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"SOIL-STRUCTURE-HALF SPACE" MODEL FOR THE DYNAMIC ANALYSIS OF STRUCTURES

J.L. Urrutia-Galicia*, M. Arroyo** and C. Vázquez*

*Engineering Institute, National Autonomous University of Mexico,
Ciudad Universitaria, 04510 Mexico.
**Graduate Studies Division, Engineering Faculty, UNAM, 04110, Mexico.

ABSTRACT

In this work, a semidefinite ***"Soil-Structure-Half space model"*** is proposed for the seismic analysis of buildings. The model allows us first, to input the seismic signal to a mobile Half Space with the soil deposits on top and the excitation is provided from an accelerogram recorded on firm ground (on the **UNAM** campus). The response spectrum is then compared with a real spectrum recorded on free ground. Once the signal is calibrated, the seismic analysis is performed with the ***"Soil-Structure-Half space model"*** complete which leads to novel and new points of view of the up to now simply termed Soil-Structure interaction problem. The final validation of the semidefinite model proposed will be the objective of further research on both the theoretical and the experimental grounds.

KEYWORDS

Soil-Structure-Half Space Interaction model; seismic analysis; forced vibrations; signal filtering; response spectrum; frequency density.

INTRODUCTION

One of the most vexing problems in seismic engineering is the dynamic coupling of a structure and the soil beneath it. However, the soil itself is also resting on a bedrock (a half space) that is the one that actually transmits the seismic signal to the soil-structure interface. In this paper a semidefinite model will be introduced that allows us to perform accurate dynamic analyses that in turn will shed light upon many important aspects of the, up to now, simply termed the *"Soil-structure interaction problem"* that does not mention how and what would be the role of a moving bedrock (the half-space) transmiting seismic induced motion. It will also be shown that the ***"Soil-Structure-Half space model"*** is worked out as a single semidefinite structure with the half space, the soil and the structure dynamically coupled. Once this model is set up, then the analysis of the modes and frequencies of the complete model can be analyzed and discussed in relation to the frequency density[1 and 2] of the rigidly fixed building and the frequency changes that occur when the soil and the half space are included in the analysis. With regard to the seismic excitation it is a common practice to use seismic records like that of the **SCT** accelerogram which is assigned to each and every one of the building's masses. In this work the authors propose to adopt as the prymary seismic signal the one obtained on september 19th of 1985 at the National Autonomous University of Mexico at the front yard of the Engineering Institute, that is located on firm rock and which from now on will be termed the **UNAM** signal. This **UNAM** signal will be applied to a theoretical **Half-space** with a soil deposit of 42.3 m. that simulates similar local conditions at the **SCT** Ministry of Communications and Transportation. The objective of this analysis was to find the actual signal that applied to the moving bedrock would give the spectrum at the top of the soil deposit that matched the one obtained at **SCT** on september 19, 1985. Once the correct seismic signal was found then a seismic analysis was done of the Half-space, the soil deposit and a building joined together.

MATHEMATICAL MODEL

We shall not dwell on reviewing the details of the theory of vibrations of semidefinite systems, for that purpose the reader is invited to review reference [3] by the first author.

In general any system positive definite or semidefinite is governed by the following two differential equations, for the participation factors $\eta_0(t)$ for the rigid body motion:

$$\ddot{\eta}_0(t) + C\dot{\eta}_0(t) = F_0(t) \tag{1}$$

and for the rest of the modes:

$$\ddot{\eta}_k(t) + 2\xi_k\omega_k\dot{\eta}_k(t) + \omega_k^2\eta_k(t) = F_k(t) \tag{2}$$

When the direct and the inverse Lapalce transforms are applied to the two equations for harmonic forcing functions $\sin(\omega t)$ and $\cos(\omega t)$ including the case for $\omega=0$ (the rigid body motion), the following participation factors (or time dependent Fourier coeficients) were obtained, a.) for the rigid body motion $\eta_0(t)$ for constant excitation of amplitude A_0:

$$\eta_{0A_0}(t) = A_0\left[\frac{t}{c} - \frac{1}{c^2} + \frac{1}{c^2}\exp^{-ct}\right] \qquad \textbf{(3-a)}$$

for a forcing force $\sin(\omega t)$, the participation factor will be:

