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STOCHASTIC LONG-TERM DAMAGE PREDICTION OF MULTI-STORY BUILDINGS

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ABSTRACT

A stochastic model is proposed to predict the lifetime of existing multi-story buildings in seismically active regions. Variability in earthquakes is described as that which fluctuates with both short-term and long-term. The damage state prediction (DSP) model is introduced to quantify multiple levels of damage state in terms of the stiffness degradation or strength deterioration of structural system. The DSP model is based on the Iwan's distributed element model and capable of predicting permanent system damage in terms of the acceptable maximum or cumulative ductility ratio of each element. For different earthquake intensities, conditional damage transition probability (CDTP) matrices are constructed by Monte Carlo simulation using the DSP model. Then an overall damage transition probability (ODTP) matrix is constructed by multiplying the element of each CDTP matrix by the corresponding earthquake occurrence rate and integrating the result with respect to all possible earthquake intensities. The ODTP matrix is used to predict the future damage state of structural system in the Markov chain model. For illustration, the damage evolution of multi-story steel and reinforced concrete buildings is presented for different soil conditions.

KEYWORDS

Damage prediction, Monte Carlo simulation, damage model, Markov chain model, multi-story building, cumulative damage, first passage failure

INTRODUCTION

To predict the lifetime of multi-story buildings located in the seismic regions, it is useful to construct a mathematical model which can deal with the evolution of structural damage in time and the time to reach a prescribed level of damage. However, there are many uncertainties and randomnesses in the modeling of earthquake excitation, dynamic characteristics of structural system, nonlinear response analysis and also in the description of damage state. Under these uncertain circumstances, a stochastic model can provide the theoretical basis for dealing with uncertainties explicitly and synthesize different sources of uncertainty systematically.

Subjected to strong ground motion, structures are often likely to undergo response in the inelastic range and the stiffness and strength of structural system occasionally degrade or deteriorate particularly at significant level of damage. A number of models have been proposed for investigating nonlinear response, ranging from a simple bilinear model to more sophisticated hysteretic models. In the stochastic response analysis, one of the most frequently used hysteretic models is a differential equation model which is mathematically motivated (Wen,1976). Another popular hysteretic model is a distributed element (DE) model which is physically motivated (Iwan,1966). The original DE model was only composed of a series of bilinear element. The DE model has been improved to reproduce the deteriorating behavior of structural system by adding a series of slip element (Iwan,1973) or by cutting the bilinear element one by one if the element reaches a specified level of plastic deformation (Iwan and Cifuentes,1986).

To quantify the seismic damage of existing structures, it is necessary to select a pertinent measure indicating the structural damage state as a nonlinear response process. Structural damage has been described in terms of ductility factor and hysteretic energy for reinforced concrete buildings (Banon and Veneziano,1982; Park and Ang,1985), whereas cumulative ductility factor or hysteretic energy for steel buildings (Kato and Akiyama, 1982) at the system level.

This paper introduces a damage state prediction (DSP) model which is based on the Iwan's distributed element model and capable of reproducing the system deterioration in terms of the yielding and failure of each element. Multiple levels of damage state are described by the stiffness and strength deterioration of the system. For a prescribed earthquake intensity, a conditional damage transition probability (CDTP) matrix is constructed by a series of nonlinear response analyses using the DSP model. To construct an overall damage transition probability (ODTP) matrix, the element of the CDTP matrix is multiplied by the occurrence rate associated with the earthquake intensity and the result is integrated with respect to all possible earthquake intensities. The ODTP matrix is used to predict the future damage state in the Markov chain model (Bogdanoff and Kozin,1985). To illustrate the applicability of this method, the damage evolution of multi-story steel and reinforced concrete buildings is presented for different soil conditions.

DAMAGE MODEL

The DSP model consists of a system composed of multiple pairs of bilinear and slip elements, a linear spring element and a viscous damping element as shown in Fig.1. The yielding of a pair of elements causes the instant stiffness reduction of the system. The breaking of a pair of elements causes the permanent stiffness loss and strength deterioration of the system. Consequently, the system behavior may be completely controlled at the element level. This is convenient to quantify the system deterioration.

