} \tag{2a}$$

$$C_D = V_0 \left(\sum_{i=1}^{N} \sum_{K=1}^{S} \beta_{XiK} d_{XiK}{}^2 + \sum_{i=1}^{N} \sum_{K=1}^{S} \beta_{YiK} d_{YiK}{}^2 \right) \tag{2b}$$

where d_{XiK} is the distance between the K-th representative bending point in ith story in X horizontal loading direction and the target Q-D relation of the shear building model and d_{YiK} is that in Y horizontal loading direction (see Fig.2). S is the number of representative bending points. β_{XiK} and β_{YiK} are the weight coefficients on the distances d_{XiK} and d_{YiK} and V_0 is a parameter for matching dimensions of C_W and C_D.

4.2 Optimization for Continuous Design Variables:

For continuous design variables, there exists the steepest descent method as an optimization algorithm. In the steepest descent method, the design variable vector is updated by

$$\mathbf{x}^{i+1} = \mathbf{x}^{i} - \varepsilon \mathbf{g} \tag{3}$$

where ε is the step length and the gradient $\mathbf{g}$ of the objective function is expressed as

$$\mathbf{g} = \left\{ \frac{\partial C}{\partial x_1} \cdots \frac{\partial C}{\partial x_j} \cdots \frac{\partial C}{\partial x_f} \right\}^T \tag{4}$$

The objective function C is a function of the representative bending points in Fig.2. The design sensitivities of the coordinates of the representative bending points can be evaluated by the proposed total quantity expression-based design sensitivity analysis [Uetani and Takewaki, 1998].

Let (D_{iK}, Q_{iK}) be the coordinates of the Kth representative bending point in the ith story. Variations of (D_{iK}, Q_{iK}) due to variation of $\mathbf{x}$ can be evaluated approximately by

$$D_{iK}(\mathbf{x} + \Delta\mathbf{x}) = D_{iK}(\mathbf{x}) + \sum_{j} \frac{\partial D_{iK}}{\partial x_j} \Delta x_j \tag{5}$$

$$Q_{iK}(\mathbf{x} + \Delta\mathbf{x}) = Q_{iK}(\mathbf{x}) + \sum_{j} \frac{\partial Q_{iK}}{\partial x_j} \Delta x_j \tag{6}$$

The theory for continuous design variables provides a theoretical background. However, some problems should be resolved resulting from computational efficiency, accuracy and availability of the obtained cross-sections.

4.3 Optimization for Discrete Design Variables:

For practical building design, member sections available are not continuous and some member sections on the usual manual are often unavailable. As for discrete optimum design, there exist many useful algorithms (see, for example, [Huang and Arora, 1997]). One of the most frequently used methods is to find first an approximate solution in terms of continuous design variables, to assign several discrete member sections around the approximate solution and to use some discrete optimization algorithms. The problem of computational efficiency may arise in finding an approximate solution in terms of continuous design variables. To overcome this difficulty, a procedure is introduced where designers choose available member sections from the manual and prepare the member section list. The proposed algorithm is illustrated in Fig.3.

4.4 Simultaneous Optimization for Horizontal Two Directions:

Building frames are three-dimensional structures and design constraints under horizontal two-directional excitations have to be satisfied. It is assumed here that two-directional excitations are inputted independently and the responses under one-directional excitation are not affected by the other directional excitation. Consider box-type cross-sections for columns and wide-flange cross-sections for beams.

5. NUMERICAL EXAMPLE

5.1 Inelastic Design for Shear Building Models:

Consider a 22-story building frame structure as shown in Fig.4. Only one loading direction is considered here. The building structure consists of seven plane frames. The story stiffnesses and the corresponding story strengths of a shear building model are determined so that the designed shear building model would exhibit a specified distribution of interstory drift angles under a set of design severe earthquakes.

