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CLOSED-FORM SOLUTIONS FOR PROBABILISTIC SEISMIC HAZARD ASSESSMENT IN DIFFUSE SEISMICITY ZONES

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ABSTRACT

Probabilistic seismic hazard assessments are currently made using specialized computer codes. The purpose of the paper is to show that simple closed-form solutions exist for the computation of return periods for usual ground motion parameters (acceleration, velocity, spectral ordinates) in diffuse seismicity zones, i.e. zones in which earthquakes of a given size can occur anywhere with equal probability. Such zones are quite often encountered in intraplate, moderate seismicity, tectonic regimes.

These closed-form solutions are aimed at :

- providing a simple means for the validation of computer codes,
- enabling the analyst to perform at little cost parametric studies of the influence of certain quantities.

Three types of closed-form solutions have-been obtained :

- return period for any ground motion parameter under the most general assumptions i.e. arbitrary values of the constants a and b, the maximum magnitude and the magnitude and distance exponents in the attenuation law,
- return period for acceleration at any point of an extended site (large city, long tunnel or pipeline) under particular assumptions (no maximum magnitude, $b = 0.75$, 1 or 1.25, distance exponent equal to 1),
- mean annual damage caused by earthquakes in the diffuse seismicity zones for a uniformly distributed population of structures having a linear fragility curve (no damage up to acceleration A_0, linear variation of damage probability between A_0 and A_1, total loss above A_1) under particular assumptions (no maximum magnitude, distance exponent equal to 1).

Comments are made on the implication of these formulas. It is shown for instance that hazard significantly increases for large or elongated sites in comparison with pinpoint sites.

KEYWORDS

Probabilistic seismic hazard assessment, closed-form solutions, return period, ground motion, extended sites.

1. INTRODUCTION

Diffuse seismicity zones, i.e. zones in which earthquakes of a given size can occur anywhere with equal probability, are quite often encountered in intraplate, moderate seismicity, tectonic regimes. In such zones, it is possible to derive closed-form solutions for the return period of the overstepping of a given value for any ground motion parameter, provided that the following assumptions are satisfied :

- The frequency distribution of earthquakes obeys the Gutenberg-Richter law :

$$\log_{10} N(M) = a - bM \quad or \quad N(M) = 10^{a-bM} \tag{1}$$

  $N(M)$ being the annual number of earthquakes with magnitude equal to or greater than M.

  If M is restricted to the range $M_1 \le M \le M_2$, eq. (1) is replaced by :

$$N(M) = 10^{a} \frac{10^{-bM} - 10^{-bM_2}}{1 - 10^{-b(M_2 - M_1)}} \tag{2}$$

- The attenuation law for the ground motion parameters has the following form :

$$P = c10^{M/n} R^{-\alpha} \tag{3}$$

  P being the parameter of interest (acceleration, velocity, spectral ordinate), R the focal distance and c, n, α three constants.

- The depth of focus varies between a lower bound h_1 and an upper bound h_2.

- The probability of occurrence of an earthquake of a given magnitude is uniformly distributed throughout the volume $\Sigma\ (h_2 - h_1)$, Σ being the surface of the diffuse seismicity zone.

Under these assumptions, the return period T_p for the overstepping of the value P of the parameter of interest at a given pinpoint site is given by :

$$\frac{1}{T_p} = \int_{h_1}^{R_m(P)} N\left[M(P,R)\right] \frac{\sigma(R)dR}{\sum(h_2 - h_1)} \tag{4}$$

$R_m(P)$ being the maximum focal distance (corresponding to maximum magnitude M_2 producing P at the site), $M(P, R)$ the magnitude which produces P at the site for a given value of the focal distance R and $\sigma(R)$ the surface of the part of the sphere of radius R centered at the site which is inside the seismogenic volume.

The integral in eq. (4) can be computed in closed-form using elementary, but sometimes tedious, algebra. Due to the shortage of pages, these calculations are not reproduced in the paper, which presents only the results and some comments on their implications.