$$\eta_{0\sin}(t) = \frac{-c}{\omega(c^2+\omega^2)}\cos(\omega t) - \frac{1}{(c^2+\omega^2)}\sin(\omega t) + \frac{1}{c\omega} - \frac{\omega}{c(c^2+\omega^2)}\exp^{-ct} \qquad \textbf{(3-b)}$$

for a forcing force $\cos(\omega t)$, the participation function leads to:

$$\eta_{0\cos}(t) = \frac{-1}{c^2+\omega^2}\cos(\omega t) + \frac{c}{\omega(c^2+\omega^2)}\sin(\omega t) + \frac{1}{c^2+\omega^2}\exp^{-ct} \qquad \textbf{(3-c)}$$

b.) for modes different from the rigid body motion and for a constant forcing of amplitude A_0, the participation factor will be:

$$\eta_{kA_0}(t) = \left[\sin(\phi_k) - \exp^{\xi_k\omega_k t}\sin(\gamma_k t+\phi_k)\right]\frac{A_0}{\omega_k\gamma_k} \qquad \textbf{(4-a)}$$

with ϕ_k given by:

$$\phi_k = \tan^{-1}\frac{\gamma_k}{\xi_k\omega_k}$$

for the cosine forces $\cos(\omega t)$ we find:

$$\eta_{kcos}(t)=\frac{1}{A}\Big[\ \frac{\sin(\phi_k)}{\gamma_k}\big[\xi_k\omega_k\cos(\omega t)\ +\ \omega\ \sin(\omega t)\big]\ -\ \cos(\phi_k)\ \cos(\omega t)$$
$$+\ \frac{\exp^{-\xi_k\omega_k t}}{\gamma_k}\big[\gamma_k\cos(\gamma_k t\ +\ \phi_k)-\xi_k\omega_k\ \sin(\gamma_k t+\phi_k)\big]\ \Big] \tag{4-b}$$

with **A** and ϕ_k given by:

$$A\ =\ \omega_k^2\left[\left(2\xi_k^2-1+\frac{\omega^2}{\omega_k^2}\right)^2+\left(2\frac{\xi_k\gamma_k}{\omega_k}\right)^2\right]^{1/2}\quad ,\ \phi_k\ =\ \tan^{-1}\frac{2\frac{\xi_k\gamma_k}{\omega_k}}{2\xi_k^2-1+\frac{\omega^2}{\omega_k^2}}$$

and for sinus forcing forces sin(ωt) the participation factor will be:

$$\eta_{ksin}(t)\ =\ \frac{1}{A}\Big[\ \sin(\omega t-\phi_k)\ +\ \frac{1}{\gamma_k}\exp^{-\xi_k\omega_k t}\sin(\gamma_k t)\big[\xi_k\omega_k\sin(\phi_k)-\omega\ \cos(\phi_k)\big]$$
$$+\exp^{-\xi_k\omega_k t}\sin(\phi_k)\cos(\gamma_k t)\ \Big] \tag{4-c}$$

with **A** and ϕ_k given by:

$$A\ =\ \omega_k^2\left[\left(1-\frac{\omega^2}{\omega_k^2}\right)^2+\left(2\xi_k\frac{\omega}{\omega_k}\right)^2\right]^{1/2}\quad ,\ \phi_k\ =\ \tan^{-1}\frac{2\xi_k\frac{\omega}{\omega_k}}{1-\frac{\omega^2}{\omega_k^2}}$$

The dynamic analysis of buildings that follows will be done by assuming discrete masses interconnected by springs and the seismic signal will be located at the bottom of the ***Soil-Structure-Half space model*** to the moving half space (FIGURE 1). The seismic signal "S" will be decomposed in a Fourier series, thus:

$$S(t)\ =\ A_0\ +\ \sum_{n=1}^{\infty}\big[A_n\cos(\omega_n t)\ +\ B_n\sin(\omega_n t)\big]$$

with $\omega_n\ =\ n\pi/(1.5T)$ where **T** stands for the time duration of the acceleration record. The **1.5 T** was chosen, in general, to appreciate the vibrations decay after the seismic event was over. The frequencies ω are the excitation frequencies of equations (3-b), (3-c), (4-b) and (4-c).
Provided that the modal forms ϕ_0 for rigid body and the rest of the modal forms ϕ_k of the semidefinite structure are available, the displacements **U** for the solution to the seismic analysis of buildings is formulated as follows

$$U\ =\ \eta_{0A_0}(t)\ \tilde{\phi}_0+\ \sum_{n=1}^{\infty}\big[B_n\eta_{0\sin}(t)+A_n\eta_{0\cos}(t)\big]\tilde{\phi}_0\ +$$
$$\sum_{k=1}^{N}\eta_{kA_0}\tilde{\phi}_k\ +\ \sum_{n=1}^{\infty}\sum_{k=1}^{N}\big[B_n\eta_{ksin}(t)\ +A_n\ \eta_{kcos}\big]\phi_k \tag{5}$$

