

## STRUCTURAL SYSTEM IDENTIFICATION WITH ACTIVE MASS DAMPER USED FOR PROVIDING INPUT EXCITATION

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### ABSTRACT

The present investigation deals with the issue of estimating the dynamic characteristics of a building structure utilizing AMD(active mass damper) as a tool providing an input excitation for system identification. For conducting effective and reliable structural control, it is desirable to design the structural control system along with more precise informations regarding the controlled building structure. In order to get such informations, AMD, originally installed on the top floor for structural control, is utilized to provide the structure to a random white input. With the acceleration output measurements, system identification is performed with the linear regressive formulation. To verify the presented methodology, computed and experimental examples are demonstrated.

### KEYWORDS

linear regression model, direct term, acceleration measurement, white-noise input

### INTRODUCTION

Along with the development of modern technologies, it has become increasingly significant to conduct system identification for constructed or existing building structures. The operation of system identification is expected to provide the accurate informations of the structural motion characteristics such as its natural frequency and damping, etc. These informations will help build more suitable mathematical model representing the building structure, thus providing more accurate and reliable estimation of its seismic responses. Furthermore, by judging how properly and realistically the design model used in the stage of structural design and analysis reflects the actual building behavior and having these judgement experiences, more improved and sophisticated methodology of structural design of buildings is expected to be established.

According to the nature and availability of an input excitation and to what is to be identified, structural identification is classified in several categories. The identification technique utilized in this paper is based on a white-noise input, and is to identify the transfer function of a structure.

The recent researches regarding this type of identification were published by Suzuki and Kawanobe(1987), and Kawanobe and Suzuki (1993). Utilizing the linear regressive model formulation employing the acceleration measurement records both for the input and output data, Kanazawa *et al.* pointed out that the significance of the direct term involvement.

Accounting for the recent development of actively-controlled buildings, the authors(Yamada, *et al.* 1994) have discussed the structural identification technique utilizing an actuator for structural control as a tool providing a white-noise input excitation. The present investigation characteristically carries out the structural identification related to the two mode (lateral and torsional) responses with one input excitation.

## LINEAR REGRESSION MODEL

The difference equation of a linear regressive representation for the single-input-output relationship for a given structure can be written as:

$$y_k = \sum_{i=1}^n a_i y_{k-i} + \sum_{i=0}^n b_i u_{k-i} + r_k \quad (k = 1, 2, \dots, N) \quad (1)$$

where  $y_k$  = discrete output observation at the  $k$ th time;  $u_k$  = discrete input observation at the  $k$ th time; and  $r_k$  denotes the prediction error at the  $k$ th time. In this representation the coefficients,  $a_i$  and  $b_i$ , are to be determined so as to minimize the following sum of the error squares:

$$J = \sum_{k=1}^N r_k^2 \quad (2)$$

With a zero-mean white-noise  $r_k$ , the coefficients thus determined are known to be consistent estimation with the limiting values as  $N \rightarrow \infty$  (Nakamizo, 1988). Once they have been appropriately evaluated, the output predicted at the  $k$ th time can be given by (1).

In the right-hand side of (1), there exists the specific term,  $b_0 u_k$ , which reflects the effect of the input  $u_k$  on the output  $y_k$ . This term, which is called the direct term, is of significance for the representation employing the acceleration response as the output observation, while it is not necessary for the formulation dealing with the velocity output observation. In such a sense (1) represents more generalized case. Although this paper mainly deals with the acceleration response output, it will be convenient, from the practical point of view, to use the same-style formulation both for the velocity and acceleration outputs. To accomplish this,  $\hat{y}_k$  is introduced in the following fashion:

$$\hat{y}_k = y_k - b_0 u_k \quad (3)$$

With this notation the formulation (1) leads to:

$$\hat{y}_k = \sum_{i=1}^n a_i y_{k-i} + \sum_{i=1}^n b_i u_{k-i} + r_k \quad (4)$$

