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Paper No. 955. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

RESPONSE STATISTICS AND RELIABILITY OF UNCERTAIN NONLINEAR SYSTEMS SUBJECTED TO RANDOM LOADS

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ABSTRACT

A probabilistic methodology is developed for calculating the response statistics and reliability of nonlinear structures with uncertain parameters, subject to seismic loads. Parametric uncertainties are modeled by random variables with prescribed probability density function. An asymptotic expansion is used to compute the probability integrals that arise in the evaluation of the overall statistics and failure probabilities in terms of conditional statistics and failure probabilities given the system parameters. A formulation is outlined for computing these conditional quantities for MDOF hysteretic systems subjected to loads that are modeled by filtered white-noise stochastic processes. The method is applied to 1-DOF and 2-DOF hysteretic structures in order to examine the accuracy of the proposed asymptotic expansion and to gain understanding of the effects of both structural and loading uncertainties on the structural response and reliability. The results are compared to simulated ones based on an importance sampling technique. Results from the second-order perturbation method are also included for comparison.

KEYWORDS

Uncertain dynamics, stochastic moments, structural reliability, random vibration, hysteretic structures

INTRODUCTION

The environmental loads, such as earthquake and wind loads, that a structure can experience during its lifetime are highly uncertain time varying loads that can best be modeled by stochastic processes. The response statistics and reliability of a structure can then be obtained using well-known techniques, usually approximate, from random vibration theory (Soong and Grigoriou, 1993). Recent studies (Spencer and Elishakoff, 1988, Cherng and Wen, 1994) have shown that uncertainties in the parameters of the structural models may also significantly influence the response predictions and structural reliability. The statistics and failure probability of structures with uncertain parameters can be computed by the total probability theorem as integrals over all uncertain parameters. These parameter uncertainties are quantified and incorporated in the analysis by modeling them as random variables with a prescribed probability density function.

The evaluation of the resulting multi-dimensional integrals requires a large number of repeated random vibration analyses for different values of the system parameters. The second-order perturbation method is computationally the least expensive method for computing approximately the response moments of uncertain nonlinear dynamic systems. However, it works well only for limited cases and for relatively small levels of uncertainties. Often it fails to give satisfactory and consistent results, such as in the case of linear primary-secondary systems, even if the level of uncertainties is small (Papadimitriou *et al.*, 1995). Approximate methods for computing structural reliability for uncertain dynamic systems subjected to stochastic loads are based on first-order (FORM) and second-order (SORM) reliability methods (Igusa and Der Kiureghian, 1988, Cherng and Wen, 1994).

A new asymptotic method is presented for evaluating the type of probability integrals encountered in the analysis of response statistics and reliabilities of uncertain nonlinear systems subject to stochastic excitations. The methodology applies when the conditional statistics or reliabilities of response quantities are available for given values of the uncertain system parameters. For example, they can be obtained from conventional random vibration theories. The methodology is applied to a general class of hysteretic systems subjected to earthquake-like excitation models. The stochastic equivalent linearization method is used as the solution technique. The accuracy and efficiency of the method is successfully illustrated by application to 1-DOF and 2-DOF hysteretic structures. To provide a basis for evaluating this accuracy, accurate numerical solutions for the failure probabilities and response moments are computed by an importance sampling technique developed for these type of integrals (Papadimitriou *et al.*, 1995).

UNCERTAIN DYNAMIC SYSTEMS

The present work deals with two sources of uncertainties. The first is time-invariant system uncertainties that can be modeled by random variables, designated by the vector $\underline{\theta}$. Examples includes uncertainties in stiffness, mass, and damping matrices of structural models, as well as member capacities, yield strengths, etc. The joint probability density function $p(\underline{\theta})$ of the random vector $\underline{\theta} \in \Theta$ indicates the relative plausibilities of the possible values of the uncertain parameters in the set Θ. The second source of uncertainties is the loading time histories which can be modeled as stochastic processes.