Table 1: Given or specified parameters

story number	floor mass ($\times 10^3 kg$)	yield story drift ($\times 10^{-2} m$)	second bending point story drift ($\times 10^{-2} m$)	second-branch stiffness ratio	third-branch stiffness ratio	story stiffness ($\times 10^8 N/m$)
22	1317	2.50	5.60	0.65	0.49	1.27
21	808	2.27	5.60	0.64	0.58	2.01
20	800	3.09	5.60	0.61	0.35	2.50
19	800	2.53	5.59	0.66	0.54	2.96
18	806	2.29	5.60	0.69	0.53	3.36
17	810	2.40	5.60	0.70	0.52	3.72
16	815	3.00	5.60	0.63	0.50	4.04
15	815	3.59	5.59	0.57	0.46	4.33
14	822	3.39	5.79	0.57	0.46	4.64
13	824	3.52	5.81	0.57	0.38	4.92
12	854	3.20	5.80	0.61	0.36	5.22
11	840	3.70	6.19	0.62	0.42	4.99
10	1079	3.01	6.20	0.60	0.26	5.33
9	904	2.77	6.21	0.63	0.19	5.58
8	1032	3.20	6.19	0.59	0.11	5.80
7	1032	2.76	6.00	0.63	0.087	6.06
6	1033	3.21	6.06	0.57	0.074	6.24
5	1033	3.50	6.00	0.56	0.082	6.42
4	1033	3.19	6.00	0.54	0.088	6.66
3	1038	2.97	5.01	0.61	0.13	6.87
2	1534	3.18	5.02	0.62	0.23	9.36
1	884	2.99	4.50	0.72	0.34	11.9

A set of spectrum-compatible earthquakes for the Newmark and Hall design response spectrum [Newmark and Hall, 1982] (maximum ground velocity = 0.5m/s) is used as the design severe earthquakes. Normal trilinear hysteresis loops are used for the shear building model. Yield interstory drifts and post-yield stiffness ratios to the initial stiffnesses have been evaluated by static push-over analysis under a set of horizontal loads based on the Japanese earthquake resistant design code. These yield interstory drifts and post-yield stiffness ratios are treated as given parameters during redesign for the shear building model. Since the yield story drifts and post-yield stiffness ratios are dependent on the member cross-sections, a few cycles of redesign have been taken to re-set these values. The final values are shown in Table 1. The specified interstory drift angles and computed initial story stiffnesses are shown in Fig.5 or Table 1. To investigate accuracy of the present inelastic design method for the shear building models, elastic-plastic time-history response analysis has been conducted on the designed shear building models. Ten spectrum-compatible artificial ground motions have been inputted for the shear building models. The results are shown in Fig.5. Due to a rather large variability of post-yield stiffness ratios along the height, the accuracy is somewhat deteriorated.

5.2 Determination of Frame Member Sections via Reduced Model-Frame Inverse Transformation:

Second moments of area are the design variables and design variable grouping is employed, i.e. common member sections are used throughout every several stories. The design variable grouping is shown in Table 2. Candidates member sections consist of 20 candidates for each design variable. Interior columns and interior beams are assumed to have a common member section, respectively.

Table 2: Initial member sections

	column (box-section)		beam (wide-flange section)	
story number	outer column	inner column	outer beam	inner beam
1	900x900x28	900x900x28	925x600x19x32	925x600x19x32
2	900x900x28	900x900x28	925x600x19x32	925x600x19x32
3-5	825x825x25	825x825x25	925x400x16x36	925x400x16x36
6-8	825x825x25	825x825x25	925x400x16x36	925x400x16x36
9-11	800x800x25	800x800x25	925x400x16x28	925x400x16x28
12-14	800x800x25	800x800x25	925x400x16x28	925x400x16x28
15-17	750x750x22	750x750x22	900x300x19x28	900x300x19x28
18-20	750x750x22	750x750x22	900x300x19x28	900x300x19x28
21, 22	650x650x19	650x650x19	800x300x14x22	800x300x14x22

Table 3: Final member sections

	column (box-section)		beam (wide-flange section)	
story number	outer column	inner column	outer beam	inner beam
1	900x900x28	750x750x22	925x600x19x32	925x500x16x36
2	900x900x28	825x825x28	925x600x19x22	925x400x16x28
3-5	825x825x25	700x700x22	925x600x19x40	850x300x14x25
6-8	825x825x25	700x700x19	925x500x16x36	900x300x16x25
9-11	800x800x22	700x700x22	925x500x16x36	850x300x14x25
12-14	800x800x22	650x650x19	925x400x16x32	800x300x14x22
15-17	750x750x22	650x650x19	900x300x19x32	750x250x14x25
18-20	750x750x22	650x650x12	750x200x12x25	750x200x12x25
21, 22	700x700x19	600x600x12	750x200x12x25	750x200x12x25