For some closed-form solutions (for instance for extended sites, as defined in 3) some restrictions have been put to the above mentioned assumptions (suppression of maximum magnitude, limitation to certain values of parameter b, value 1 for the distance exponent) in order to achieve analytical integration. These restrictions are defined in the relevant paragraphs.

2. CLOSED-FORM SOLUTIONS FOR A PINPOINT SITE

With the notations :

P_2 = maximum possible value for the parameter of interest (corresponding to maximum magnitude M_2 occurring at the focal distance h_1).

$x = \frac{P_2}{P}$, P being the given value for the parameter of interest.

$\delta = \frac{h_2}{h_1}$, $\beta = \alpha n b$, $T_r = \frac{\sum (h_2 - h_1)}{2\pi h_1^3} 10^{-a} (10^{bM_2} - 10^{bM_1})$

the following formulas are obtained for the return period T_p :

- if $1 \le x \le \delta^{\alpha}$

$$\frac{T_r}{T_p} = \frac{x^{nb}}{(2-\beta)(3-\beta)} + \frac{\beta x^{3/\alpha}}{3(3-\beta)} - \frac{\beta x^{2/\alpha}}{2(2-\beta)} - \frac{1}{6} \tag{5}$$

- if $x > \delta^{\alpha}$

$$\frac{T_r}{T_p} = \frac{1-\delta^{3-\beta}}{(2-\beta)(3-\beta)} x^{nb} + \frac{\beta(\delta-1)}{2(2-\beta)} x^{2/\alpha} + \frac{1}{6}\left(\delta^3 - 1\right) \tag{6}$$

Eqs (5) and (6) do not apply if β equals either 2 or 3. They are replaced by :

- if $\beta = 2$ and $1 \le x \le \delta^{\alpha}$

$$\frac{T_r}{T_p} = \frac{2}{3} x^{3/\alpha} - \frac{1}{2}\left(1 + \frac{2}{\alpha} Lnx\right) x^{2/\alpha} - \frac{1}{6} \tag{7}$$

- if $\beta = 2$ and $x \ge \delta^{\alpha}$

$$\frac{T_r}{T_p} = (\delta - 1)\left[\frac{1}{2} + \frac{1}{\alpha} Lnx - \frac{\delta\, Ln\delta}{\delta - 1}\right] x^{2/\alpha} + \frac{1}{6}\left(\delta^3 - 1\right) \tag{8}$$

- if $\beta = 3$ and $1 \le x \le \delta^{\alpha}$

$$\frac{T_r}{T_p} = \frac{3}{2} x^{2/\alpha} - \left(\frac{4}{3} - \frac{1}{\alpha} Lnx\right) x^{3/\alpha} - \frac{1}{6} \tag{9}$$

- if $\beta = 3$ and $x \ge \delta^{\alpha}$

$$\frac{T_r}{T_p} = x^{3/\alpha} Ln\delta - \frac{3}{2}(\delta - 1) x^{2/\alpha} + \frac{1}{6}\left(\delta^3 - 1\right) \tag{10}$$

For a numerical application, the following values are considered :

$\alpha = 1 \quad n = 4 \quad b = 1$

The choice for α and n corresponds to an attenuation law which is very close to the acceleration attenuation laws of Joyner-Boore (1981-1988) or Ambraseys-Bommer (1991). The parameter P is therefore acceleration and T_A is written instead of T_P.

For $\alpha = 1$, $n = 2$, $b = 1$, the attenuation law is very close to the velocity attenuation laws of Joyner-Boore (1981-1988) or Betbeder-Matibet (1995). The parameter P is therefore velocity and T_V is written instead of T_P.