Let I denote a probabilistic response descriptor. Examples of response descriptors I include, but are not limited to, the response statistics and the failure probability defined herein as the probability of exceeding a specified level. Let $h(\underline{\theta}) = I \mid \underline{\theta}$ denote the conditional statistics or conditional probability of failure of a structure for a given value of $\underline{\theta}$. Using the total probability theorem, the overall statistics or failure probability I considering the uncertainties in the structural parameters $\underline{\theta}$ is given in the integral form

$$I = \int_{\Theta} h(\underline{\theta})\ p(\underline{\theta})\ d\underline{\theta} \tag{1}$$

Rarely, if ever, can one integrate (1) analytically. Numerical integration can be very costly and usually unaffordable for more than a few variables. Simulation methods may also require a very large number of integrand evaluations in order to get accurate results. Each integrand evaluation requires $h(\underline{\theta})$ to be calculated for some $\underline{\theta}$ value, and this often requires a computationally expensive structural analysis. Next, methods for approximately computing the probability of failure and the response moments of structures are presented based on the approximation of the integral (1). In particular, a new asymptotic formula is outlined for approximating the integral (1) which requires only a small number of $h(\underline{\theta})$ function evaluations. Then methods for calculating $h(\underline{\theta})$ for general hysteretic systems and general stochastic loading models are presented.

PROBABILITY INTEGRAL APPROXIMATIONS

The second-order perturbation method offers an approximation of the integral (1) with minimal computational effort. It is based on expanding $h(\underline{\theta})$ into a Taylor series about the mean $\overline{\underline{\theta}}$ of $\underline{\theta}$. As it will be seen in the applications, the second-order perturbation method suffers from inaccuracy, especially when applied to the computation of the total failure probability. A more accurate and yet computationally efficient approximation of the integral (1) is based on an expansion of the logarithm of the integrand about the point that corresponds to the maximum of the integrand. It is assumed that the integral has a single global maximum $\underline{\theta}^*$ which is the case in a number of applications. The idea is to rewrite the integral in the form

$$I = \int_{\Theta} \exp[\ell(\underline{\theta})] \; d\underline{\theta} \tag{2}$$

where $\ell(\underline{\theta}) = \ln h(\underline{\theta}) + \ln \; p(\underline{\theta})$, and expand $\ell(\underline{\theta})$ about the point $\underline{\theta}^*$ which also maximizes $\ell(\underline{\theta})$. The value of $\underline{\theta}^*$ is obtained by applying a standard minimization algorithm to $-\ell(\underline{\theta})$. Expanding $\ell(\underline{\theta})$ about $\underline{\theta}^*$ and applying Laplace's method of asymptotic expansion to the integral (Bleistein and Handelsman 1986), the following asymptotic approximation for I is obtained (Papadimitriou *et al.*, 1995)

$$I(\underline{\theta}^*) \sim (2\pi)^{n/2} h(\underline{\theta}^*) \; p(\underline{\theta}^*) \; / \sqrt{\det[\mathbf{L}(\underline{\theta}^*)]} \tag{3}$$

where $\mathbf{L}(\underline{\theta})$ is the Hessian matrix of $-\ell(\underline{\theta})$ with the (i,j) component given by $L_{ij}(\underline{\theta}) = -\partial^2 \ell(\underline{\theta})/\partial\theta_i \partial\theta_j$. The expansion is valid for $\lambda > 0$, where $\lambda = \min_i[\lambda_i(\mathbf{L}(\underline{\theta}^*))]$, and $\lambda_i(\mathbf{L}(\underline{\theta}^*))$ is the i-th eigenvalue of the Hessian matrix $\mathbf{L}(\underline{\theta})$ evaluated at $\underline{\theta}^*$. Furthermore, the approximation is asymptotically correct as $\lambda \rightarrow \infty$. Specifically, the larger the value of λ, the sharper the peak of the integrand at $\underline{\theta}^*$ and therefore the more accurate the value of the asymptotic approximation is expected to be. The method is computationally much less intense than simulations which, in order to yield satisfactory results, require a significantly larger number of random vibration analyses conditional on $\underline{\theta}$ to be performed. Note that when used to compute failure probabilities, the new asymptotic method is simpler than existing second-order reliability methods developed to treat these types of problems (Cherng and Wen, 1994, Papadimitriou *et al.*, 1995).

The accuracy of the estimate I in (3) can be improved by using simulation methods. In particular, importance sampling (Schueller and Stix, 1987) offers advantages in efficiently computing the reliability integrals using a significantly smaller number of simulations than that used in the straightforward Monte-Carlo simulation. The reader is referred to the work by Papadimitriou *et al.* (1995) for details about the application of the importance sampling method on the type of probability integral (1) used herein. The importance sampling technique proves to be the most efficient way to obtain accurate numerical solutions. On the other hand, the proposed asymptotic method usually gives acceptable accuracy with much less computation.