This idea pre-eliminating the direct term was presented by Kanazawa *et al.*(1993). In putting this idea into practice, the coefficient  $b_0$  must be evaluated in advance. Their way of evaluation is based on the precise knowledge of the mass value of the top story. In the present study, however, the orthogonal projection principle is employed to evaluate the coefficient of the direct term. The principle provides the condition as follows:

$$(\mathbf{u}, \mathbf{y} - b_0 \mathbf{u}) = 0 \quad (5)$$

where

$$\mathbf{u} = [u_1, u_2, \dots, u_{N-1}, u_N]^T ; \quad \mathbf{y} = [y_1, y_2, \dots, y_{N-1}, y_N]^T \quad (6)$$

and  $(\cdot, \cdot)$  denotes the inner product of two vectors. From the condition of (5),  $b_0$  is derived:

$$b_0 = \frac{(\mathbf{u}, \mathbf{y})}{(\mathbf{u}, \mathbf{u})} \quad (7)$$

Alternately, the same result is obtainable under the following two assumptions:  $u_k$  is a zero-mean white-noise; and  $u_k$  and  $r_k$  are mutually independent. Multiplying both sides of (1) by  $u_k$  and taking the expectation, one has

$$b_0 = \frac{R_{uy}(0)}{R_{uu}(0)} = \frac{\frac{1}{N} \sum_{k=1}^N u_k y_k}{\frac{1}{N} \sum_{k=1}^N u_k u_k} = \frac{\sum_{k=1}^N u_k y_k}{\sum_{k=1}^N u_k u_k} = \frac{(\mathbf{u}, \mathbf{y})}{(\mathbf{u}, \mathbf{u})} \quad (8)$$

The above discussion indicates that (7) necessitates the implicit assumption of a white-noise input. In determining the coefficients after eliminating the direct term by means of (7), the recursive least square algorithm is employed in this paper for the sake of the computer storage space saving as well as keeping away from the inverse matrix calculation.

## NUMERICAL ANALYSIS

Numerical example is demonstrated for a four-story building model structure. Each story has two degrees of freedom: one is with respect to the lateral direction and another the torsional direction and they are assumed uncoupled. AMD is assumed to be placed with eccentricity such that input excitation from AMD induce both the lateral and torsional movements simultaneously. The parameters of this model building are as follows: mass of each story = 2.00 (kg), mass of AMD=0.77 (kg); mass moment of inertia of each story=0.03497(kg · m); viscous damping of each story with respect to the lateral response= 12.96 (N·sec/m), viscous damping of each story with respect to the torsional response= 0.35 (N·m·sec/rad); stiffness of each story with respect to the lateral response= 27000(N/m), stiffness of each story with respect to the torsional response= 1000.0(N·m/rad).

Gaussian white-noise excitation is applied as an input to the top floor of the building through AMD installed at the top floor, and the acceleration input of AMD and the acceleration response outputs of the top floor with respect to the lateral and torsional directions are calculated by assuming the sampling rate  $\Delta = 0.008[\text{sec}]$  and  $N = 1024$ .

For the purpose of considering the significance of the direct term involvement, these three models are examined and compared:

[Model #1] , which is based on the formulation (4) pre-extracting the direct term.

[Model #2] , which is based on the formulation (1) with the direct term involved.

[Model #3] , which is based on the formulation (1) with the direct term initially excluded.

For comparison, the residual sum of the error squares,  $J$ , for each Model with respect to the lateral response is displayed in Fig.2. This figure shows the tendency that the residual sums for Models #1 and #2 get smaller with larger  $N$ , while Model #3 does not. The coefficient  $b_0$  is evaluated as 0.503 and 0.500 by Models #1 and #2, respectively. The similar calculations are also made for the torsional outputs. Since they have, however, exhibited the same trends, they are not shown here schematically.

**Figs.3, 4, and 5** show the magnitudes of the transfer functions of the top floor acceleration response in the lateral direction, which are based on Models #1, #2 and #3, respectively. These transfer functions are calculated by means of the Z-transform in the following manner (Kanazawa *et al.*, 1993):

$$G(z) = b_0 + \frac{\sum_{i=1}^n b_i z^{-i}}{1 - \sum_{i=1}^n a_i z^{-i}} = \frac{b_0 + \sum_{i=1}^n (b_i - b_0 a_i) z^{-i}}{1 - \sum_{i=1}^n a_i z^{-i}} \quad (9)$$

In these figures, the solid lines are obtained by (9) with  $n = 8$ , with the dashed-lines denoting the theoretical transfer function. The identified results given by Models #1 and #2 are in fairly good agreement with the theoretical one, whereas with less effectiveness Model #3 anticipatedly identifies the structural system dynamics than Models #1 and #2. This fact indicates how an important role the direct term  $b_0 u_k$  plays in the stage of the structural system identification based on the acceleration output observations.

