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RESPONSE SPECTRA BASED ON PARAMETRIC TIME SERIES MODELS

SÍMON ÓLAFSSON and SVEIN REMSETH
Department of Structural Engineering, University of Trondheim,
Rich. Birkelands vei 1a, N 7034 Trondheim, Norway

RAGNAR SIGBJÖRNSSON
Engineering Research Institute, University of Iceland
Hjarðarhagi 2-6, 107 Reykjavík, Iceland

ABSTRACT

A parametric time series model (ARMA model) has been developed using measured strong motion in South Iceland. In this paper the Fourier spectra obtained using parametric time series model is compared to results using source models. The parametric time series model is used for simulations for studying the non-linear response of a stiffness degrading SDOF-system. The extreme value distributions of the response are investigated. Probabilistic inelastic response spectra obtained using empirical cumulative distribution functions are presented.

KEYWORDS

ARMA; Bouc-Wen model; ductility ratio; empirical distributions; inelastic response; parametric time series model; probabilistic response spectra; seismic source model; stiffness degrading; strong motion duration.

INTRODUCTION

Based on available earthquake data recorded in South Iceland a low order ARMA model has been found to give reasonable fit to the ground motion (Ólafsson and Sigbjörnsson, 1995a,b). The main emphasis is on developing a model that is capable of simulating consistent acceleration series, which reflect the variability in the data. The available data is however limited, and in this paper it is therefore suggested to use a seismic source model (see for example Haskell, 1964 and Kasahara, 1981) to estimate the ground motion outside the range of the data. The ARMA model is then applied to a non-linear stiffness degrading system, in order to obtain probabilistic response spectra

MODELS FOR SIMULATION

ARMA models

In this study it is assumed that the earthquake induced ground acceleration can be represented by the following simplified model, formulated in discrete time:

$$\alpha_k = A_k\, x_k \quad \text{for} \quad k = 1,2,3,....,N \tag{1}$$

Here, x_k is a stationary zero mean Gaussian process, A_k is an amplitude modulation function and $N = T/T_s$, where T denotes the duration and T_s the sampling interval. Furthermore, it is assumed that the stationary part can be represented by means of the ARMA models presented in the following equation (see for instance Ljung, 1987 or Söderström and Stoica, 1989):

$$x_k + a_1 x_{k-1} + \ldots + a_p x_{k-p} = w_k + b_1 w_{k-1} + \ldots + b_q w_{k-q} \qquad \text{for} \quad k = 1,2,3,\ldots,N \tag{2}$$

where, w_k is a 'white' (Gaussian) noise process with zero mean and finite standard deviation σ_w, $a_1,\ldots,a_p$ and $b_1,\ldots,b_q$ are the model parameters, the AR- and MA-parameters, respectively. The left hand side of eq. (2) is known as the autoregressive or AR part of order p while the right hand side is known as the moving average or MA part of order q. The notation ARMA(p,q) is conventionally used to indicate an ARMA model with p autoregressive and q moving average parameters.

The amplitude modulation function is assumed modelled by

$$A_k = (k/N)^e \exp(-c(k/N)^m) \quad \text{for} \quad k = 1,2,3,\ldots,N \tag{3}$$

where, e, c and m denote model parameters obtained from data.

Scaling formulas

The scaling of the model is done by using the standard deviation, σ_α, and duration, T, of the ground acceleration, which are assumed related to the earthquake magnitude and distance to source by the following formulas:

$$log(\sigma_\alpha) = \theta_1 + \theta_2 M - log(r) + \theta_3 r + \sigma P \tag{4}$$

$$log(T) = \varphi_1 + \varphi_2 M + \varphi_3 log(r) + \sigma_T P \tag{5}$$

Here θ_1, θ_2, θ_3, φ_1, φ_2 and φ_3 are regression coefficients, σ and σ_T are the standard error of $log(\sigma_\alpha)$ and $log(\sigma_T)$, P is a suitable fractile in the standardized normal distribution, M is the magnitude of the earthquake and $r = \sqrt{d^2 + h^2}$, where d is the shortest distance to the causative fault and h is a 'depth' parameter.