The dynamic analysis of the ***"Soil-Structure-Half Space model"*** was done via the Holzer method which requires the modal forms to satisfy dynamic equilibrium balance.

EXAMPLE

The next illustrative analysis considers a soil deposit of 42.3 m. according to the information of TABLE 1.

TABLE 1 SOIL PROPERTIES

from (m)	to (m)	soil type#	h (cm)	γ (Kg/cm^3)	G (Kg/cm^2)	M (T s^2/cm)	K (T/cm)
0	4.6	1	460	0.00152	50.3	71.27	13122
4.6	7.8	2	320	0.00114	38.1	37.19	14288
7.8	9.6	3	180	0.00111	35.8	20.37	23867
9.6	10.6	4	100	0.00120	69.2	12.23	83040
10.6	11.4	2	80	0.00118	30.0	0.62	45000
11.4	14.2	5	280	0.00138	27.5	39.39	11786
14.2	17.6	3	340	0.00117	35.6	40.55	12565
17.6	20.4	2	280	0.00125	42.5	35.68	18214
20.4	26.9	2	650	0.00125	42.0	82.82	7754
26.9	36.0	2	910	0.00128	48.5	118.74	6369
36.0	42.3	4	630	0.00120	69.2	77.06	131.81

#1) Silt sandy-clayey, 2) Silt clayey, 3) Clay silty, 4) Sand silty, 5) Silt clayey-sandy.

The general model employed in this research is shown in Figure 1. It is seen that the upper masses represent the building masses and springs, the intermediate masses represent the soil deposits and the bottom mass stands for the half-space which (as stated before) is completely free to move according to the **UNAM** seismic signal.

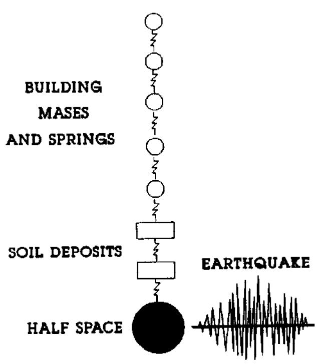

Fig. 1. General semidefinite model.

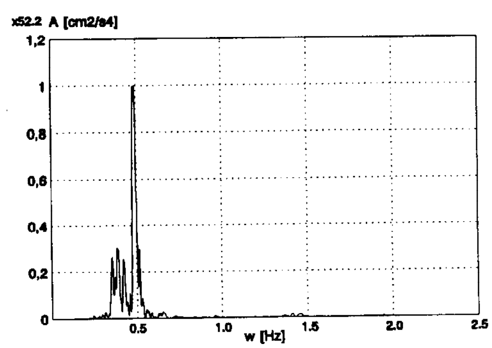

Fig. 1. SCT(N90E) spectrum,sept. 19, 1985.

FREQUENCY DENSITY

The natural frequencies of the three main components of the ***Soil-Structure-Half space model*** were calculated separately and are shown in TABLE 2.

TABLE 2 Natural frequencies

Natural frquencies, in Hz.		
HALF SPACE-SOIL	HALF SPACE-BUILDING	HALF SPACE-SOIL-BUILDING
0	0	0
0.37042	0.42662	0.36231
1.01195	1.27078	0.43318
1.73536	2.10286	1.01092
2.34291	2.91349	1.27231
2.69319	3.69428	1.73540