Table 1 gives T_r / T_A and T_r / T_V as functions of x :

Table 1. Comparison of T_r / T_A with T_r / T_V

x	T_r / T_A	T_r / T_V
2	1.167	0.394
3	13.33	3.446
4	58.50	12.32
5	169.9	30.08
6	388.5	58.38
7	763.9	98.33
8	1354	150.9
9	2228	216.8
10	3460	296.8

It is seen from Table 1 that T_A is always smaller than T_V and T_A / T_V decreases if x increases. This means that uniform hazard response spectra are shifted towards the high frequencies, as compared with spectra originating from one single earthquake and this finding is consistent with results of probabilistic seismic hazard assessments. The high frequency side of the spectrum is mostly controlled by local, moderate magnitude and relatively frequent earthquakes, while the medium and low frequency side is controlled by larger and less frequent events. A similar trend can be observed for b-values different from 1.

3. CLOSED-FORM SOLUTIONS FOR AN EXTENDED SITE

Extended sites are defined as having in plane dimensions of the order of the focal depths. Examples are huge cities and very long lifelines (tunnel, pipeline, highways...). It is clear that the probability of overstepping a given level of ground motion is higher for an extended site than for a pinpoint side.

For an extended site having the shape of a l-side convex polygon the surface σ (R) which appears in eq. (4) is no longer spherical, but consists of l cylindrical parts, l fractions of a portion of a sphere which together reconstruct the complete portion and, if $R < h_2$, a horizontal surface equal to that of the site. In that case the analytical integration is more difficult than in 2 and some limitations have to be imposed to the assumptions in order to obtain a closed-form solution. These limitations are :

- no maximum magnitude
- b = 3/4, 1 or 5/4
- α = 1
- n = 4 which means that the parameter of interest is acceleration.

Under these assumptions, it can be shown that the acceleration level must be multiplied by the factor F given hereafter in order to obtain the same return period for an extended site as for a pinpoint site. This factor depends only on the depth bounds h_1 and h_2, on the area S and the half perimeter L of the extended site.

The factor F is given by the following expressions :

$$b=\frac{3}{4},\ F=\left[1+\frac{1}{\pi}\frac{L(h_2-h_1)}{h_1\ h_2\ Ln\ h_2/h_1}+\frac{1}{4\pi}\frac{S\left(h_2^2-h_1^2\right)}{h_1^2\ h_2^2\ Ln\ h_2/h_1}\right]^{1/3} \quad (11)$$

$$b=1,\ F=\left[1+\frac{1}{4}\frac{L(h_1+h_2)}{h_1\ h_2}+\frac{1}{3\pi}\frac{S\left(h_1^2+h_1\ h_2+h_2^2\right)}{h_1^2\ h_2^2}\right]^{1/4} \quad (12)$$

$$b=\frac{5}{4},\ F=\left[1+\frac{4}{3\pi}\frac{L\left(h_1^2+h_1\ h_2+h_2^2\right)}{h_1\ h_2\ (h_1+h_2)}+\frac{3}{4\pi}\frac{S\left(h_1^2+h_2^2\right)}{h_1^2\ h_2^2}\right]^{1/5} \quad (13)$$

Table 2 gives the values of F for various dimensions of lifelines and a number of square cities ; the values chosen for h_1 and h_2 are 5 *km* and 20 *km* respectively.

Table 2. Amplification factor F for extended sites

L (*km*)	S (km^2)	$F, b=\frac{3}{4}$	$F, b=1$	$F, b=\frac{5}{4}$
2	0	1.022	1.030	1.034
5	0	1.054	1.070	1.078
10	0	1.104	1.129	1.138
20	0	1.191	1.225	1.230
50	0	1.397	1.425	1.409
4	4	1.047	1.062	1.070
10	25	1.118	1.152	1.167
20	100	1.240	1.294	1.308
40	400	1.480	1.547	1.541

This table shows that the "size effect" of an extended site can be quite significant ; for a 50 *km* long lifeline, it corresponds to a 40 % increase of the acceleration ; for a 20 × 20 km^2 city, the increase is of the order of 50 %. The influence of b-values is of minor importance.