Stiffness degrading structural model

The elastic plastic hysteresis model has been widely applied along with the closely related bi-linear hysteresis model. A more flexible model is the so-called Bouc-Wen model (see for instance Bouc, 1967, and Wen, 1989). The restoring force in a hysteric degrading version of this model can be expressed as follows (see Baber and Wen, 1981):

$$G_k = a\rho\omega_o^2 y_k + (1-a) K_u z_k \tag{6}$$

where

$$\eta y_u z_k' + \beta |z_k|^\upsilon y_k' + \gamma \left| y_k' z_k^{\upsilon-1} \right| z_k - B y_k' = 0 \tag{7}$$

Here, ρ is the mass of the system (taken as a unit mass in the following), K_u, is the yield force, y_u, is the corresponding yield level, a, B, β, γ, υ and η are (time-varying) parameters which define the hysteresis loop and degradation of the system, and primes are used to denote time derivatives. For a stiffness degrading system the parameter, η, can be assumed to increase as function of the total energy dissipated by the hysterisis, ε, according to $\eta(\varepsilon) = 1 + \delta_\eta \varepsilon$, where δ_η is a nonnegative parameter.

Probabilistic response spectra

For a given set of ARMA parameters a stationary time series is readily obtained using eq. (2) and a computer generated sample of 'white' Gaussian noise. An accelerogram, α_k, is obtained by multiplying the ARMA series with a suitable amplitude modulation function (see eq. (3)). Then, earthquake induced structural response of time invariant single degree-of-freedom (SDOF) structural systems may formally be obtained using a discrete time series model of the following type:

$$y_k = H(\alpha_k, y_{k-1}, y_{k-2}, \ldots | \omega_o, \zeta, \ldots) \tag{8}$$

where $H(\ldots)$ is a system function depending on undamped natural frequency, ω_o, critical damping ratio, ζ, and if required, parameters representing non-linear properties.

A probabilistic definition of the earthquake response spectrum can be based on the statistical properties of $y_{peak} = max(y_k)$ as an extreme value from a filtered Gaussian process. If the filters involved are linear it can be shown that y_{peak} follows asymptotically a type I extreme value distribution. For non-linear systems this may hold as an approximation in some cases (see for instance Clough and Penzien, 1993). Alternative methods of dealing with probability distributions of non-linear response include transforming the non-Gaussian variable to a Gaussian one (Winterstein, 1989), or simply using empirical distribution functions either derived from recorded accelerograms (Mirinda, 1993), or by stochastic simulation.

In probabilistic terms the displacement spectrum, S_d, for inelastic systems, taking the yield force, K_u, as a parameter, can formally be expressed as follows:

$$P\left[\max(y_k) \le S_d \middle| \omega_0, \zeta, K_u\right] = F_{S_d}(S_d) \tag{9}$$

Here $P[\ ]$ denotes the probability operator and $F_S(S)$ is the probability distribution of S. Correspondingly the acceleration spectrum, S_a, for inelastic systems, taking the ductility ratio μ, as a parameter, is given as:

$$P\left[\max(G_k) \le S_a \middle| \omega_0, \zeta, \mu\right] = F_{S_a}(S_a) \tag{10}$$

where the ductility ratio is defined as $\mu = \max(\mathrm{y}) / \mathrm{y_u}$, where y_u is the yield displacement. The seismic coefficient is given as $C = S_a / g$.

For inelastic response spectra it is common to plot the ductility against the initial undamped natural period of the system, $T_0 = 2\pi\sqrt{\rho y_u / K_u}$. In accordance with this the inelastic response spectrum can either be calculated as a constant strength spectrum (eq. (9)), or a constant ductility spectrum (eq. (10)). The latter however requires considerably more computational effort, as it involves iteration until a certain ductility is achieved (Mahin and Bertero, 1981, Miranda, 1993, Ólafsson and Sigbjörnsson 1995b).

RESULTS

Strong motion data

Results based on available data from South Iceland suggest ARMA(4,1) as an optimum model. A representative model was obtained by taking average values of the model parameters derived from 54 accelerograms from 6 earthquakes with magnitudes in the range 4 to 6 and epicentral distances in the range 0 to 80 km (Ólafsson and Sigbjörnsson, 1995a,b). The resulting parameters for the ARMA model (eq. (2)) and the envelope (eq. (3)) parameters for horizontal ground motion are shown in Table 1. Revised values for the scaling formulas of eq. (4) and (5) are also given in Table 1, and in Fig. 1 the fitting of the curves to the data is displayed.

Table 1. Model parameters, for eq. (2), (3), (4) and (5), with sampling period, $T_S = 0.02$ s.