It is immediately noted that while the natural frequencies of the soil and the building are wide spread, the natural frequencies of the complete ***Soil-Structure-Half space model*** are gattered in a more compact group with the highest frequency density. For example, from 0 to 1.73540 the first column for the soil deposit only presents four natural frequencies, the second column for the building shows only three natural frequencies and the whole system shown in the third column exhibits six natural frequencies and therefore has the highest frequency density of all. In fact the reader can verify that all of the six frequencies of the complete model would be included within the domain of the spectrum of the **UNAM** signal shown in FIGURE 3 and would make them sususceptible to be excited by the accelerations of the record of the **UNAM** signal. The influence of this six frequencies will be noted in due time, when the analisis of the response will be done for the complete system under earthquake motion.
To continue on, it was decided to investigate if the **SCT** accelerogram whose spectrum is shown in Figure 2 could be approximately reproduced by applying the **UNAM** signal, whose spectrum is shown in Figure 3, at the bottom of the half space and passing through the soil deposit of Table 1, with no building on top of these two elements. To accomplish this, it was decided to use the original **UNAM** signal to a Half Space 100 times larger than the soil deposits and the building together. Within this context it was assumed that the **UNAM** signal (being recorded on firm ground) was a seismic signal rich in frequency content and it could therefore be used as input for the signal filtering searched.

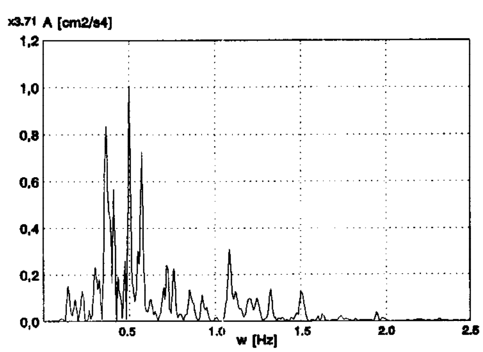

Fig. 3. UNAM IP spectrum.

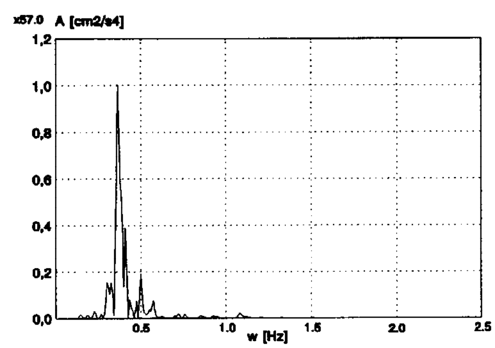

Fig. 4. Spectrum at the top of the soil deposit.

The result is seen in Figure 4 where as it can be observed that the spectrum at the top of the soil deposit approximately matches the SCT record, in shape, but it is shifted towards the left hand side by the influence of the first natural frequency of the soil deposit of 0.37 Hz. However, the record of Figure 3 at the top mass has an energy which ammounts to only 16% of the energy of the original SCT record and

the maximum amplitude of the spectrum of 57 (cm^2/seg^4) that overshuts the 52.2 value of the original SCT record by only 9%. Regarding the accelerations the SCT (N90E) record had a maximum acceleration of 167.9 gals and the spectrum of FIGURE 4 is associated with a maximum acceleration of 60.64 gals.
By using the same model and earthquake UNAM signal , the next step was to calculate the dynamic response of the entire system with the 15 storey building located as indicated in FIGURE 1. The corresponding spectra for the top of the soil deposit, the first floor and the upper floor are shown in FIGURES 5,6 and 7 as follows.

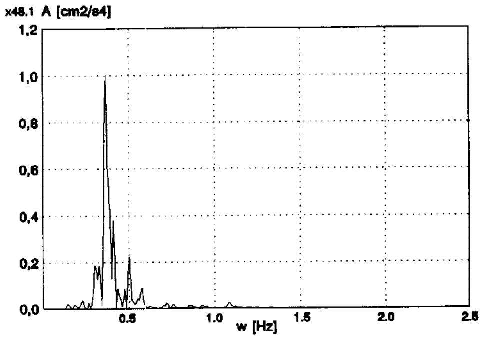

Fig. 5. Spectrum at the top of the soil deposit.

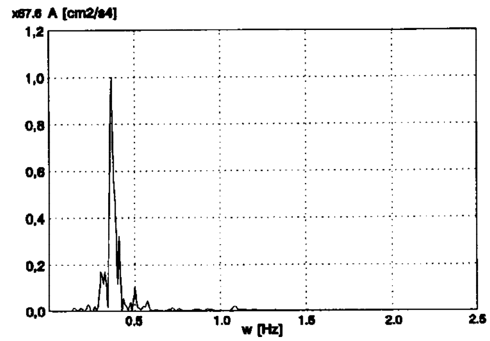

Fig. 6. Spectrum at the first floor.

