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Equivalent Linearized Method of SDOF Shear Structures Based on Equivalence of Responses Statistic Characteristics

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ABSTRACT

Considering the correlation between the second-order statistic characteristic of responses and the structural reliabiliyt, the equivalent linearized method of structures based on equivalence of responses statistic characteristics is developed. The expressions of characteristic parameters of equivalent linear system are determined according to the average power spectral density functions of responses of 21,600 SDOF shear structures calculated by time history analysis. It is illustrated that the proposed method has an adequate precision.

KEYWORDS

Equivalent linearization method; shear structure; statistic characteristic; power spectral density; equivalent frequency f_e; equivalent damping ratio ζ_e.

INTRODUCTION

It is well-known that building structure performs as a nonlinear system under strong motion earthquake. The investigation of nonlinear dynamic behavior of structures has attracted more interests of earthquake engineers and designers. Some results have been accommodated by seismic codes in many countries. The specification that, for example, there are three different levels and two stages in seismic design, states that the structures subjected to earthquakes of different intensity should be damaged in different degrees. In fact, the study of structural reliability should be based on the statistic characteristics of the nonlinear responses because of the uncertainty of strong ground motions and structures, so it is necessary to analyse the response statistic characteristics of nonlinear system excited by random loads. Monte Carlo method, though it is effective to process the above complex problem, is difficult to be used in practice because of its large quantity of calculations. Many approximate procedures have been developed. The equivalent linearized method, transforming a nonlinear system to a linear system, is most in common use. Most previor linearized methods are based on equivalent deformation or energy, it is not corresponding to the present seismic design based on structural reliability. Considering the correlation between the second-order statistic characteristic of responses (such as power spectral density function) and the structural reliability, the equivalent linearized method of structures based on equivalence of responses statistic characteristics is developed. It should provide a basis for simplified seismic design and reliability analysis of complicated structures.

According to the average power spectral density functions of responses of 21,600 SDOF shear structures calculated by time history analysis, the expressions of characteristic parameters of equivalent linear system are determined. It is illustrated that the proposed method has an adequate precision.

THE TRANSMISSION THEORY OF LINEAR SYSTEM

The output $Y_{k1}(t)$ at arbitrary point k of a linear system shown in Fig. 1 with the random stationary excitation $X_1(t)$ at point 1 can be represented in frequency domain as

$$Y_{k1}(\omega) = H_{k1}(\omega)X_1(\omega) \tag{1}$$

And its power spectral density (PSD) can be readily obtained as

$$S_{Y_{k1}Y_{k1}}(\omega) = |H_{k1}(\omega)|^2 S_{X_1X_1}(\omega) \tag{2}$$

in which $H_{k1}(\omega)$ is the frequency response function of the system and $S_{X_1X_1}(\omega)$ the PSD function of the excitation.

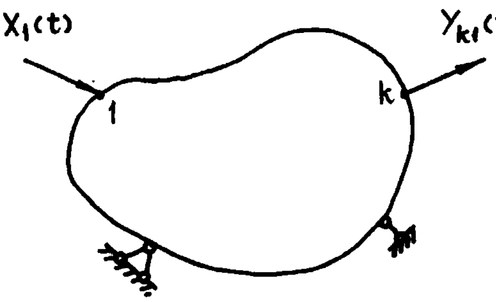


Fig. 1. Sketch of linear system

Considering the case of single input and single output, it can be obtained from eq. (2)

$$|H_Y(\omega)| = \sqrt{S_{YY}(\omega)/S_{XX}(\omega)} \tag{3}$$

or

$$|H_Y(f)| = \sqrt{S_{YY}(f)/S_{XX}(f)} \tag{4}$$

in which $f=\omega/(2\pi)$.

Let $\ddot{Y}(t)$ denotes the second derivative process of $Y(t)$, it is readily derived by the transmission theory of linear system

$$S_{\ddot{Y}\ddot{Y}}(\omega) = \omega^4 S_{YY}(\omega) \tag{5}$$

Substituting eq. (5) to eq. (3), the transfer function of $\ddot{Y}(t)$ to $\ddot{X}_g(t)$ is

$$|H_{\ddot{Y}}(\omega)| = \sqrt{S_{\ddot{Y}\ddot{Y}}(\omega)/S_{\ddot{X}_g\ddot{X}_g}(\omega)} = \omega^2|H_Y(\omega)| \tag{6}$$