ARMA parameters	Envelope parameters	Standard deviation	Duration formula
$a_1 = -0.82$	$e = 0.310$	$\theta_1 = -4.1405$	$\varphi_1 = -0.3701$
$a_2 = -0.35$	$c = 2.080$	$\theta_2 = 0.6124$	$\varphi_2 = 0.1255$
$a_3 = -0.20$	$m = 1$	$\theta_3 \approx 0$	$\varphi_3 = 0.3507$
$a_4 = 0.22$	$N = T / T_S$	$\sigma = 0.2070$	$\sigma_T = 0.2008$
$b_1 = 0.97$			

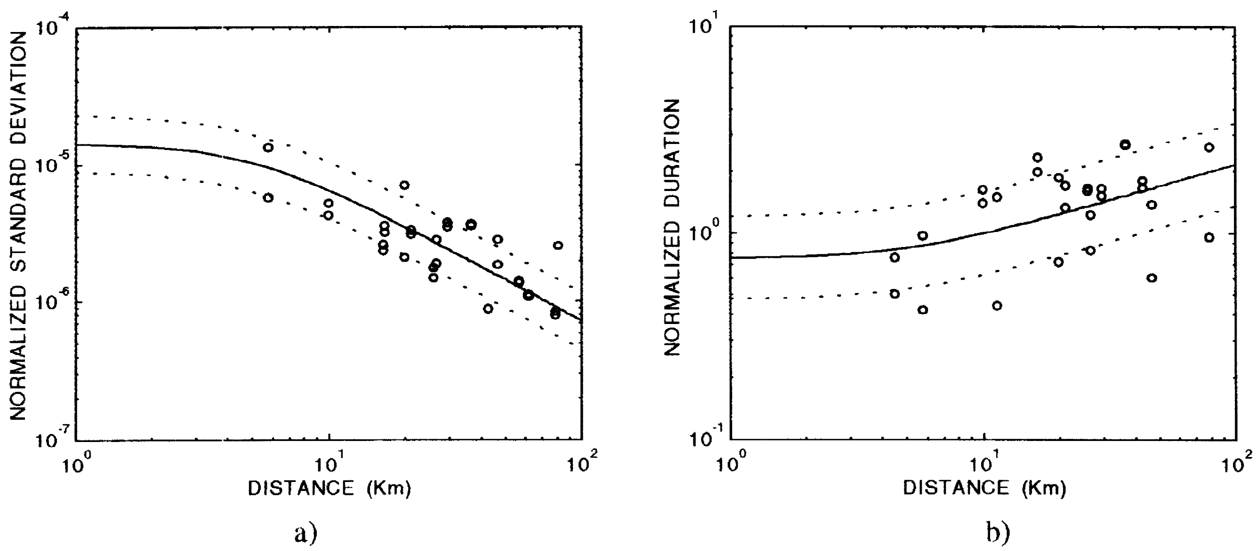


Fig. 1. Scaling formulas and observed values for earthquakes with magnitude greater than 4 and less than 6.
a) Normalized standard deviation of horizontal ground motion.
b) Normalized duration of horizontal ground motion.
The solid line represents the mean values and the dotted lines the mean values ± one standard deviation.

Comparison with source model.

The earthquake model presented has been verified by comparing it to a source model for the so-called Vatnafjöll earthquake (Bjarnason and Einarsson). The source duration derived for this earthquake is 3 s (Ólafsson et al., 1996) and 3 ± 1 s (Bjarnason and Einarsson, 1991), which fits reasonably well with eq. (5) that gives 4 s for the epicentral area, that is $d = 0$.

Fourier spectra obtained from the ARMA model is also found to be in fair agreement with spectra obtained from the source model, for magnitudes and epicentral distances which are within the limits set by the data. This is demonstrated in Fig. 2 for a site 15 km from the source, for the Vatnafjöll earthquake. In the figure a comparison is made between the average Fourier spectrum obtained from 50 simulations with the ARMA model and the Fourier spectrum obtained using the source model.