## EXPERIMENTAL MEASUREMENTS

Having demonstrated in the preceding section the effectiveness of extracting the direct term from the acceleration-output based representation through the computer simulations, this section consequently presents experimental measurements utilizing the model building depicted in **Fig.6**. In conducting experiments, the sampling rate  $\Delta$  is set to be 0.001 [sec] and the data number  $N$  is 1024. Except for the top floor, the plan of each floor of the model building is of symmetry, thus making the center of rigidity to coincide with the center of mass. By referring to **Fig.7**, it is found for the top floor the existence of three Accelerometers A, B and C and AMD produces the eccentricity between the centers of rigidity and mass. To prevent the occurrence of the mode coupling resulting from this eccentricity, an auxiliary or extra mass has been placed.

The following steps are taken for the experiment. (i) The first step is to apply a white-noise input excitation to the top floor through AMD driven by the actuator. Following the excitation, the discrete-time input force is obtained as  $u_k$  by multiplying by the mass of AMD the recorded data of the acceleration of AMD observed via Accelerometer C. (ii) Secondly, the measurements of the acceleration responses are made through Accelerometers A and B. The mean values of the data of Accelerometers A and B are regarded as the lateral output data whereas the torsional output data are obtained by dividing by the distance of Accelerometers A and B the difference between the data from Accelerometers A and B. When the velocity output is to be used for the regressive formulation, the data of acceleration are obtained through integral operations.

Since Model #1 is, as explained in the previous section, based on the implicit assumption of a white-noise input, the input excitation should qualify the white-noise requirement. In the present study, twelve experiments are performed using twelve samples of white-noise inputs. To examine the quality of white-noise, the normalized autocorrelation function for one of them, which is for the typical simulated sample input, is shown in **Fig.8**. The dashed lines in **Fig.8** indicate the 95 [%] confidence interval. This figure indicates that the input can be viewed as a white-noise.

Considering the behavior of the residual sum of error squares,  $n$  is taken 20. Following the measurements of twelve inputs and outputs, twelve kinds of the coefficients  $a_i$  and  $b_i$  are evaluated respectively for the lateral and torsional mode responses. Then the experimentally identified transfer functions for each mode response, to be presented in the following figures, are determined with the mean values of the coefficients. As well as the acceleration output-based formulation, the transfer functions based on the velocity outputs are also provided. **Figs.9** and **10** show the transfer functions of the lateral acceleration

and velocity responses, then representing a good agreement between the peak frequencies denoting the 1st to 4th natural frequencies. On the other hand, **Figs.11** and **12** show the experimentally identified functions of the torsional mode responses for the acceleration and velocity outputs, respectively.

To investigate the reason for such poor agreement, **Figs.13** to **16** present the coherence functions. From these figures it is found that the low coherences with respect to the torsional acceleration responses result in the unsatisfactory agreement.

## CONCLUSIONS

The investigation of this paper has discussed the structural identification for a building structure with AMD providing a white-noise input excitation. The identification technique employs the linear regressive representation based on the acceleration output, identifying the transfer functions. The technique pre-extracting the direct term from the formulation with the white-noise input can provide the satisfactory results.

## ACKNOWLEDGEMENT

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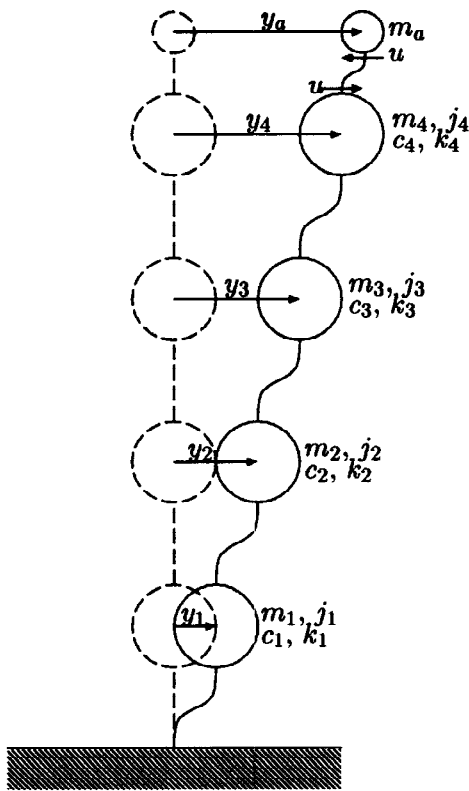