The bilinear element in the i-th pair has a yielding force equal to $k_i X_{yi}$, where k_i is the spring constant and X_{yi} is the yielding displacement. If the maximum displacement becomes larger than βX_{yi} (β *is* the acceptable maximum ductility ratio

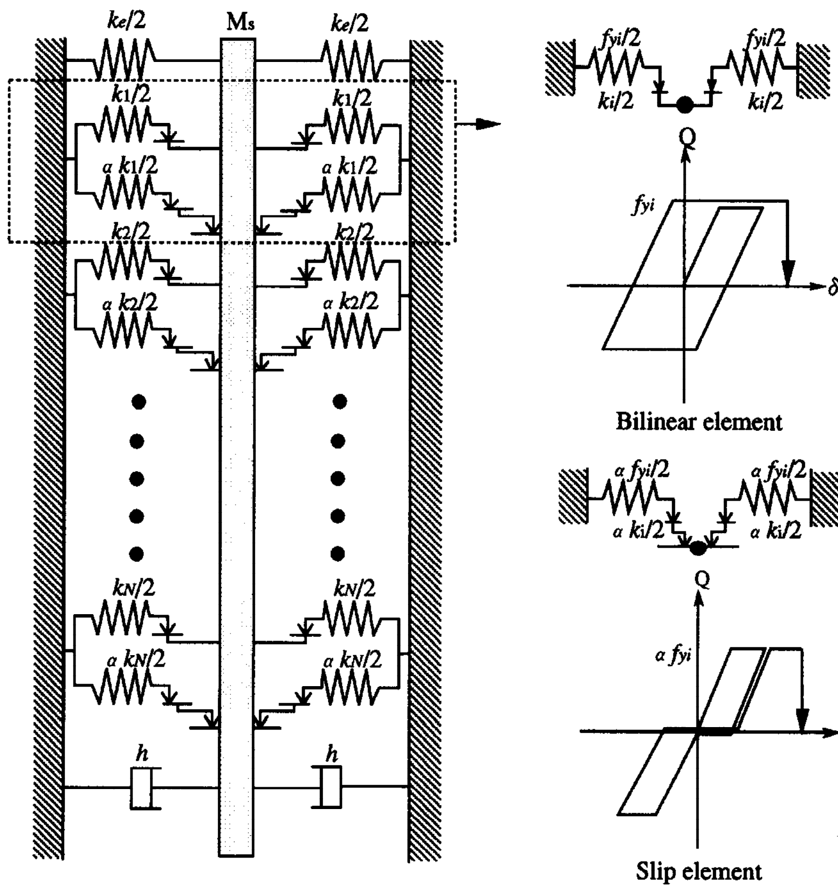


Fig.1 DSP model

for first passage failure at the element level) or if the cumulative plastic displacement becomes larger than γX_{yi} (γ is the acceptable cumulative ductility ratio for cumulative damage at the element level), the i-th pair of elements breaks and the contribution of the pair to the total restoring force becomes zero. The slip element in the i-th pair is set to have a spring constant αk_i (α is the stiffness ratio which represents the contribution of slip element). The DSP model accounts for the progressive loss of stiffness and strength of structural system at large amplitude of oscillation. Each pair of elements is arranged so that $X_{y1} < X_{y2} < \bullet\bullet\bullet < X_{yN}$. Since each pair of elements breaks when the maximum displacement exceeds the value βX_{yi} or the cumulative plastic displacement exceeds the value γX_{yi}, they will break in ascending order. Each pair of elements is completely defined in terms of two parameters, k_i and X_{yi}. Since there are N pairs of elements, there will be $2N$ parameters. It is assumed that the yielding force is constant for all pairs of elements. This reduces the number of parameters from $2N$ to $N+1$. The initial stiffness, K_0, of structural system can be related to the parameters of the DSP model by the relationship

$$K_0 = k_e + \sum_{i=1}^{N} (1+\alpha) k_i \quad , \tag{1}$$

in which k_e is the stiffness of a linear spring. The ultimate strength, F_u, of the DSP model can be expressed as

$$F_u = k_e X_{yN} + \sum_{i=1}^{N} (1+\alpha) f_{yi} \quad , \tag{2}$$

in which f_{yi} is the yielding force of the i-th pair. Thus, the initial stiffness and ultimate strength of a DSP model may be expressed by the contribution of N pairs of elements and a linear spring element.