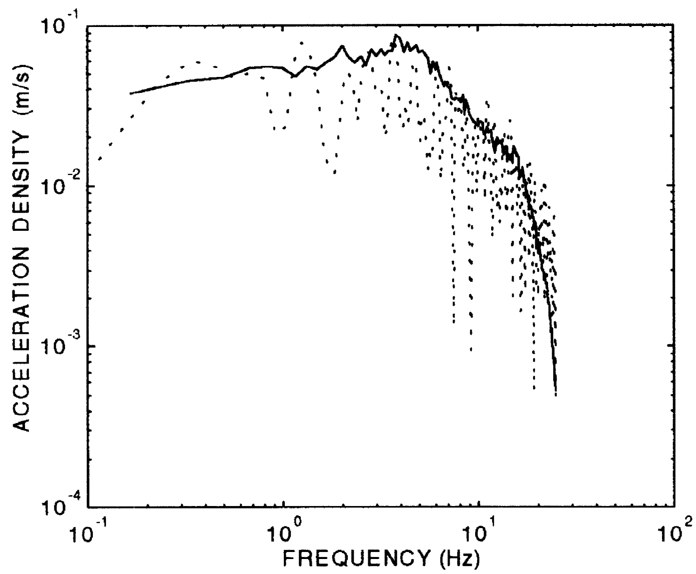


Fig. 2. A Fourier spectrum derived from ARMA model (using an average of 50 simulations - solid line), and a Fourier spectrum obtained from a source model (dotted line). Magnitude of earthquake, $M = 5.9$, distance to causative fault, $d = 15$ km.

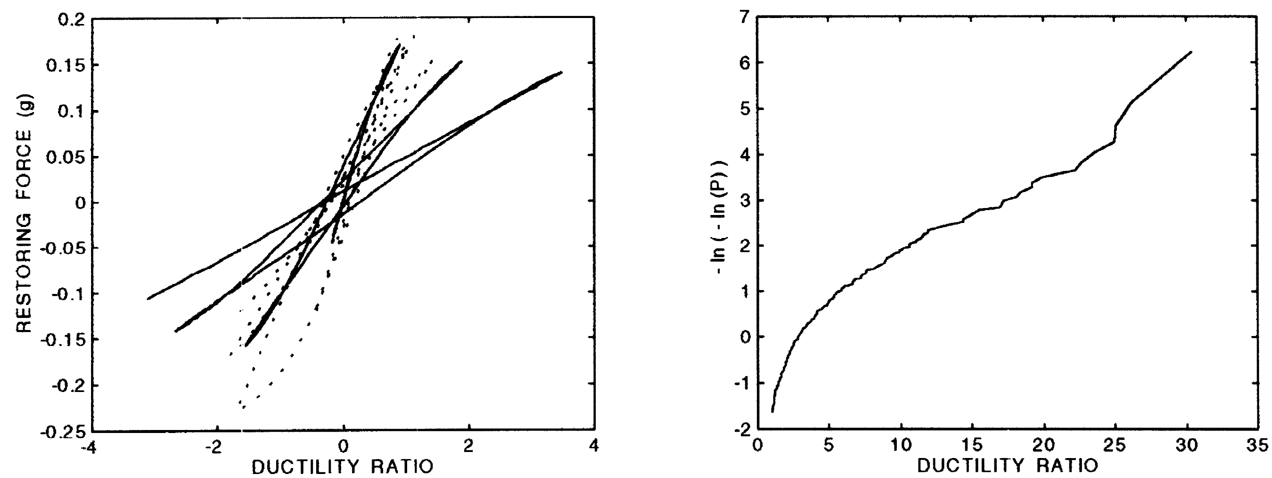
The ARMA model is an empirical model based on still rather limited data. It is suggested that using the ARMA model based on measured data and the theoretical source model outside the range of the data could lead to a hybrid model (Johansen and Foss, 1995). Further analysis along these lines would explore the relation between the parameters of the source model and the ARMA parameters or the zero-pole locations, in order to include in the ARMA model a change in predominant period of ground motion, with regard to epicentral distance and magnitude.

Probabilistic response spectra

The earthquake model presented here has been applied to simulate probabilistic response spectra. In the simulation account is made of the observed variability of standard deviation of ground acceleration and duration of shaking. In studying the variability of the ground motion, the duration, T, and the standard deviation, σ_α, are assumed approximated by a truncated log-normal distribution. The response of a SDOF-system, with degrading stiffness, using the Bouc-Wen model to model the hysterisis, was calculated for an ensemble of simulated earthquakes. A sample is shown in Fig. 3a. In all cases considered the extreme values of the simulated response data (see Fig. 3b) could not be described by classical asymptotic distributions without exceeding the 95% confidence limits. A Hermite moment model was applied to the response (Winterstein, 1989), reducing the non-linear analysis to transformations of results for Gaussian responses. The resulting distributions were, however, not sufficiently close to the empirical distributions, possibly due to non-stationarity in the data or the order of the transformation was not high enough.