APPLICATION TO HYSTERETIC STRUCTURES

For large structures and several uncertain parameters, the computation of $h(\underline{\theta})$ required in (2) or (3) can be excessive if it is based on simulations since it requires a large number of response analyses corresponding to different realizations of the stochastic excitation process. Analytical results from classical random vibration theory provide a convenient alternative for performing relatively inexpensive computations of the conditional quantities $h(\underline{\theta})$. More specifically, consider an N-DOF hysteretic structure subjected to a stochastic excitation process $\mathbf{f}_e(t)$. The equation of motion is

$$\mathbf{M}_s \; \ddot{\mathbf{x}}_s \; (t) + \mathbf{C}_s \; \dot{\mathbf{x}}_s \; (t) + \mathbf{K}_s \mathbf{x}_s(t) + \mathbf{R}_s \mathbf{z}_s(t) = \hat{\mathbf{B}}_s \; \mathbf{f}_e(t) \tag{4}$$

where $\mathbf{x}_s$ is the generalized displacement vector, $\mathbf{M}_s = \mathbf{M}_s(\underline{\theta}_s)$, $\mathbf{C}_s = \mathbf{C}_s(\underline{\theta}_s)$, $\mathbf{K}_s = \mathbf{K}_s(\underline{\theta}_s)$, $\hat{\mathbf{B}}_s = \hat{\mathbf{B}}_s(\underline{\theta}_s)$ are the mass, damping, stiffness and input matrices, $\mathbf{R}_s = \mathbf{R}_s(\underline{\theta}_s)$ is a transformation matrix, and $\mathbf{z}_s = \mathbf{z}_s(\mathbf{x}_s, \dot{\mathbf{x}}_s, \underline{\theta}_s)$ accounts for the hysteretic nonlinearities assumed to be a function of $\mathbf{x}_s$ and $\dot{\mathbf{x}}_s$. The set $\underline{\theta}_s$ include all uncertain structural parameters and it is a subset of the set $\underline{\theta}$.

The statistical equivalent linearization method (Roberts and Spanos, 1990) can be used to obtain the evolution of the response covariance. According to this method the nonlinear system is replaced by an equivalent linear one with parameters obtained by minimizing the mean-square error between the nonlinear and the equivalent linear system. Carrying out the minimization and approximating the non-Gaussian response process by a Gaussian one, the vector $\mathbf{z}_s(t)$ in (5) takes the linearized form

$$\dot{\mathbf{z}}_s\,(t) = \mathbf{C}_h(\mathbf{Q})\,\dot{\mathbf{x}}_s\,(t) + \mathbf{K}_h(\mathbf{Q})\,\mathbf{z}_s(t) \tag{5}$$

where $\mathbf{C}_h(\mathbf{Q})$ and $\mathbf{K}_h(\mathbf{Q})$ are matrices which depend on the second-order statistics of the response. These statistics are components of the matrix $\mathbf{Q}$ which is defined later. These statistics are obtained by solving the nonlinear equation for the covariance response. To enable an analysis of the covariance response, the state vector $\mathbf{y}_s = (\mathbf{x}_s^T, \dot{\mathbf{x}}_s^T, \mathbf{z}_s^T)^T$ is introduced and the equations (4) and (5) are conveniently written in the state space form

$$\dot{\mathbf{y}}_s(t) + \mathbf{A}_s(\mathbf{Q})\,\mathbf{y}_s(t) = \mathbf{B}_s\,\mathbf{f}_e(t) \tag{6}$$

where

$$\mathbf{A}_s(\mathbf{Q}) = \begin{bmatrix} \mathbf{0}_{nn} & -\mathbf{I}_{nn} & \mathbf{0}_{nn} \\ \mathbf{M}_s^{-1}\,\mathbf{K}_s & \mathbf{M}_s^{-1}\,\mathbf{C}_s & \mathbf{M}_s^{-1}\,\mathbf{R}_s \\ \mathbf{0}_{nn} & -\mathbf{C}_h(\mathbf{Q}) & -\mathbf{K}_h(\mathbf{Q}) \end{bmatrix} \quad \text{and} \quad \mathbf{B}_s = \begin{bmatrix} \mathbf{0}_{np} \\ \mathbf{M}_s^{-1}\,\hat{\mathbf{B}}_s \\ \mathbf{0}_{np} \end{bmatrix} \tag{7}$$

Usually, one is interested in computing a response quantity $r(t) = \mathbf{D}_s\,\mathbf{y}_s(t)$, where $\mathbf{D}_s = \mathbf{D}_s(\underline{\theta}_s)$ is a given matrix.