When an SDOF linear system is subjected to a ground motion acceleration $\ddot{X}_g(t)$, the equation of motion is written as

$$\ddot{Y}(t) + 2\xi_1\omega_1\dot{Y}(t) + \omega_1^2 Y(t) = -\ddot{X}_g(t) \tag{7}$$

and the system transfer function, $H_Y(i\omega)$, between the stationary input acceleration $\ddot{X}_g(t)$ and the output displacement is given by

$$H_Y(i\omega) = (\omega_1^2 - \omega^2 + 2i\zeta_1\omega_1\omega)^{-1} \tag{8}$$

Combining with eq. (6), the transfer function, $H_{\ddot{Y}}(\omega)$, between $\ddot{X}_g(t)$ and the output acceleration can be expressed as

$$|H_{\ddot{Y}}(\omega)| = \{[(\omega_1/\omega)^2 - 1]^2 + 4\zeta_1(\omega_1/\omega)^2\}^{-\frac{1}{2}} \tag{9}$$

or

$$|H_{\ddot{Y}}(f)| = \{[(f_1/f)^2 - 1]^2 + 4\zeta_1(f_1/f)^2\}^{-\frac{1}{2}} \tag{10}$$

Obviously, the responses of linear system excited by a stationary input process is also stationary, its PSD function is only the product of PSD function of the excitation and the square of the system transfer function. The above theory can not, unfortunately, be directly used to solve the nonlinear responses of practical earthquake resistant structures because of the nonstationary of ground motions and the nonlinearity of the structures. For its practical utilization, the earthquake ground motion was taken as a stationary process and the structure was linearized in past, and the linearized method was usually depended on equivalent deformation or energy.

Xia (1991) has shown that the nonlinear responses of SDOF shear structures with excitation by nonstationary ground motion acceleration would be simplified by

$$\bar{S}_{\ddot{Y}\ddot{Y}}(f) = |H_{\ddot{Y}}(f)|^2 \cdot \bar{S}_{\ddot{X}_g\ddot{X}_g}(f) \tag{11}$$

in which, $\bar{S}_{\ddot{Y}\ddot{Y}}(f)$ presents the ensemble average value of PSD function of acceleration responses of nonlinear system, $\bar{S}_{\ddot{X}_g\ddot{X}_g}(f)$ the ensemble average value of PSD function of nonstationary acceleration inputs, and $H_{\ddot{Y}}(f)$ the transfer function of equivalent linear system, defined by

$$|H_{\ddot{Y}}(f)| = \{[(f_e/f)^2 - 1]^2 + 4\zeta_e(f_e/f)^2\}^{-\frac{1}{2}} \tag{12}$$

Where f_e, ζ_e are the equivalent natural frequency and equivalent damping ratio of the equivalent linear system, respectively. Then the proposed linearization of practical structures, based on the statistic characteristics of structural nonlinear responses, is not depended on the assumption that the inputs must be stationary, and the results will be more practical.

NUMERICAL CALCULATION AND STATISTIC ANALYSIS OF NONLINEAR RESPONSES

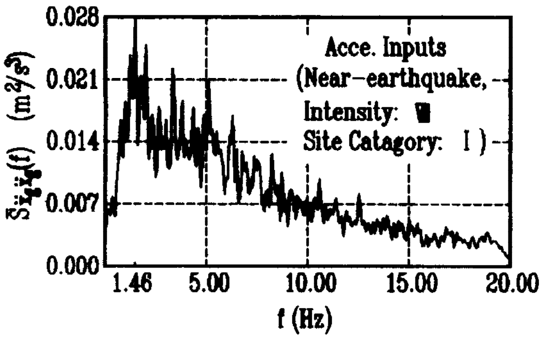
Analysing the statistic characteristics of responses obtained by a great number of numerical calculations should be done for the equivalent linearization of nonlinear structures according to eq. (11). Employing time history analysis method, the $\bar{S}_{\ddot{Y}\ddot{Y}}(f)$ and $\bar{S}_{\ddot{X}_g\ddot{X}_g}(f)$ of various SDOF shear structures under different earthquakes are determined. The inputs and the structural parameters are considered as following.

For earthquakes, 8 groups of 30 artificial earthquake accelerations, corresponding to the near/far-earthquakes at four site catagories with fortification intensity of Ⅶ specified in Code for Seismic Design of Building GBJ11—89 in China, are used.