Initial member sections are shown in Table 2. Design sensitivity analysis in terms of total quantity expressions in Q-D relations requires the expressions of member cross-sectional properties in terms of design variables (I_C, I_B). The column cross-sectional area A_C and beam cross-sectional area A_B are assumed to be expressed as

$$A_C = 0.8412\sqrt{I_C} \tag{7a}$$

$$A_B = 0.8496\sqrt{I_B} \tag{7b}$$

The beam fully plastic moment M_P is assumed to be expressed as

$$M_P = 0.0382\sigma_Y I_B / d_0 \quad (d_0 = 1cm) \tag{8}$$

Furthermore employ the following given parameters; $\sigma_Y = 2.35\times10^4 (N/cm^2), E = 2.06\times10^7 (N/cm^2)$, $Q_0 = 980$ $(\times10^4 N)$, $D_0 = 1(cm)$. The weight coefficients for representative bending points have been chosen so that the weight for the first bending point is large.

In the present design example, only the term C_D has been employed as the objective function in Eq.(1). This is due to the fact that the number of design variables is smaller than the number of requirements on coincidence of Q-D relations. The final member sections are shown in Table 3.

Fig.6 shows the variation of the objective function with respect to the step number. It can be observed that the objective function was reduced rapidly in the first ten steps. Fig.7 illustrates the variation of member second moments of area of the inner beams. It can be understood from Fig.7 that some member sizes are still changing after the tenth step. Fig.8 shows the story shear-interstory drift relations in the 6th and 16th stories for the initial design, the target design and the optimal design. It can be observed that the story shear-interstory drift relations of the optimal frame coincide fairly well with the target relations. Fig.9 illustrates the mean peak interstory drift angles of the shear building model reduced from the optimal building structure. The distribution of the specified interstory drift angles is also plotted in Fig.9. It can be concluded that the final frame (optimal frame) modeled as a reduced shear building exhibits the specified distribution of interstory drift angles within a reasonable accuracy.

6. CONCLUSIONS

The conclusions may be summarized as follows:

- A method can be developed for designing member sections of a frame which would satisfy the design constraints on inelastic responses under design severe earthquakes. This method consists of two steps. In the first step, a shear building model is used as a reduced model and story stiffnesses and story strengths of the shear building model are determined so that the designed shear building model would exhibit a specified distribution of interstory drift angles (or ductility factors) under the design severe earthquakes. In the second step, member sections of the frame are transformed from the story stiffnesses and story strengths of the designed shear building model. In this transformation, the total member weight may be minimized as well as the equivalent transformation of the restoring force characteristics in the story level. This method does not require any elastic-plastic time-history response analysis for frame models.
- The total quantity expression-based design sensitivity analysis proposed by the present authors enables one to conduct the reduced model-frame inverse transformation within reasonable computational resources.
- Three-dimensional building frames have been designed via the proposed method and it has been demonstrated that the present method is highly efficient and can design building frames taking into account elastic-plastic response constraints under severe design earthquakes.

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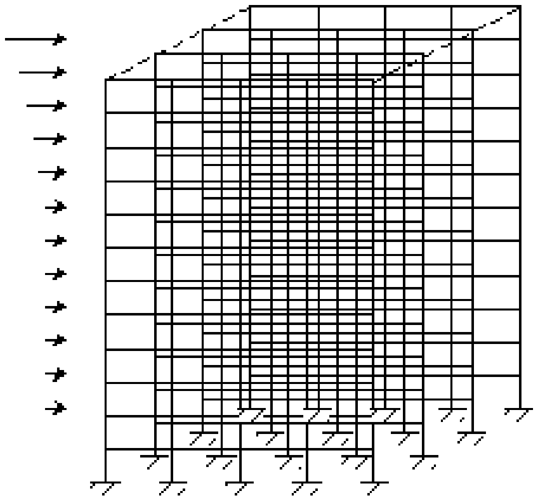