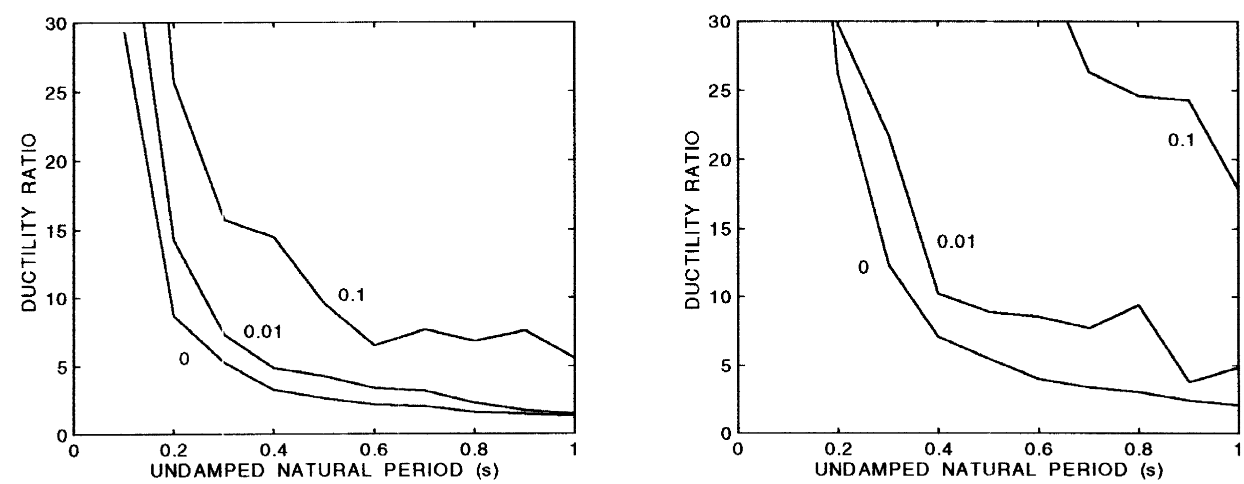
In Fig. 4 the probabilistic response spectra for a stiffness degrading system is shown, as obtained from the empirical distributions of an ensemble of 100 simulation. The damping ratio, $\zeta = 0$, stiffness degrading parameter, $\delta_\eta = 0.01$ and hysterisis loop parameters, $a = 0$, $\beta = \gamma = 0.5$, $\nu = 2$, and $B = 1$. Magnitude of earthquake, $M = 7$, distance to causative fault, $d = 10$ km, depth parameter, $h = 5$ km. The average peak acceleration was 0.47 g and the yield strength, $K_u = 0.25$ g. Fig. 5 shows the corresponding spectra as in Fig. 4, with displacement instead of ductility.

The ductility demand increases dramatically for lower periods, and the of risk of exceedance. For example at a period of $T_0 = 0.3$ s and $\delta_\eta = 0.01$ the ductility demand increase from 7 to 22 by decreasing the risk of exceedance from 50% to 16%. It can be seen in Fig. 5 that the displacement increases with higher periods. In Fig. 7 the yield strength is taken as a parameter instead of δ_η. For a design yield strength of 0.25 g, and a period of 0.2 s, the risk is 50% that a ductility level will be 14 or higher and 16% that it will be 30 or higher. The examples presented here are taken to demonstrate the effect of a catastrophic ground motion on a stiffness degrading structure. Further research is need to improve the strong motion model in order to obtain realistic probabilistic response spectra.



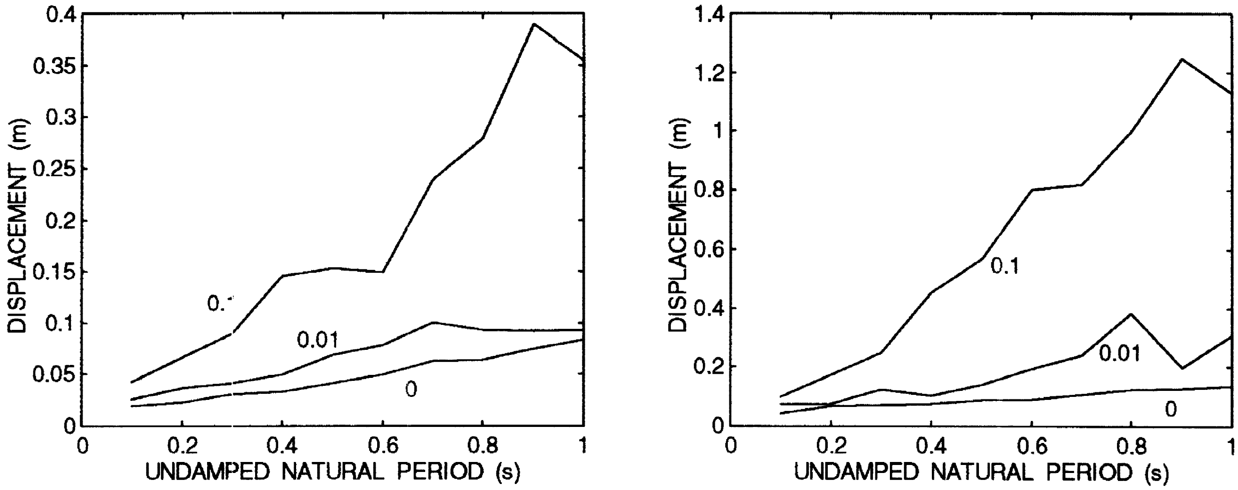
a) b)