Figure 2 shows the hysteretic behaviors of the DSP model, which are compared with published experimental results on steel and RC frames (Wakabayahi,1981). It can be seen that the DSP model is capable of reasonably reproducing the hysteretic and deteriorating behavior of both steel and RC frames.

DESCRIPTION OF EARTHQUAKES

The future nature of the earthquake ground motion expected at a particular site is unpredictable in the deterministic sense and realistically represented by a stochastic point of view. According to the time scale of fluctuation, the variability of earthquake force may be described as that which fluctuates with short-term and long-term. The long-term variation may be approximately described by a pulse process characterized by random occurrence. A seismic hazard curve is used to calculate the occurrence rates of all relevant earthquake intensities over a specified time period. The short-term variation, on the other hand, may be described by a continuous random process. A ground motion model is constructed as a quasi-nonstationary random process which is characterized by a stationary power spectral density function and a deterministic envelope function. Sample ground motions are generated with all possible earthquake intensities and used as the inputs to structural system.

A seismic hazard curve is described through a plot of annual exceedance probability versus peak ground acceleration, A_p. The hazard curve of the type II extreme value distribution is given by

$$P[A_p > a] = 1 - \exp[-(u/a)^k] , \tag{3}$$

in which u and k are the size and shape parameters of the distribution, respectively. The peak ground acceleration corresponding to a T year return period can be obtained from the following equation :

$$P[A_p < a] = 1 - \frac{1}{T(a)} . \tag{4}$$

The method used for artificial ground motion generation is the superposition of sinusoids having random phase angles and amplitudes devised from a stationary power spectral density function. The Kanai-Tajimi