4. COMPARISON OF DETERMINISTIC AND PROBABILISTIC PROCEDURES FOR SEISMIC HAZARD ASSESSMENT OF CRITICAL FACILITIES

For critical facilities such as nuclear power stations, seismic hazard has been assessed either using deterministic or probabilistic methods. In deterministic methods, earthquakes similar to historical earthquakes are supposed to occur anywhere inside the tectonic domains. A diffuse seismicity zone is a tectonic domain and therefore all the historical earthquakes are considered as if they can occur beneath the site. In some variants of deterministic methods, an additional safety factor is applied to the greatest historical earthquake.

In probabilistic methods, the ground motion level corresponds to a given return period, usually 10 000 years.

Various attempts have been made to compare the conservatism of the two approaches. In the case of diffuse seismicity zones, it is relatively easy to undertake this comparison. Usin nuclear terminology the return period used in probabilistic methods is noted T_{PSSE} and the estimated return period for the maximum historical earthquake is noted T_{DSSE}.

The respective conservatisms of the two methods are mostly controlled by the ratio T_{PSSE}/T_{DSSE}, the safety factor s applied to the maximum historical earthquake and the area Σ of the zone.

Taking the same values as in 3 (b = 3/4, 1 or 5/4, $\alpha = 1$, $n = 4$, $M_2 = \infty$) and reasoning on accelerations only, it can be shown that there is one value Σ_0 of Σ which gives the same result for the two methods ; Σ_0 has the following expressions :

$$b = \frac{3}{4} \quad , \quad \Sigma_0 = \frac{4\pi}{s^3} \frac{T_{PSSE}}{T_{DSSE}} \frac{h_1^2 h_2^2}{h_2^2 - h_1^2} Ln \frac{h_2}{h_2} \tag{14}$$

$$b = 1 \quad , \quad \Sigma_0 = \frac{3\pi}{s^4} \frac{T_{PSSE}}{T_{DSSE}} \frac{h_1^2 h_2^2}{h_1^2 + h_1 h_2 + h_2^2} \tag{15}$$

$$b = \frac{5}{4} \quad , \quad \Sigma_0 = \frac{4\pi}{3s^5} \frac{T_{PSSE}}{T_{DSSE}} \frac{h_1^2 h_2^2}{h_1^2 + h_2^2} \tag{16}$$

If $\Sigma > \Sigma_0$ the deterministic method is more conservative while the opposite conclusion is obtained if $\Sigma < \Sigma_0$.

With the numerical values T_{PSSE} = 10 000 years, T_{DSSE} = 500 years, s = $\sqrt{2}$, h_1 = 5 km, h_2 = 20 km one obtains for Σ_0 :

$$b = \frac{3}{4} \qquad \Sigma_0 = 3285\ km^2$$

$$b = 1 \qquad \Sigma_0 = 898\ km^2$$

$$b = \frac{5}{4} \qquad \Sigma_0 = 348\ km^2$$

The influence of b on Σ_0 is quite significant.

5. OPTIMAL CHOICE OF DESIGN ACCELERATION

The diffuse seismicity zone is supposed to be uniformly urbanized, with a number ν of buildings per unit of area. The buildings are all identical and are designed in such a way that :

- if the ground acceleration A is less than a given value A_0, the probability of collapse P_c is zero,
- if A is greater than a given value $A_1 > A_0$, $P_c = 1$,
- if $A_0 \leq A \leq A_1$, $P_c = (A - A_0)/(A_1 - A_0)$

Under the assumptions $M_2 = \infty$ and $\alpha = 1$ in eq. (3), it is possible to compute in closed-form the annual number N_d of destroyed buildings ; noting $\beta = nb$, one obtains :

$$N_d = \frac{2\pi\nu 10^a}{(\beta-1)(\beta-2)(\beta-3)} \frac{c}{A_1 - A_0} \left[\left(\frac{A_0}{c}\right)^{1-\beta} - \left(\frac{A_1}{c}\right)^{1-\beta} \right] \frac{h_1^{3-\beta} - h_2^{3-\beta}}{h_2 - h_1} \tag{17}$$