Fig. 1 Structural model

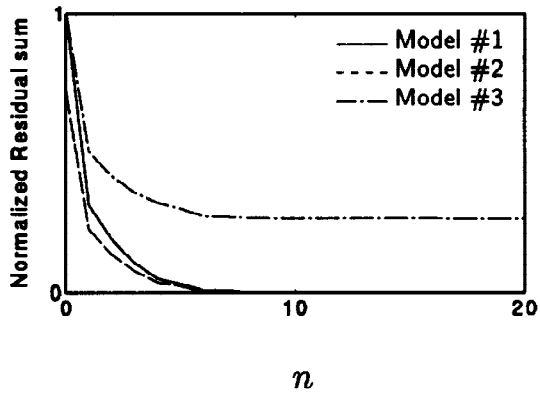


Fig. 2 Residual sum of squares (lateral acceleration)

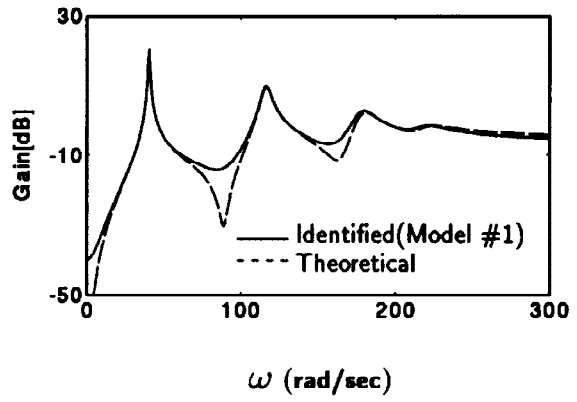


Fig. 3 Transfer function identified by Model #1

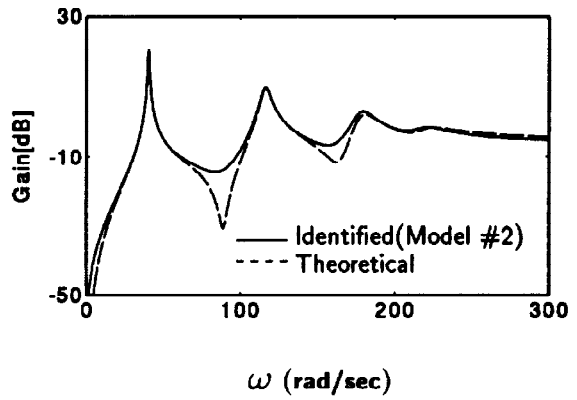


Fig. 4 Transfer function identified by Model #2

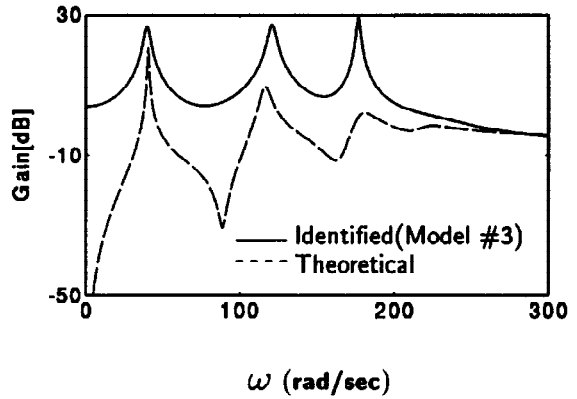
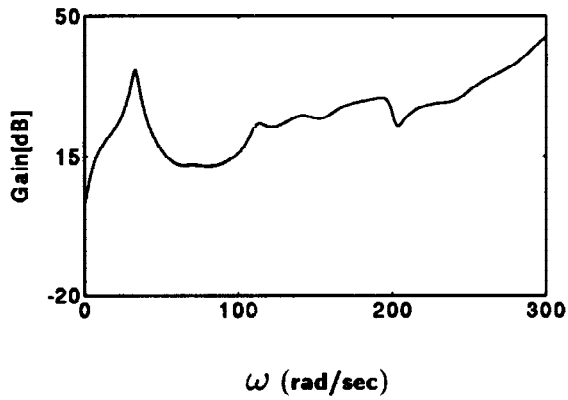
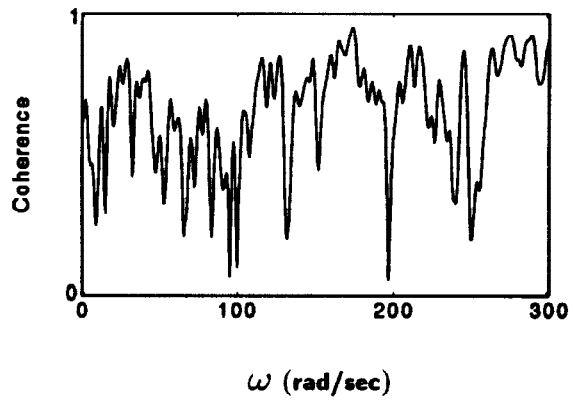