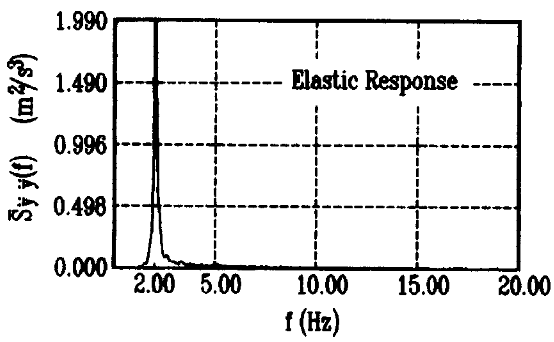
For structural parameters, their practical interpretations should be properly considered. The structural natural period, $T_0(=1/f_0)$, is selected from 0.3s to 1.2s with the interval 0.1s; the structural damping is specified as Rayleigh's model; the damping ratio, ζ_0, is taken as 0.05; the yielding strength ratio, ζ_y, is considered to be 0.2, 0.3 and 0.4 for practical reason shown by Yang (1991); and the structural hysteretic behavior is modeled by the well-known bilinear. It should be noticed that, the following values are used as the strain hardening/softening ratio, p, of the bilinear model: −0.02, 0.0, and 0.1. The lower limit value of p choosen to be −0.02 prevents the structural maximum elastoplastic displacement from being an unacceptable value (Yang, 1991).

So the nonlinear responses time history analyses of 21,600 cases have been performed.

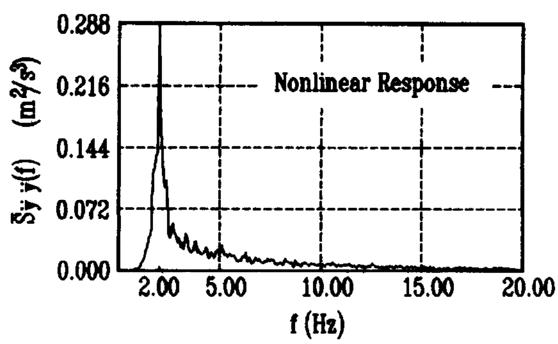
Figure 2 shows the average PSD functions of inputs and outputs of an elastic SDOF system and a nonlinear SDOF system. It states that the equivalent linearization according to eq. (11) is proper.



(a) Average PSD function of inputs



(b) Average PSD function of outputs of an elastic system



(c) Average PSD function of outputs of a nonliear system

Fig. 2. Average PSD functions of inputs and outputs of an elastic SDOF system and a nonlinear SDOF system (With $f_0 = 2$ Hz, $\xi_y = 0.3$ and $p = -0.02$)

Comparing the nonlinear responses with the elastic responses, it could be found that the maximum nonlinear displacement is greater than the maximum elastic displacement while the maximum nonlinear acceleration is less than the maximum elastic acceleration. That fact should be considered in the expressions of the equivalent structural parameters.

EQUIVALENT LINEARIZATION OF NONLINEAR STRUCTURES

It needs two steps to draw the rules on which the equivalent frequency f_e and equivalent damping ratio ζ_e vary with the practical structural parameters, such as f_0, ξ_y, p, and ζ_0.

The first is to determine the values of f_e and ζ_e corresponding to various structures with different inputs. Namely, using the $|H_Y(f)|$ defined by eq. (12) to fit $\sqrt{S_{\ddot{Y}\ddot{Y}}(f)/S_{\ddot{X}_g\ddot{X}_g}(f)}$ with the constraint conditions

$$0 < f_e \leqslant f_0 \quad \text{and} \quad \zeta_e \geqslant \zeta_0 .$$

Then 720 groups of (f_e, ζ_e) can be estimated by one of nonlinear parameter identification methods, the Complex Form Optimization method.

The second is to determine the approximate expressions of f_e and ζ_e. Analysing the effects of practical structural parameters on the equivalent frequency f_e, Xia (1991) has shown the following properties of f_e: f_e increases with the natural frequency f_0 and the yielding strength ratio ξ_y increasing; f_e has a small range with the variation of the strain hardening/softening ratio p; and the value of f_e has hardly been influenced by the site catagories and near/far-earthquakes which can be neglected. Then the expression of f_e can be properly described as

$$f_e = f_0[a_1\xi_y + a_2(1+p) + a_3] \tag{13}$$

in which a_1, a_2 and a_3 are constants. In terms of parameter indentification, the evaluated expression of f_e is

$$f_e = k_e \cdot f_0 = [0.365551\xi_y - 0.079799(1+p) + 0.833459]f_0 \tag{14}$$

in which k_e is termed as reduction factor of equivalent frequency.