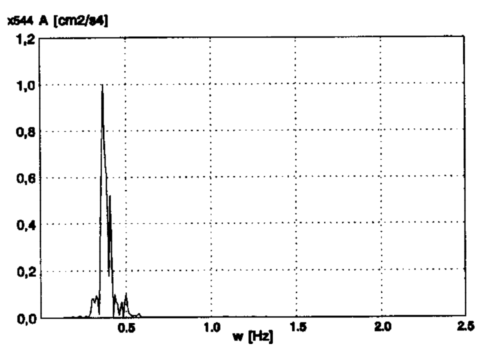

Fig. 7. Spectrum at the top floor.

When the maximum relative accelerations were calculated it was found that the total maximum accelerations and the time of occurance for the top of the soil deposit, the first floor and the upper floor are as follows.

TOP OF SOIL DEPOSIT, MAXIMUM ACCELERATION 80.43 cm/sec^2 , t=14.32 sec.
1st, LEVEL OF BUILDING, MAXIMUM ACCELERATION 77.06 cm/sec^2, t=14.34 sec. TOP OF BUILDING, MAXIMUM ACCELERATION 162.90 cm/sec^2, t=14.92 sec.

In the same way the maximum accelerations a_{max} and displacements d (both relative and total) are shown in TABLE 2 and in graphic form in FIGURE 8.

TABLE 2 Relative and total accelerations a_{max} and displacements d.

Model's level.	Relative			Total		
	t(sec)	a_{max} cm/sec^2	d cm	t(sec)	a_{max} cm/sec^2	d cm
Top of soil deposit.	14.32	80.43	7.11	27.44	59.04	23.18
First building's level.	14.34	77.06	6.91	27.48	66.39	24.98
Top floor.	14.92	162.9	20.1	14.92 27.48	153.85 121.7	22.56 38.25