The excitation is assumed to be a filtered white-noise stochastic process. These processes have been used in the past to model both earthquake (Beck and Papadimitriou, 1993) and wind excitations. Specifically, $\mathbf{f}_e(t)$ is given by $\mathbf{f}_e(t) = \mathbf{D}_e\,\mathbf{y}_e(t)$, where $\mathbf{y}_e(t)$ satisfies the filter equation

$$\dot{\mathbf{y}}_e(t) + \mathbf{A}_e\,\mathbf{y}_e(t) = \mathbf{B}_e\,\mathbf{w}_e(t) \tag{8}$$

in which $\mathbf{w}(t)$ is a nonstationary white-noise vector process. $\mathbf{A}_e = \mathbf{A}_e(\underline{\theta}_e)$ and $\mathbf{B}_e = \mathbf{B}_e(\underline{\theta}_e)$ are given filter matrices which depend on a set $\underline{\theta}_e$ of filter parameters which may also be treated as uncertain in order to reflect parametric uncertainties in the excitation process.

The state space formulation for the combined structure-excitation system can be derived by introducing the state vector $\mathbf{y}^T = (\mathbf{y}_s^T, \mathbf{y}_e^T)$. The following equations for $\mathbf{y}$ are readily derived

$$\dot{\mathbf{y}}\,(t) + \mathbf{A}(\underline{\theta})\,\mathbf{y}(t) = \mathbf{B}(\underline{\theta})\;\mathbf{w}_e(t) \tag{9}$$

where

$$\mathbf{A}(\underline{\theta}) = \begin{bmatrix} \mathbf{A}_s(\mathbf{Q}) & -\mathbf{B}_s\,\mathbf{D}_e \\ \mathbf{0}_{pn} & \mathbf{A}_e \end{bmatrix} \quad \text{and} \quad \mathbf{B}(\underline{\theta}) = \begin{bmatrix} \mathbf{0}_{nq} \\ \mathbf{B}_e \end{bmatrix} \tag{10}$$

The vector $\underline{\theta}^T = (\underline{\theta}_s^T, \underline{\theta}_e^T)$ includes uncertainties in both the system and excitation parameters. Note that $\mathbf{A}$ depends on $\mathbf{Q}$ and $\underline{\theta}$ in the following way: $\mathbf{A} \equiv \mathbf{A}(\mathbf{Q}(\underline{\theta}), \underline{\theta})$. It can be shown that the conditional covariance $\mathbf{Q}(t) = E[\mathbf{y}(\mathbf{t})\mathbf{y}^T(t)]$ of the response $\mathbf{y}(t)$ given $\underline{\theta}$ satisfies the nonlinear matrix equation

$$\dot{\mathbf{Q}}\,(t) + \mathbf{A}(\underline{\theta})\;\mathbf{Q}(t) + \mathbf{Q}(t)\;\mathbf{A}^T(\underline{\theta}) = 2\pi\mathbf{B}(\underline{\theta})\;\mathbf{S}_e(t)\;\mathbf{B}^T(\underline{\theta}) \tag{11}$$

where it was taken that $E[\mathbf{w}(t)\mathbf{w}^T(t-\tau)] = 2\pi\mathbf{S}_e(t)\delta(\tau)$.

The conditional second-order statistics of $r(t, \underline{\theta})$ given $\underline{\theta}$, designated by $h(\underline{\theta})$ in (1), can readily be computed from the elements of the matrix $\mathbf{Q}(t)$. Then the overall second-order statistics including parameter uncertainties are computed approximately by (3). One can proceed further and obtain the probability of failure using approximate extreme response theories. Failure occurs when the response $r(t, \underline{\theta})$ reaches level b for the first time. The probability that a portion of duration T of the response

$r(t,\underline{\theta})$ will exceed the level b for the given $\underline{\theta}$, can be obtained using available results from random vibration theory as follows:

$$h(\underline{\theta}) = Pr(q(t) \geq b \mid \underline{\theta}) = 1 - \exp\left(-\int_0^T a(\underline{\theta}, b, t)\, dt\right) \tag{12}$$