Figure 1: Multi-plane steel building structure

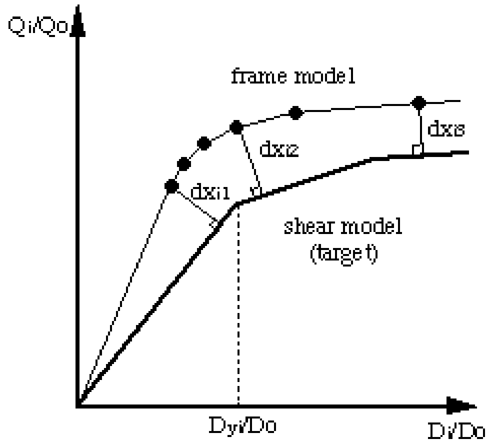

Figure 2: Story shear-interstory drift relations for a frame model and a target shear model

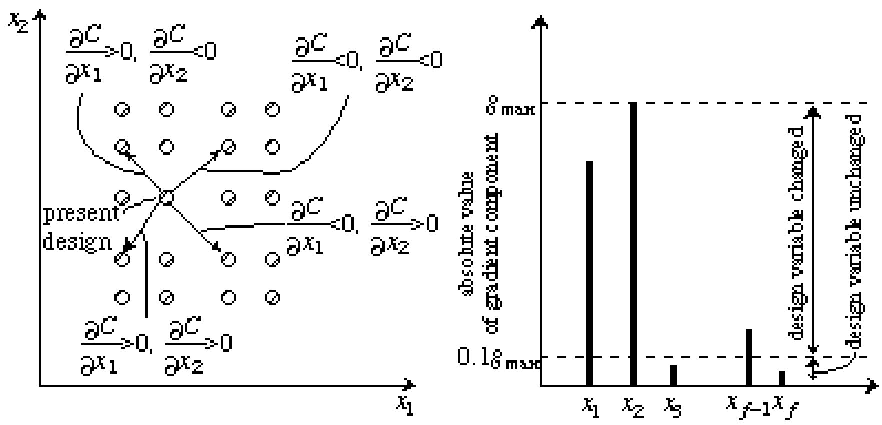

Figure 3: Proposed algorithm for discrete design variables

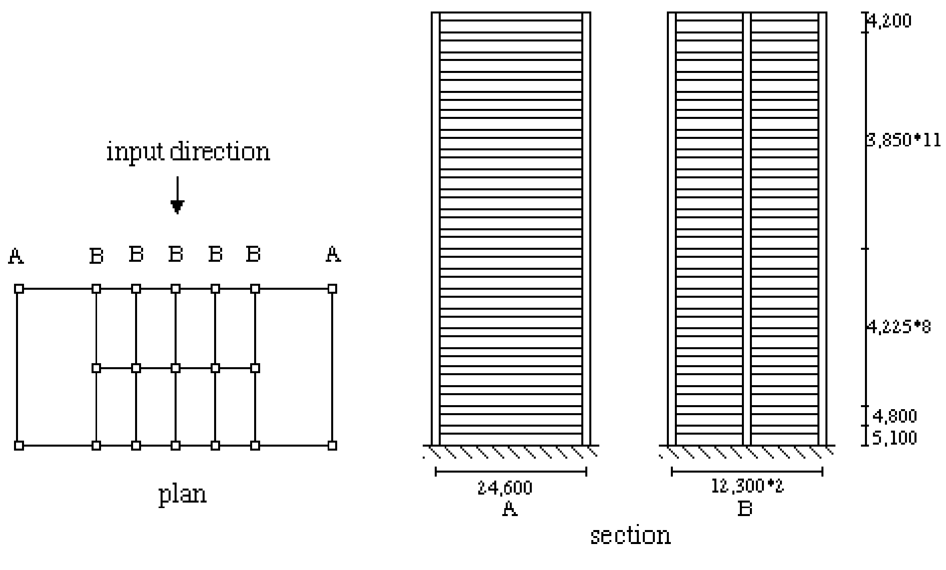

Figure 4: 22-story steel building structure consisting of seven-plane frames

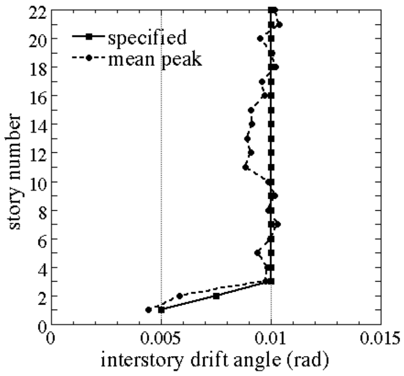

Figure 5: Mean peak interstory drift angles of the inelastic response-constrained shear building model under 10 simulated motions