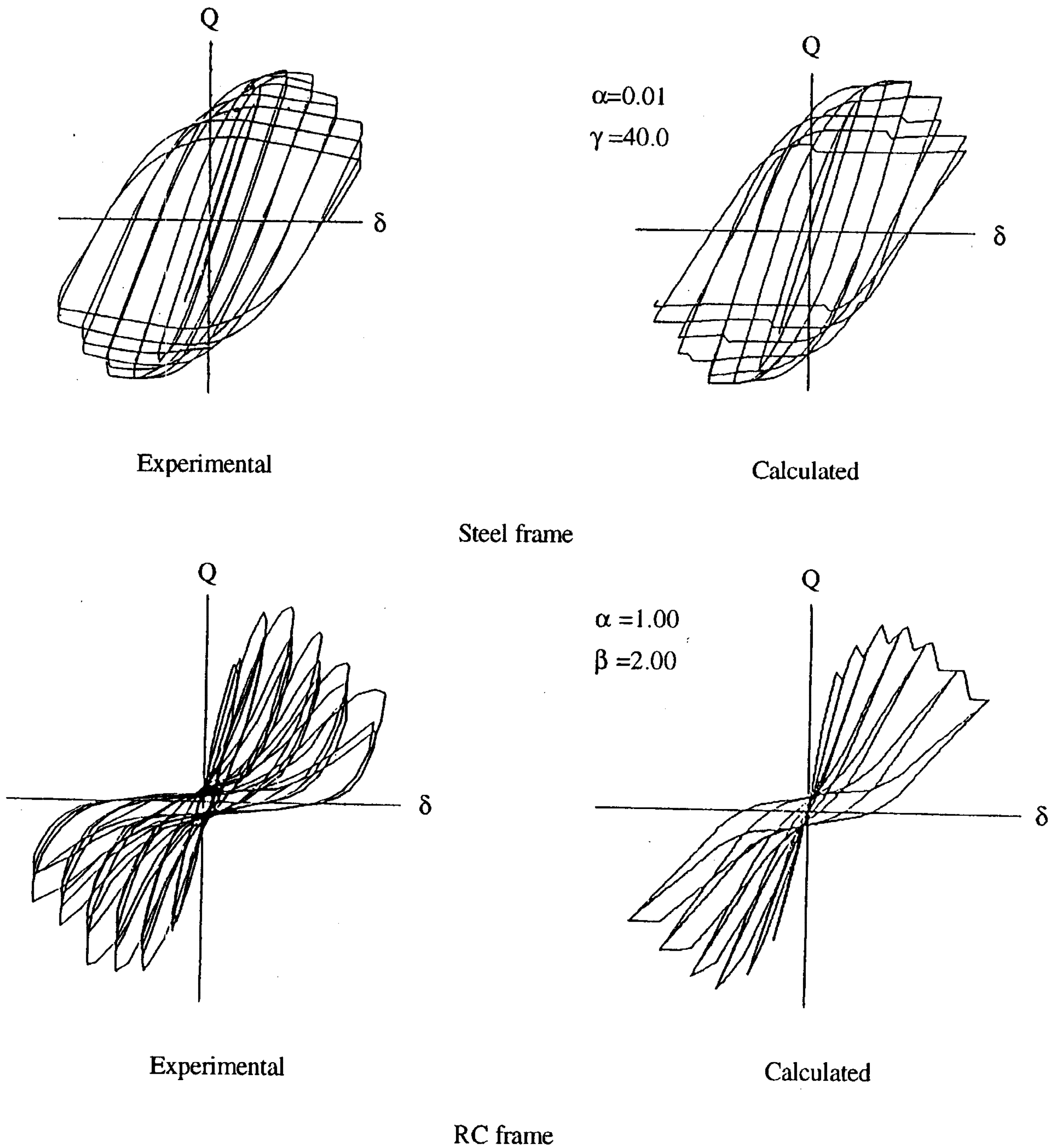


Fig.2 Hysteretic behavior of DSP model

power spectral density function of ground acceleration is given by

$$G(\omega) = G_0 \cdot \frac{1 + 4h_g^2(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2 + 4h_g^2(\omega/\omega_g)^2} \qquad (0 < \omega < \infty), \qquad (5)$$

in which ω_g is the predominant circular frequency, h_g is the damping ratio and G_0 is the spectral intensity which is a measure of earthquake intensity. The relationship between spectral intensity, G_0, and peak ground acceleration, A_p, may be obtained as

$$G_0 = \frac{4h_g(A_p / Z)^2}{\pi\omega_g(1+4h_g^2)}, \tag{6}$$

in which Z is the peak factor which is assumed here to be 3.0. The transient character of earthquake is described by the Jennings-type envelope function as

$$g(t) = \begin{cases} (t/t_1)^2 & : \quad 0 \le t \le t_1 \\ 1 & : \quad t_1 \le t \le t_2 \\ \exp\{-c(t-t_2)\} & : \quad t \ge t_2 \end{cases}, \tag{7}$$

in which t_2-t_1 is the duration of stationary part and c is the decay coefficient.

SHORT-TERM PREDICTION

The range of damage state is divided into N discrete damage levels corresponding to the number of broken pairs of elements in the DSP model. A two-dimensional Markov matrix which is referred to as a conditional damage transition probability (CDTP) matrix is constructed. Each column and each row in the matrix contains transition probabilities, *e.g.*, element P_{ij} is the probability that the post-earthquake damage level D_{post} is level j, given that the pre-earthquake damage level D_{pre} is level i, or

$$P_{ij} = P\left(D_{post} = D_j \middle| D_{pre} = D_i\right). \tag{8}$$

The elements in the matrix may be calculated by Monte Carlo simulation. For each earthquake intensity level, sample ground motions are generated and used as the inputs to structural system. Using a DSP model, a series of nonlinear response analyses are carried out to find the post-earthquake damage state for each response time history. To construct a CDTP matrix, the pre-earthquake damage state is varied from level 1 to N. Depending on earthquake intensity, the stiffness and strength of structural system deteriorate during earthquakes. The damage level gradually evolves in time from the pre-earthquake damage state to the post-earthquake damage state. Since structural damage is an irreversible process, the resulting CDTP matrix becomes a triangular matrix.