Assuming that the response is Gaussian and that the events of crossings a level b are independent, $a(\underline{\theta}, b, t)$ is approximated by $a(\underline{\theta}, b, t) = 2\nu^+(\underline{\theta}, b, t)$, where $\nu^+(\underline{\theta}, q, t)$ is the expected level of upcrossings of level b at any time t. For illustration, only the case of stationary response will be presented for which $\nu^+(\underline{\theta}, b, t) = \nu^+(\underline{\theta}, b)$ is independent of t, given by

$$\nu^+(\underline{\theta}, b) = \left[\frac{\sqrt{\sigma_{\dot{r}}(\underline{\theta})}}{2\pi\sqrt{\sigma_r(\underline{\theta})}}\right] \exp\left[\frac{-b^2}{2\sigma_r(\underline{\theta})}\right] \tag{13}$$

where $\sigma_r(\underline{\theta})$ and $\sigma_{\dot{r}}(\underline{\theta})$ are the conditional second moments of response $r(t,\underline{\theta})$ and its derivative for a given $\underline{\theta}$. The total failure probability including parameter uncertainties is computed from (3) with $h(\underline{\theta})$ given by (12).

ILLUSTRATIVE EXAMPLES

The accuracy of the asymptotic expansion is investigated by computing the failure probabilities of a single DOF hysteretic structure, and the second-order response moments and failure probabilities of a two DOF hysteretic primary - linear secondary system, when both are subjected to a stationary Gaussian white-noise base excitation with spectral density S.

The hysteretic model proposed by Wen (1976) is adopted. The nonlinearity is introduced by an auxiliary variable z and parameters A, n, β and γ that control the shape of the hysteretic loop. It is more informative to write equations of motion of the SDOF oscillator in terms of the ductility $\eta = x/x_y$, where x_y is the yield displacement, given by $x_y = [A/(\beta+\gamma)]^{1/n}/A$. Defining a more convenient re-parameterization in terms of $\beta_s = \beta x_y^n$, $\gamma_s = \gamma x_y^n$, $\omega = \sqrt{k/m}$, $\zeta = c/(2m\omega)$, the equation of motion of a single DOF structure is

$$\ddot{\eta} + 2\zeta\omega\,\dot{\eta} + \alpha\omega^2\eta + (1-\alpha)\omega^2 z_s = \frac{1}{x_y} w(t) \tag{14}$$

where $\alpha\omega^2\eta$ is the linear part of the restoring force, $(1-\alpha)\omega^2 z_s$ is the nonlinear hysteretic part, $w(t) = \ddot{a}(t)/g$ is the ground acceleration measured in g's, g is the accelaration of gravity, and the normalized auxiliary variable $z_s = z/x_y$ satisfies the differential equation

$$\dot{z}_s = A\,\dot{\mu} - \beta_s \left|\dot{\mu}\right| |z_s|^{n-1} z_s - \gamma_s\,\dot{\mu}\,|z_s|^n \tag{15}$$

The system can be completely described by the following quantities: ω, ζ, A, n, x_y and β_s or, equivalently, γ_s since $\beta_s + \gamma_s = 1/A^{n-1}$. Assuming that $\beta_s = \gamma_s$, then $\beta_s = \gamma_s = 1/(2A^{n-1})$ and thus the parameters β_s and γ_s are not explicitly involved in the description of the system. Assuming in addition that $n = 1$, one obtains that $\beta_s = \gamma_s = 0.5$.

The response $r(t,\underline{\theta})$ is chosen to be the displacement of the mass of the hysteretic single DOF system. Uncertainties in the natural frequency ω, damping ratio ζ and excitation spectral density S are considered. These uncertainties are described by lognormal distributions with most probable values (MPV) given by $\underline{\mu}_\theta = \{\mu_\omega, \mu_\zeta, \mu_S\} = \{2\pi, 0.05, 1.0\}$. The level of uncertainty for a variable θ is measured by the ratio σ_θ/μ_θ, where σ_θ denotes the standard deviation (STD) of θ. The threshold level b in (13) is chosen to be a multiple of the standard deviation $\sigma_r(\underline{\mu}_\theta)$ of the response $r(t,\underline{\theta})$ computed at the most probable values $\underline{\mu}_\theta$ of the system parameters, i.e. $b = d\sqrt{2}\sigma_r(\underline{\mu}_\theta)$, where d denotes a normalized measure of the threshold level. The duration T of the response is taken to be $T = 10\ (2\pi/\mu_\omega)$. Results are computed