Fig. 3. a) Hysterisis loops, $T_0 = 0.5$ s, $K_u = 0.25$ g, and $\delta_\eta = 0.01$. A system with nondegrading stiffness is represented by dotted line. Same system with degrading stiffness is represented by solid line.
b) Empirical probability distribution of extreme values of response, $K_u = 0.25$ g, and $\delta_\eta = 0.01$.



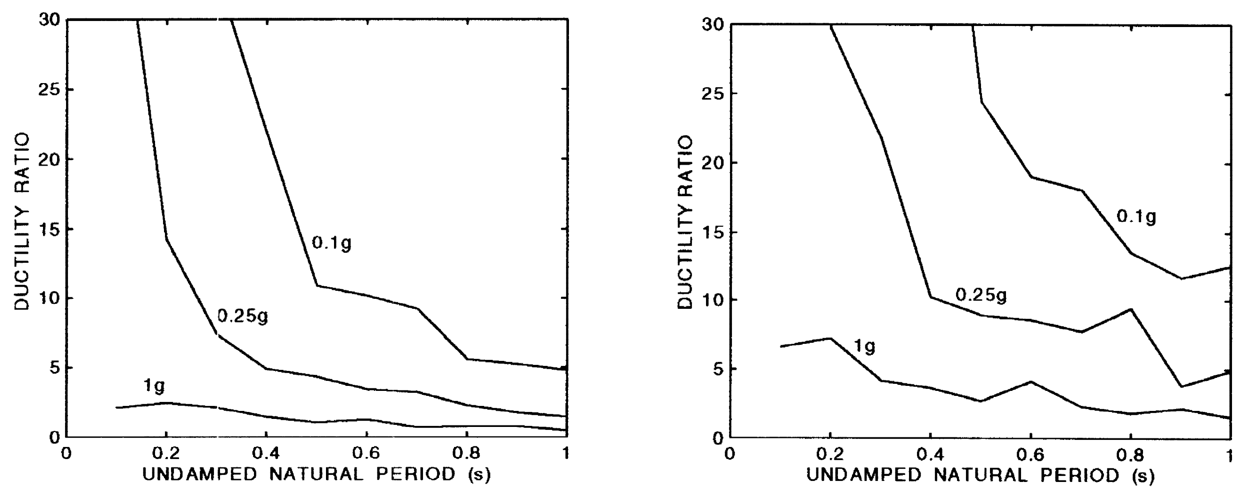
a) b)

Fig. 4. Constant strength response spectra, with $K_u = 0.25$ g, and $\delta_\eta = 0, 0.01, 0.1$ a) 50% probability of non-exceedance. b) 84% probability of non-exceedance.



a) b)

Fig. 5. Constant strength response spectra, with $K_u = 0.25$ g, and $\delta_\eta = 0, 0.01, 0.1$. a) 50% probability of non-exceedance. b) 84% probability of non-exceedance.



a) b)

Fig. 6. Constant strength response spectra, with $\delta_\eta = 0.01$, and three levels of yield strength $K_u = 0.1$ g, 0.25 g and 1 g. a) 50% probability of non-exceedance. b) 84% probability of non-exceedance.

CONCLUSIONS

A preliminary comparison of the Fourier spectra of the ARMA model and a seismic source model suggest a fair match in the range where there is available data. Furthermore there is good agreement between the source duration obtained from the source model and the duration obtained from the empirical duration formula.

Simulation studies of the response of a stiffness degrading system produce ensembles of peak values which can only be considered to follow theoretical asymptotic extreme distributions in a very limited way. This resulted in the use of empirical distributions, in order to obtain a probabilistic response spectra. The results of this paper confirms the necessity of considering the effect the uncertainties in the earthquake motion on the response.

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