LONG-TERM PREDICTION

The CDTP matrix is conditional on a specific level of earthquake intensity. To construct a ODTP matrix, which is an overall damage transition matrix and independent of earthquake intensity, the element P_{ij} in the CDTP matrix is multiplied by the occurrence rate associated with the earthquake intensity level and then the result is integrated with respect to all possible earthquake intensities. Assuming that the occurrence of earthquake follows the Poisson process, the probability of damage state after t years is given by

$$\{P(t)\} = \sum_{K=0}^{\infty} \{P(0)\}[\Phi]^K \frac{(\nu t)^K}{K!} \exp(-\nu t), \tag{9}$$

in which $\{P(0)\}$ is the initial damage state vector (N×1), $\{P(t)\}$ is the t-year future damage state vector (N×1), $[\Phi]$ is the ODTP matrix (N×N) whose element Φ_{ij} gives the probability of being in damage level j having started in damage level i, ν is the mean occurrence rate of earthquake and K is the number of occurrence.

NUMERICAL EXAMPLES

To illustrate the proposed method, the seismic damage evolution of five-story steel and four-story reinforced concrete frame buildings is investigated. Figure 3 shows typical plans of the example steel and RC buildings. The inner frames, which are circled by dotted lines, of each building are modeled as single degree of freedom systems. The initial natural period of the RC building is 0.27sec (23.3rad/sec) and steel building is 0.28sec (22.7 rad/sec). Nine levels of earthquake intensity are assumed as shown in Table 1. The failure criterion of the steel and RC buildings are assumed to be cumulative damage and first passage types, respectively. The parameters in the Type II extreme value distribution of peak ground acceleration are u=0.608m/sec^2 and k=3.3. The mean occurrence rate is ν =1.0. The parameters in the Kanai-Tajimi power spectral density