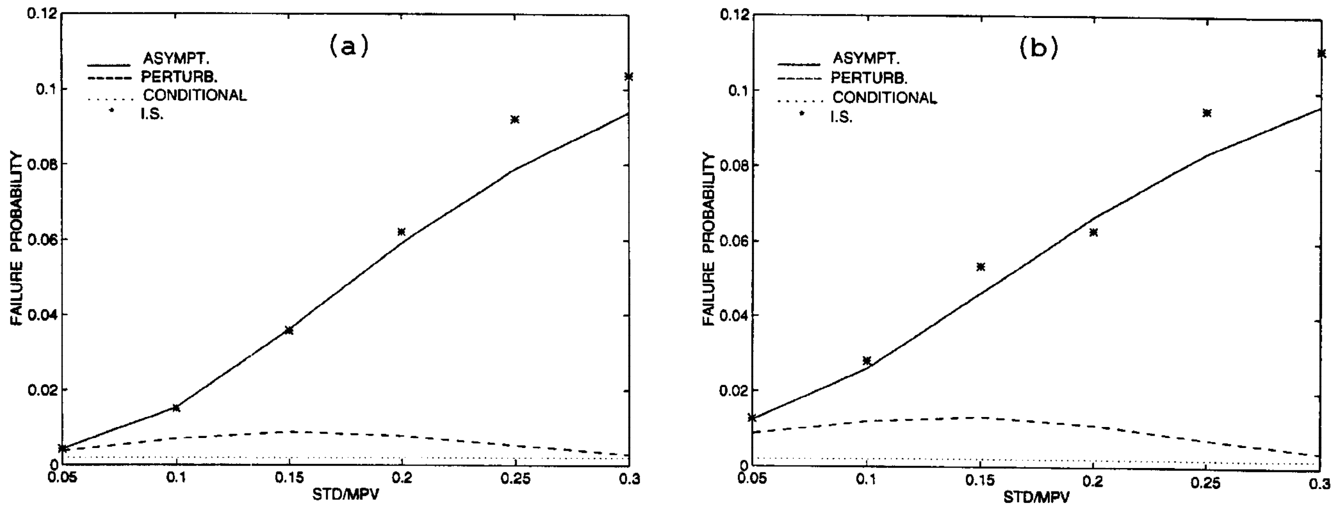

Fig. 1. Failure probability for the 1-DOF system ($d = 3$); (a) uncertain ω, (b) uncertain ω, ζ and S.

for $A = 1$ and $x_y = \rho g/\mu_\omega^2$, where $\rho = \mu_k x_y/(mg) = 8$ is a nondimensional measure of the most probable ultimate resistance, and μ_k is the most probable value of the stiffness k.

Figure 1(a) shows the failure probabilities as a function of the ratio σ_ω/μ_ω (STD/MPV) for the case of uncertain ω with the other parameters fixed at their most probable values. Figure 1(b) shows the failure probabilities as a function of the ratio σ_ω/μ_ω for the case where all three parameters ω, ζ and S are uncertain. The standard deviations of the parameters ζ and S are fixed at $\sigma_\zeta = 0.3\mu_\zeta$ and $\sigma_S = 0.2\mu_S$, respectively. All results in Figure 1 correspond to a normalized threshold level $d = 3$. Table 1 gives the failure probabilities as a function of the normalized threshold level d for the particular case of Figure 1(b) with $\sigma_\omega/\mu_\omega = 0.2$. For comparison purposes, the results obtained from the second-order perturbation method, and importance sampling (I.S.) are also included in Figure 1 and Table 1. Only 100 samples are used in Figure 1 for the importance sampling technique.