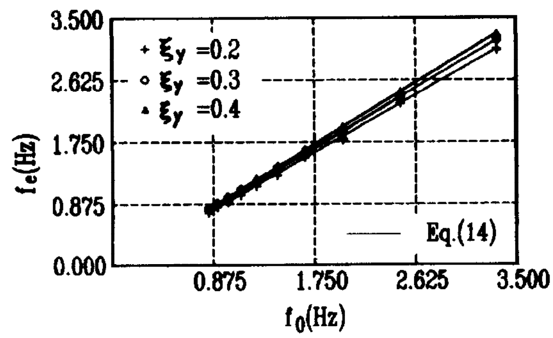
Similarly, it has been seen (Xia, 1991) that the equivalent damping ratio ζ_e decreases with f_0 and ξ_y increasing. The site catagories, near/far-earthquakes and the value of p, having a slight influence on the value of ζ_e, can also be ignored. Then the expression of ζ_e can be properly represented by

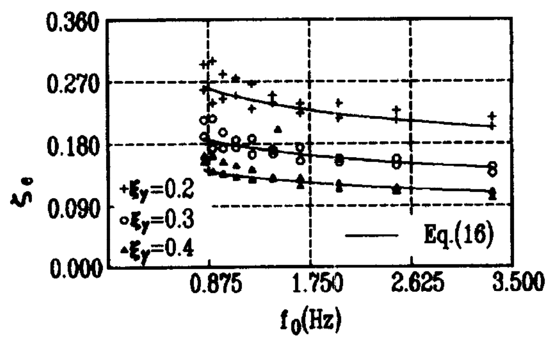
$$\zeta_e = f_0^{\alpha}(\beta\xi_y^{\gamma} + \delta)\zeta_0 \tag{15}$$

in which α, β, γ and δ are constants. Using parameter identification, the evaluated expression of ζ_e is

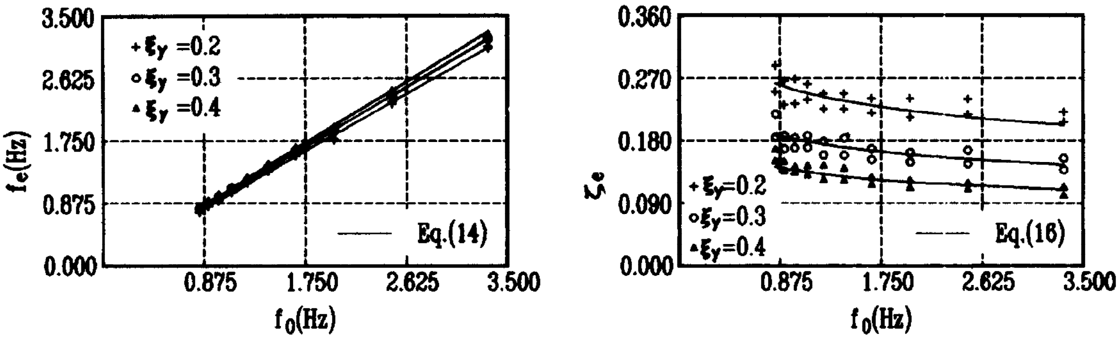
$$\zeta_e = f_0^{-0.187465}(4.53316\xi_y^{-0.429177} - 3.97696)\zeta_0 \tag{16}$$

Comparision of some estimated equivalent frequencies and equivalent damping ratios and the values calculated from eq. (14) and eq. (16) is shown in Fig. 3. It is seen that eq. (14) and eq. (16) are in good agreement with the estimated values.

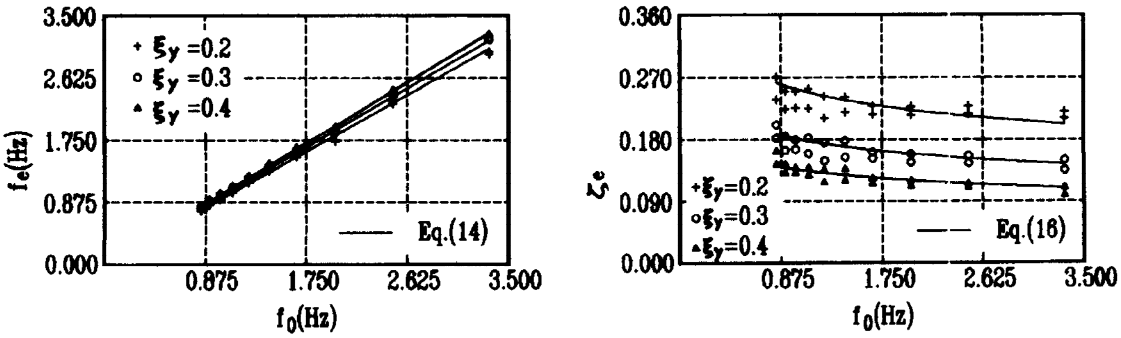




(a) Case of near/far-earthquake at site catagory of II with fortification intensity of VII, $p=-0.02$ and $\zeta_0=0.05$.