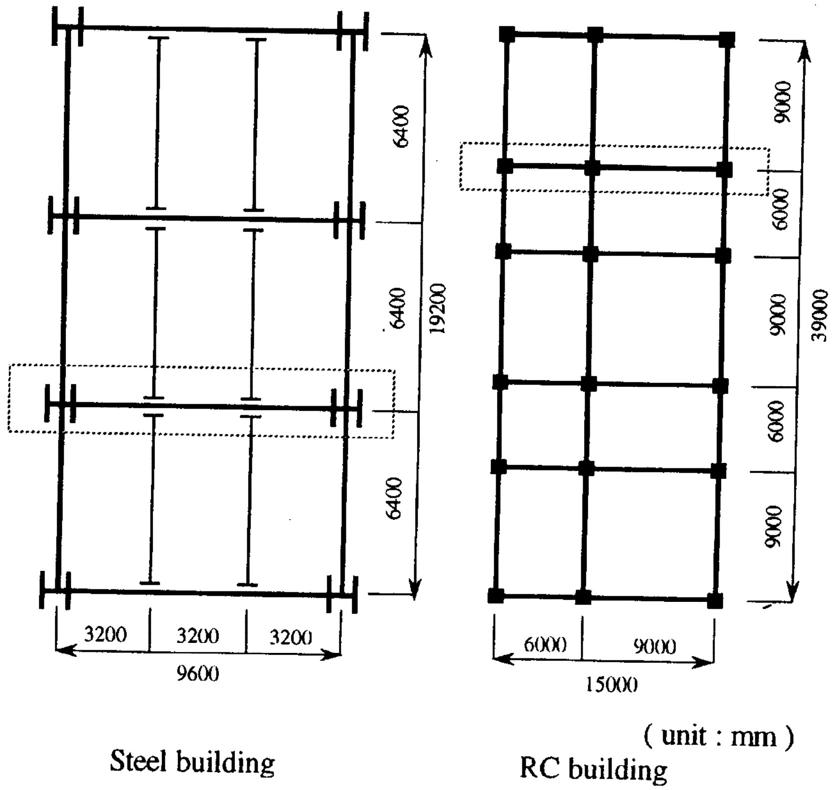


Fig.3 Typical plans of example buildings

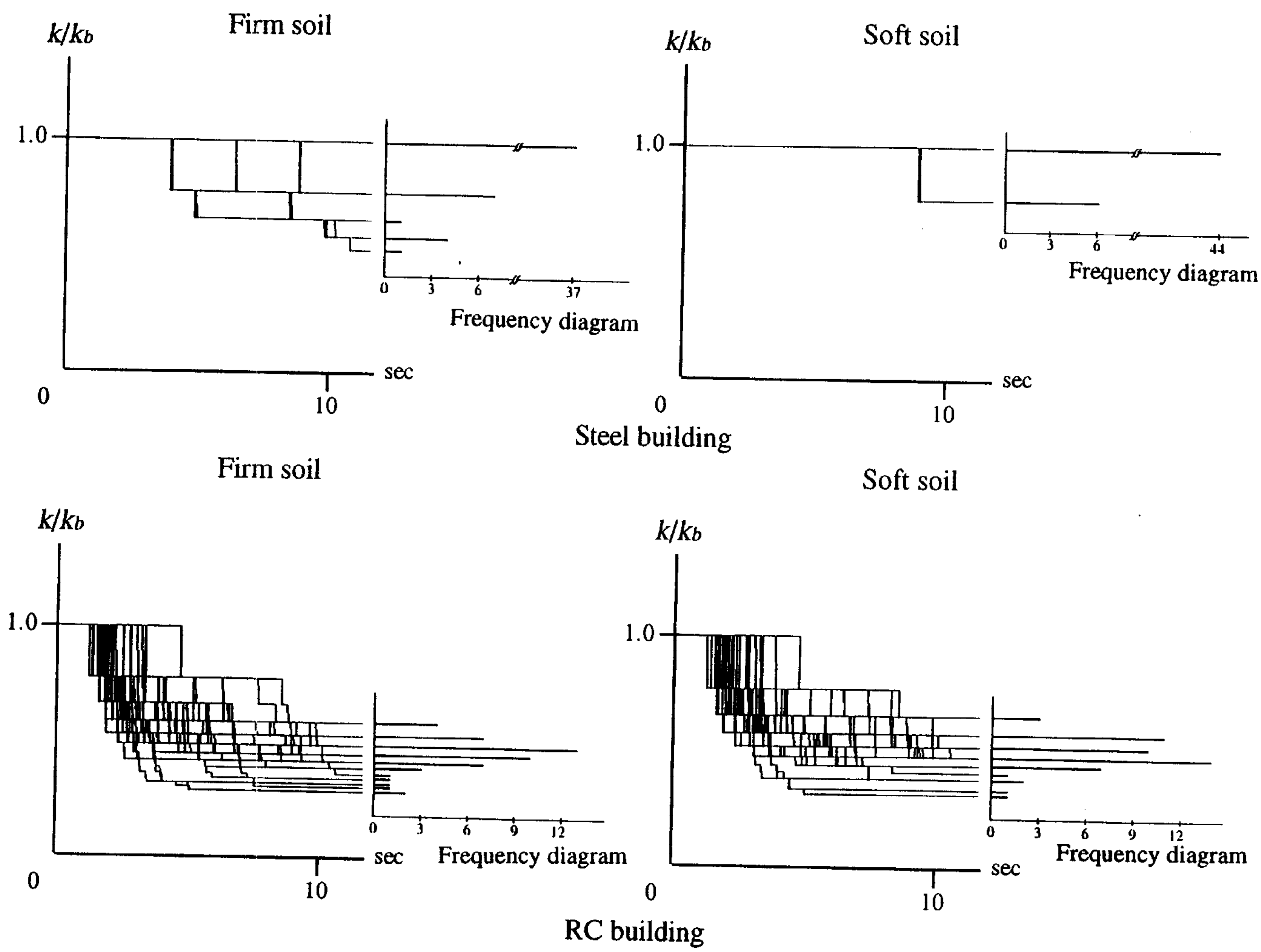
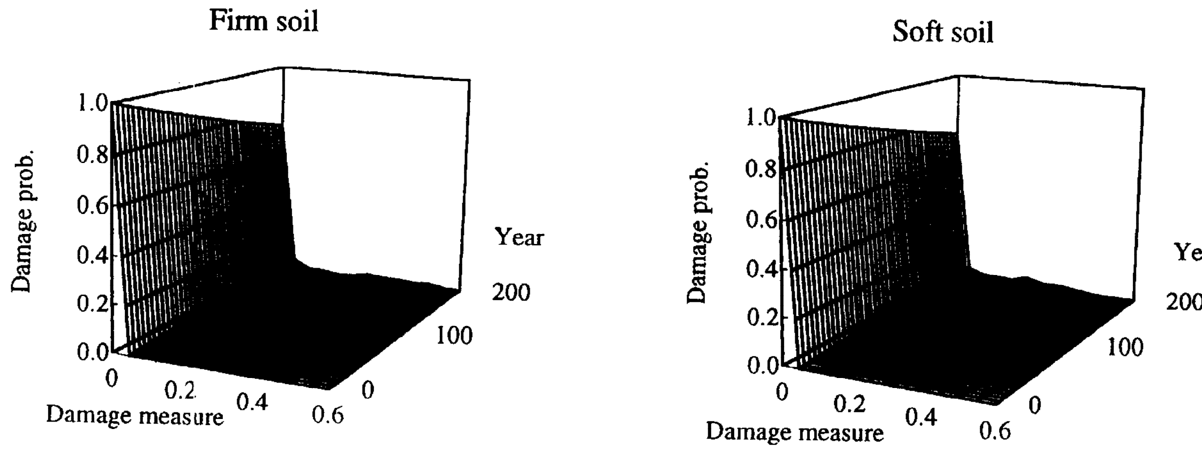