The second-order perturbation method performs well only for the case of one uncertain parameter and small levels of uncertainty. For the case of three uncertain parameters shown in Figure 1(b), the perturbation method performs poorly even for small levels of uncertainties. In all cases, the asymptotic method performs acceptably well even if the level of uncertainties are relatively large. From Table 1, the accuracy of the perturbation results deteriorates significantly as the threshold level increases. For $d = 4.0$, for example, it underestimates the failure probability by three to four orders of magnitude. Such deterioration is not observed for the asymptotic method. Comparing the failure probabilities to the conditional failure $F(\underline{\mu}_\theta)$ in Table 1 and Figure 1, it is clear that uncertainties are very important since they can change the failure probabilities by orders of magnitude. Also, studies have shown that the uncertainty in the frequency is much more important than the uncertainties in the damping and input power spectal density.

The 2-DOF system consists of a linear secondary oscillator attached to a hysteretic primary oscillator. The masses of the oscillators are assumed to be deterministic and the mass ratio $\varepsilon = m_s/m_p$ is taken to be $\varepsilon = 0.01$. The uncertain parameters are chosen to be the natural frequencies ω_p and ω_s and the damping ratio ζ_p and ζ_s of the two oscillators. Note that the subscripts p and s are used to denote properties of the primary and secondary oscillators, respectively. All uncertainties are modeled by lognormal variables which are assumed to be independent. The following most probable values are chosen for the system parameters: $\mu_{\zeta_p} = 5\%$, $\mu_{\zeta_s} = 2\%$, $\mu_{\omega_p} = 1$ and $\mu_{\omega_s} = \lambda\ \mu_{\omega_p}$. The ratios of the standard deviations to the most probable values are chosen to be 0.1 for the frequencies and 0.25 for the damping ratios. The response quantity of interest is the restoring force per unit secondary mass,

Table 1. Failure probabilities ($\times 10^{-4}$) for different threshold levels; 1-DOF hysteretic system; Uncertain ω, ζ and S

Threshold $\times\sqrt{2}\,\sigma_r(\underline{\mu}_\theta)$	**Condit.** $F(\mu_\theta)$	**Asympt.**	**Perturb.**	**I. S.** 10^2 **samples**	**I. S.** 10^3 **samples**
3.0	20.12	260.3	119.9	291.5	274.2
3.5	0.78	66.87	7.81	74.60	70.06
4.0	0.018	17.01	0.0287	18.90	17.75

$r(t,\underline{\theta}) = \omega_s{}^2[x_s(t) - x_p(t)]$, of the spring connecting the secondary mass m_s to the primary mass m_p. The following three cases, namely A, B and C, are considered. Case A corresponds to uncertainties in the frequency ratios ω_p and ω_s; case B corresponds to uncertainties in the frequencies and damping ratios ω_p, ω_s, ζ_p and ζ_s; and case C corresponds to uncertainties in A, b, S and T as well as in ω_p, ω_s, ζ_p and ζ_s. For case C, the uncertainties in A, b, S and T are modeled by independent lognormal variables with most probable values $\mu_A = 1.0$, $\mu_b = d\sqrt{2}\sigma_r(\underline{\mu}_\theta)$, $\mu_S = 1$ and $\mu_T = 10\ (2\pi/\mu_{\omega_s})$. The standard deviations are chosen as: $\sigma_A = 0.2\mu_A$, $\sigma_b = 0.1\mu_b$, $\sigma_S = 0.2\mu_S$, and $\sigma_T = 0.2\mu_T$.

The overall stationary mean-square response (MSR) $E[r^2]$, normalized by the conditional MSR $\sigma_r^2(\underline{\mu}_\theta) = E[r^2|\,\underline{\mu}_\theta]$, is plotted for the cases A and B in Figures 2(a) and 2(b), respectively, for $\rho = 10$ and for values of the frequency ratio $\lambda = \mu_{\omega_s}/\mu_{\omega_p}$ ranging from 0.5 to 2.0. Results corresponding to a linear primary structure are also shown. Note that for $\mu_{\omega_s}/\mu_{\omega_p} = 1$ the perturbation method gives unrealistic negative values for the MSR of the linear 2-DOF system. The asymptotic expansion gives better and more reliable estimates of the variance over the whole range of variation of the frequency ratio $\mu_{\omega_s}/\mu_{\omega_p}$. The deviation of the results shown in Figure 2 from unity is directly related to the effect of the uncertainties on the MSR. From the large values in Figure 2, and the similarity of Figures 2(a) and 2(b), it can be seen that the MSR is very sensitive to uncertainties in the frequencies but less sensitive to uncertainties in the damping ratios for nearly tuned conditions. Neglecting the uncertainties in the frequencies will give highly unconservative results, especially for the case of the 2-DOF linear system. Under perfectly tuned conditions, the MSR is lower than that of the deterministic system which means that analyses based on the most probable system will be conservative. A similar qualitative behavior was also computed for the failure probabilities. Finally, the failure probabilities for the case C ($d = 4$) obtained from the asymptotic expansion, importance sampling and perturbation method are 1.9×10^{-2}, 2.3×10^{-2} (4000 samples) and 2.0×10^{-4}, respectively, which again show that the asymptotic expansion performs well while the perturbation method gives values which can be orders of magnitude different from the exact ones. For comparison, the value of the conditional failure probability is 2.5×10^{-6}.