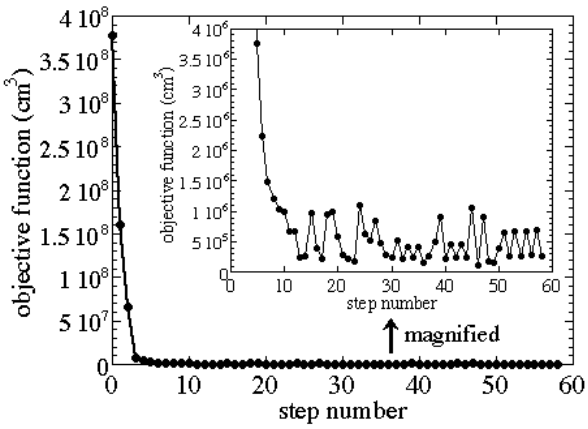

Figure 6: Variation of the objective function

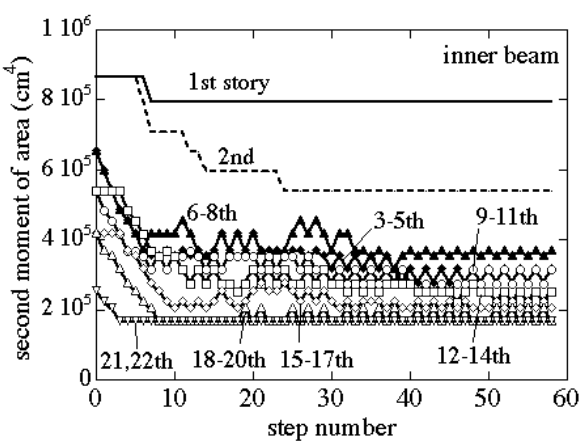

Figure 7: Variation of member second moments of area of inner beams

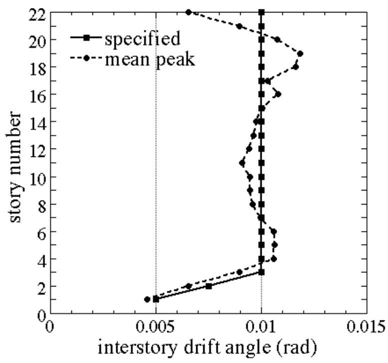

Figure 9: Mean peak interstory drift angles of the shear building model reduced from the optimal frame under 10 simulated motions

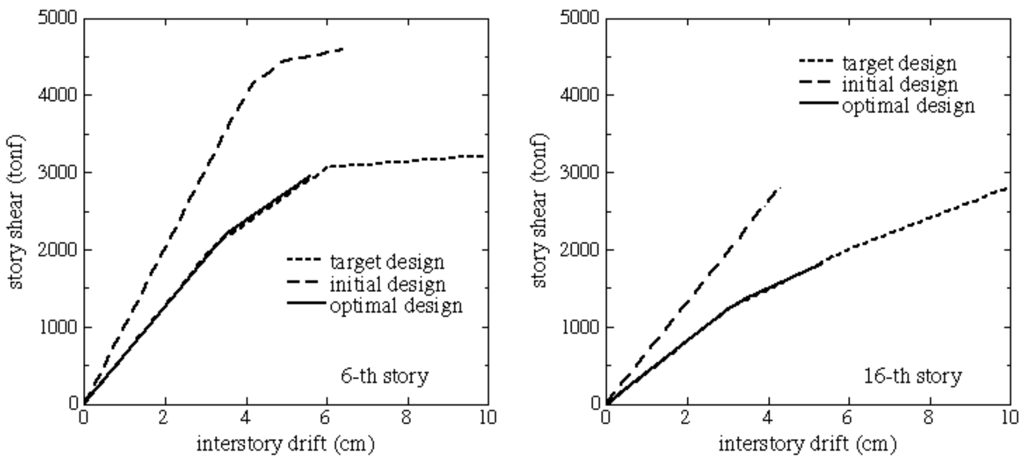

Figure 8: Story shear-interstory drift relations in the 6th and 16th stories for the initial design, the target design and the optimal design