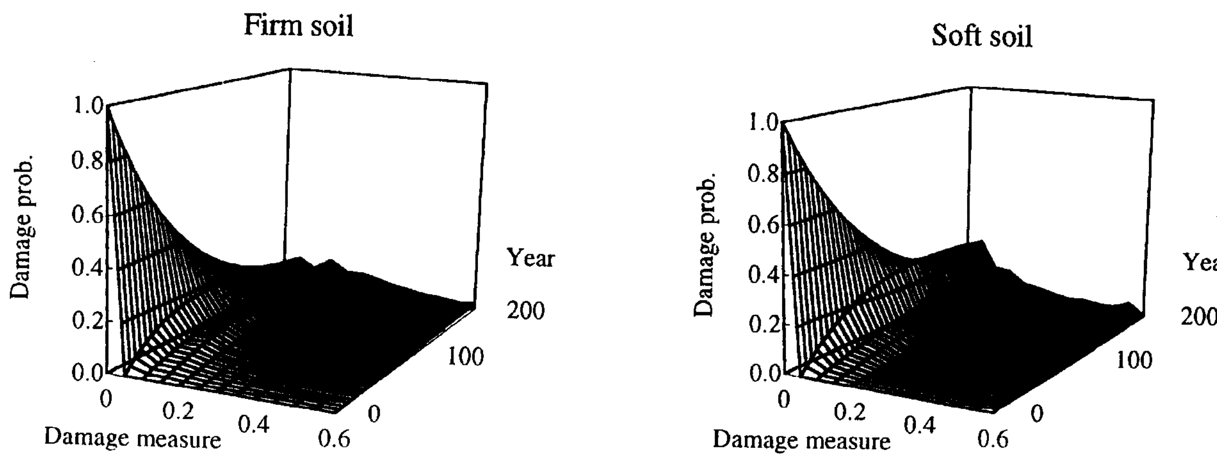


Fig.4 Stiffness degradation during simulated ground motions of earthquake level 8