CONCLUSIONS

A new asymptotic approximation for probability integrals of the type arising in the study of uncertain dynamic systems is introduced and then applied to compute the statistics and failure probability of MDOF hysteretic structures subjected to random loads. The asymptotic method performs satisfactorily and can be used to draw reliable qualitative conclusions about the behavior of uncertain linear and nonlinear hysteretic structures for both the response moments and failure probabilities. In contrast, the perturbation method performs poorly, especially when used to predict failure probabilities. Numerical studies on simple structures show that the structural uncertainties can have a substantial effect on both the statistics and the reliability of dynamic systems excited by stochastic loads. The effects of structural uncertainties are more pronounced for some critical structural configurations, such as primary-secondary systems with nearly tuned conditions. The procedure described is general and can be extended to handle geometrically nonlinear systems subject to time-varying loads that are modelled by non-white Gaussian and non-Gaussian stochastic processes.

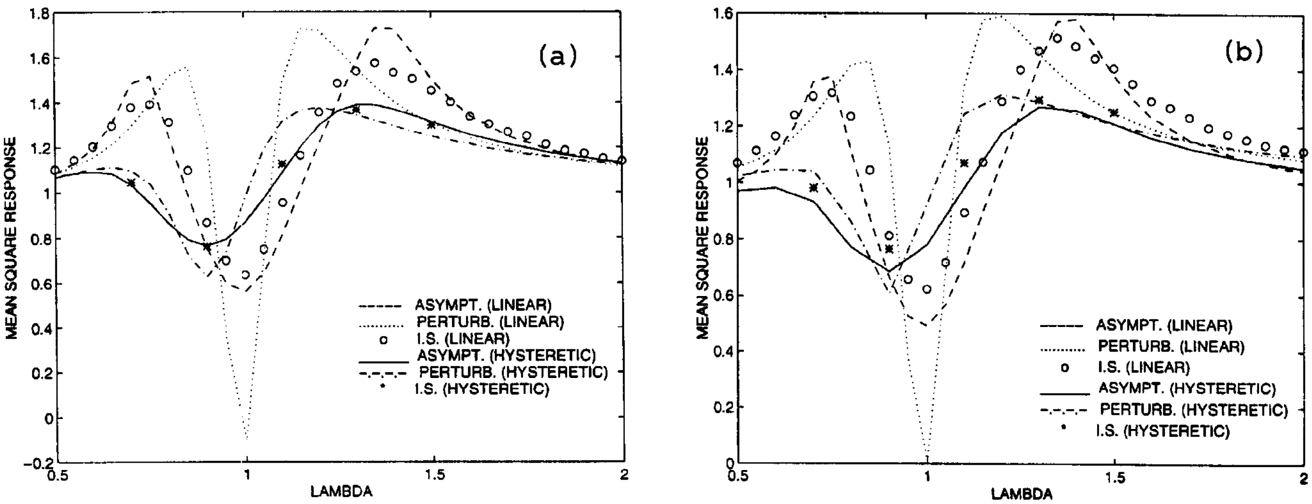

Fig. 2. Normalized mean-square response for the two DOF primary-secondary system; (a) Case A: uncertain ω_p and ω_s, (b) Case B: uncertain ω_p, ω_s , ζ_p and ζ_s.

ACKNOWLEDGMENT

This paper is based upon work partly supported by the National Science Foundation under grant BCS-9309149 and by the Hong Kong UPGC Research Infrastructure Grant, RIG 94/95.EG02. This support is gratefully acknowledged. The authors are grateful to Professor J.L. Beck for constructive comments. The assistance given by the graduate student Au Siu Kui for the numerical applications is appreciated.

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