(b) Case of near/far-earthquake at site catagory of Ⅲ with fortification intensity of Ⅶ , $p=0.0$ and $\zeta_0=0.05$.



(c) Case of near/far-earthquake at site catagory of Ⅳ with fortification intensity of Ⅶ , $p=0.1$ and $\zeta_0=0.05$.

Fig. 3. Comparision of some estimated values of f_e and ζ_e and values calculated from eq. (14) and eq. (16)

ILLUSTRATIVE EXAMPLE

Consider an SDOF shear structure with $T_0=0.45s$ ($f_0 \approx 2.22Hz$), $\xi_y=0.35$, $p=0.05$ and $\zeta_0=0.05$. The acceleration inputs are 30 artificial far-earthquakes at the site catagory of Ⅱ with fortification intensity of Ⅶ. From eq. (14) and eq. (16), the equivalent linear system of the structure is described by

$$f_e = 2.14652Hz \text{ and } \zeta_e = 0.135019$$

Its PSD function of acceleration response can be easily obtained by

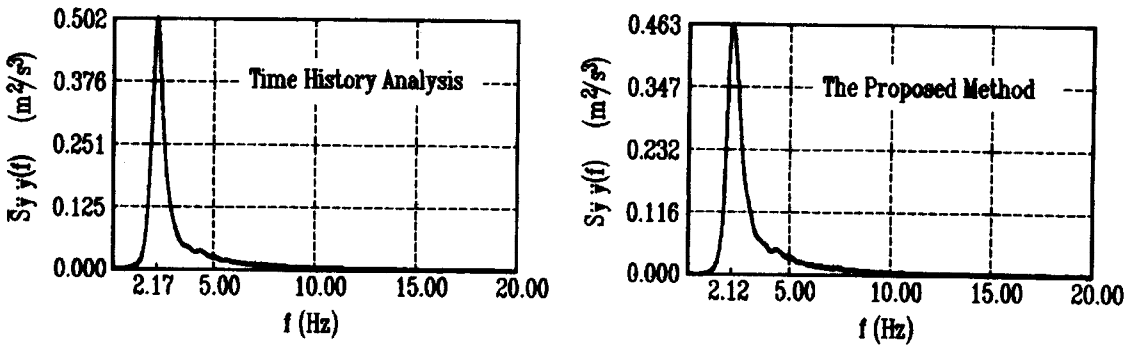
$$S_{\ddot{Y}\ddot{Y}}(f) = |H_{\ddot{Y}}(f)|^2 \cdot \bar{S}_{\ddot{X}_g\ddot{X}_g}(f)$$

in which $|H_{\ddot{Y}}(f)|$ is calculated by f_e and ζ_e from eq. (12). It is shown in Fig. 4. Simultaneously, Fig. 4 gives the average PSD function, $\bar{S}_{\ddot{Y}\ddot{Y}}(f)$, of nonliear structural acceleration responses calculated by time history analysis. $S_{\ddot{Y}\ddot{Y}}(f)$ is seen to fit $\bar{S}_{\ddot{Y}\ddot{Y}}(f)$ with higher precision. For example, the relative error of the frequencies at which the peak values occurred is

$$R_f = \left|\frac{2.12 - 2.17}{2.17}\right| \approx 2.3\%$$

while the relative error of peak values is

$$R_p = \left|\frac{0.463 - 0.502}{0.502}\right| \approx 7.8\%$$



(a) Average PSD function calculated by time history analysis

(b) PSD function calculated by the proposed method

Fig. 4. Comparision of response PSD function calculated by time history analysis and the proposed method

The above errors are mainly caused by cumulative error of numerical integral in time history analysis and the error occurred in parameter identification. The errors are so small that the presented equivalent linearized method can be used for engineering purpose.

CONCLUSIONS

It is convenient to simplify a nonlinear system to a linear. The equivalent linearized procedure based on equivalence of responses statistic characteristics can be directly concerned with the reliability of seismic structures. The expressions of equivalent structural parameters, f_e and ζ_e, are regressed from a great deal of numerical calculations. Results of the example show that the presented equivalent linearized method is suitable for designing earthquake resistant structures.

It should be noticed that the results, obtained only from various SDOF shear structures, give an important foundation for subsequent research works. Convincingly, the suggested method for SDOF shear structures will advance the study on equivalent linearization of other structures (such as bending and shear-bending structures) and various MDOF building structures.

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