Steel building



RC building

Fig.5 Damage evolution of steel and RC buildings for different site conditions

function are as follows: ω_g=15.7 rad/sec and h_g=0.6 for firm soil condition, ω_g=7.85rad/sec and h_g=0.8 for soft soil condition. The parameters in the Jennings envelope function are fixed to be t_1=1.5sec, t_2-t_1=7.0sec, c=0.18 and the total duration is 20sec for all earthquake intensities. 50 sample ground motions are generated for each earthquake intensity. The model parameters of the steel building are as follows: the stiffness of a linear element; $k_e=7.06\times10^2$ N/mm, the yielding displacement for each pair of elements; X_{y1}= 9 mm, X_{y2}= 17 mm, X_{y3}=25 mm, · · · , X_{y20}= 170 mm, the yielding force for each pair of elements; $f_y= 6.9\times10^3$ N(constant), the stiffness ratio; α=0.01, the acceptable cumulative ductility ratio; γ =40.0, the total mass of structural system; $M_s=7.0\times10^3$kg, and the damping ratio; h=0.02. The model

Table1 Earthquake levels and occurrence rates

Earthq. level	Range of Ap (m/sec²)	Median of Ap (m/sec²)	G_0(m²/sec³) $\times10^{-3}$ firm	soft	Occurrence rate
1	~0.961	0.624	0.86	1.58	0.800
2	~1.208	1.059	2.48	4.54	0.100
3	~1.507	1.325	3.89	7.11	0.050
4	~2.001	1.683	6.28	14.47	0.030
5	~2.477	2.187	10.60	19.37	0.010
6	~3.063	2.706	16.23	29.65	0.005
7	~4.055	3.417	25.87	47.29	0.003
8	~5.011	4.427	43.43	79.37	0.001
9	~8.031	6.194	85.01	1553.7	0.001

parameters of the RC building are as follows: the stiffness of a linear element; k_e=32.34×10^2 N/mm, the yielding displacement for each pair of elements; X_{y1}= 3 mm, X_{y2}= 6 mm, X_{y3}= 9 mm, · · ·, X_{y20}= 60 mm, the yielding force for each pair of elements; f_y= 5.6×10^3 N(constant), the stiffness ratio; α=1.0, the acceptable maximum ductility ratio; β=2.0, the total mass of structural system; M_s=29.4×10^{3}kg, and the damping ratio; h=0.05. The number of damage levels is set to be 21 for both models.

Figure 4 shows the change in stiffness of the completed steel and RC buildings resting on firm and soft soil for 50 simulated ground motions of earthquake intensity level 8. The vertical axis is the ratio of the instant stiffness (k) to the pre-earthquake one (k_b), whereas the horizontal axis is the earthquake duration. The change in stiffness and its variability of the steel building is relatively small, while those of the RC building is considerably large.

Figure 5 shows the evolution of damage state of the steel and RC buildings for different soil conditions. The vertical axis is the probability of being in the corresponding damage state after t years. The damage state is defined in terms of the ratio of the number of broken pairs of elements to the total number of pairs of elements. 0.0 is the undamaged state, whereas 1.0 corresponds to the totally damaged state. The probability of undamaged state gradually decreases in time and that of damage states slightly increases for the steel building. On the other hand, the probability of undamaged state rapidly decreases in time and that of damaged states becomes relatively large for the RC building. The evolution of damage is almost the same irrespective of soil condition for the steel building, while it is heavily dependent on soil condition for the RC building.

CONCLUSIONS

A stochastic model has been developed for predicting the future damage state of existing buildings in the seismic regions. The proposed model is capable of dealing with multiple levels of damage state and evolution of the damage in time in terms of stiffness degradation or strength deterioration. Based on the numerical results, it is concluded that the stochastic model is a useful and powerful tool to predict the lifetime of existing buildings for different soil conditions. In addition, it is proved that a DSP model can reasonably reproduce the nonlinear degrading dynamic behavior of both multi-story steel and reinforced concrete buildings during earthquakes by specifying the acceptable maximum or cumulative ductility ratio at the element level.

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