Fig. 5 Transfer function identified by Model #3

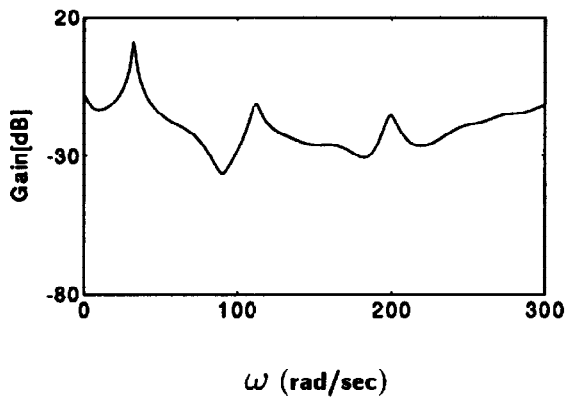




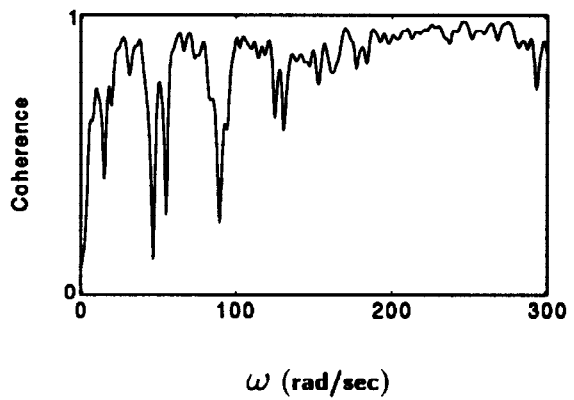
**Fig. 11 Experimentally identified transfer function with respect to torsional acceleration**



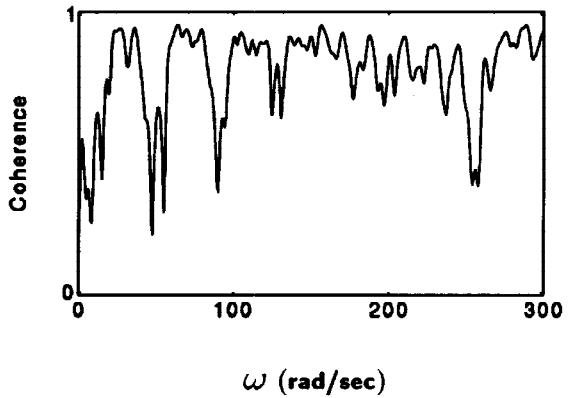
**Fig. 14 Coherence between a sample of input and torsional acceleration output**



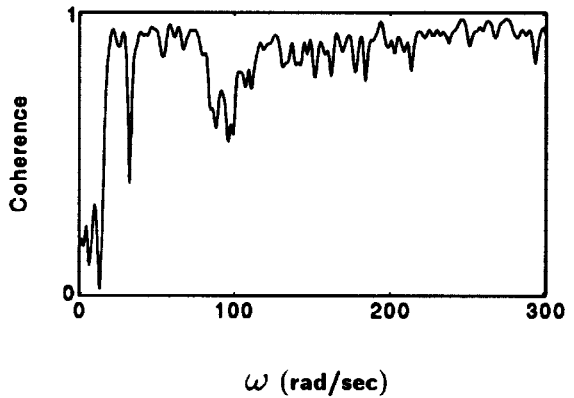
**Fig. 12 Experimentally identified transfer function with respect to the torsional velocity**



**Fig. 15 Coherence between a sample of input and lateral velocity output**



**Fig. 13 Coherence between a sample of input and the lateral acceleration output**



**Fig. 16 Coherence between a sample of input and torsional velocity output**