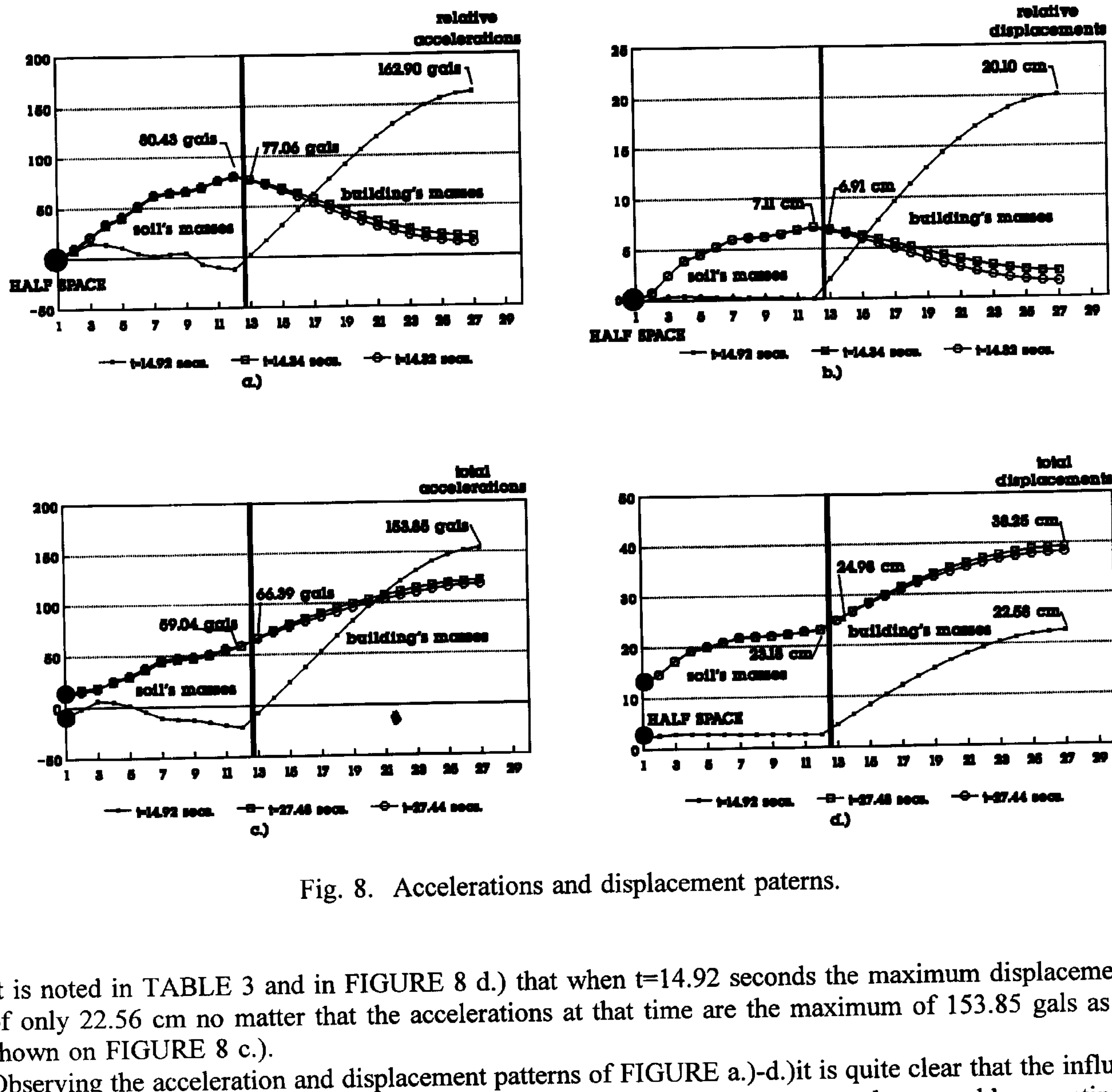

Fig. 8. Accelerations and displacement paterns.

It is noted in TABLE 3 and in FIGURE 8 d.) that when t=14.92 seconds the maximum displacement is of only 22.56 cm no matter that the accelerations at that time are the maximum of 153.85 gals as it is shown on FIGURE 8 c.).

Observing the acceleration and displacement patterns of FIGURE a.)-d.)it is quite clear that the influence of more than four modes of the complete system is present. This statement can be proved by counting the number of changes in curvature (five) of the acceleration pattern for t=14.92 seconds shown in FIGURE 8 a.).

From this kind of analyses of displacements and accelerations a rule for design could be put forward (indeed we could do so)for the ***"Soil-Structure-Half space model"*** problem. At present, we are researching a fact not addressed before. Nothing has been said about the question of ***POWER*** (in units of work done per second), a matter that in the authors'view should also be accounted for. For the time being it is left as a matter of subsequent research.

To end this paper we just recall the reader that, we have avoided the usual hipotesis of assuming that all the masses of the building experience the same accelerations than the ground $\ddot{u}_g$ (in this case the filtered **UNAM** signal) , a case that was also proved with the semidefinite model of the present investigation, the results are shown in Figure 9. In that figure it is readily seen that the influence of the soil deposits is almost nill, because the energy input is much more smaller than the energy transmited by the **UNAM** signal.

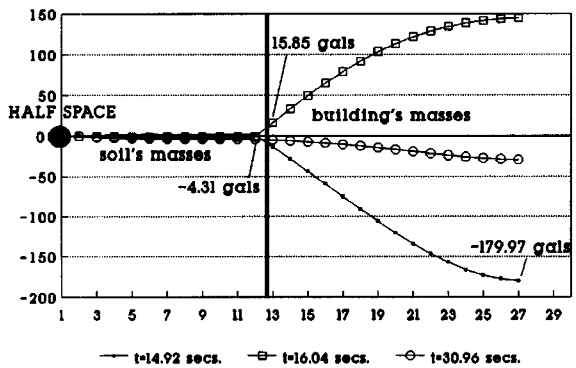

FIGURE 9

CONCLUSIONS

The ***"Soil-Structure-Half space model"*** has been introduced for the seismic analysis of buildings. For the first time in current literature the input signal (the so called UNAM signal, registered at the front yard of the Engineering Institute on firm ground) has been first filtered from the half space and through a soil deposit trying to match in shape and frequency content the spectrum of a specific signal registered on free field. In the present investigation the objective was to match the SCT (N90E) spectrum. Once the **UNAM** signal was proved to be a reasonably accurate signal for the ***"Soil-Structure-Half space model"*** the analysis was performed by applying the **UNAM** signal at the mass representing the **HALF SPACE** and allowing the rest of the masses, conforming the model, to freely vibrate according to the input signal.
At this stage of development of the ***"Soil-Structure-Half space model"*** still remains the challenge of finding ways to match the objective spectrum (the SCT spectrum) both in energy and in amplitude. This task is left for a future research.

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