The total annual cost due to earthquakes is :

$$C_e = N_d\left(C_c + C_l\right) + \nu \sum \rho \varepsilon C_c \tag{18}$$

C_c being the construction cost of one building, C_l the cost of other losses associated with the collapse of the building, ρ the ratio between the number of new buildings annually built and the total number of buildings and ε he fraction of construction cost which corresponds to seismic design.

ε can be expressed by a power-law $\varepsilon = (A_0 / A_r)^\gamma$, A_r being a reference value for acceleration ; for instance with $A_0 = 2\ m/s^2$, $A_r = 10\ m/s^2$ and $\gamma = 2$, $\varepsilon = 0.04$. It can be shown from eq. (17) and the above expression for ε that the total annual cost of earthquakes C_e has a minimum value for a particular value $A_{0,m}$ of A_0, provided that $A_1 = k A_0$, k being a constant.

Taking n = 4, $b = 1$ (i.e. $\beta = 4$), $\gamma = 2$, $k = 2$, the optimal value $A_{0,m}$ is found to be :

$$A_{0,m} = \left[\frac{7\pi}{12}\frac{10^a c^4 A_r^2}{\sum h_1 h_2 \rho}\left(1+\frac{C_l}{C_c}\right)\right]^{1/6} \tag{19}$$

Expressing the accelerations in m/s^2 and taking the values $c = 1$, $A_r = 10$, $\Sigma = 40\ 000\ km^2$, $h_1 = 5\ km$, $h_2 = 20\ km$, $\rho = 0.01$, $C_l/C_c = 1$, one obtains :

$$A_{0,m} = 2.12\, m/s^2 \textit{ for } a = 4 \ (M = 6 \textit{ centennial earthquake})$$

$$A_{0,m} = 4.57\, m/s^2 \textit{ for } a = 6 \ (M = 8 \textit{ centennial earthquake})$$

The exponent 1/6 in eq. (19) ensures that the value of $A_{0,m}$ is not too sensitive to the choice of the parameters.

It is interesting to note that the total cost C_e does not increase very fast for $A_0 > A_{0,m}$; on the contrary the increase is faster for $A_0 < A_{0,m}$; in other words, overdesign is less costly than underdesign. This is shown in Table 3, in which $C_{e,m}$ is the minimum cost.

Table 3. Variation of C_e with A_0

$A_0/A_{0,m}$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$C_e/C_{e,m}$	208	13.2	2.81	1.24	1.00	1.12	1.39	1.76	2.19	2.69

6. CONCLUSIONS

A number of closed-form solutions have been obtained for probabilistic seismic hazard assessment in diffuse seismicity zones. These solutions provide a simple means for the validation of computer codes. They should also be useful to perform low-cost parametric studies on the influence of certain parameters.

From these solutions, some comments have been made on the following points :

- for an extended site of a large size, seismic hazard can be significantly higher than for a pinpoint site,
- the area of the diffuse seismicity zone is a very important parameter when comparing the respective conservatism of probabilistic and deterministic methods,

- the optimal choice of design acceleration can be obtained in a rational way in an ideally simplified case ; it is probably not applicable to real cases, but it gives some insight on the relevance of some parameters.

REFERENCES

Ambraseys N.N., Bommer J.J. (1991). The attenuation of ground acceleration in Europe. *Earth. Eng. and Struct. Dyn.*, **20**, 1179-1202.

Betbeder-Matibet J. (1995). Attenuation laws in the nearfield. *Proceedings of the 5th Int. Conf. on Seismic Zonation*, Vol. II, 949-956.

Joyner W.B., Boore D.M. (1981). Peak horizontal acceleration and velocity from strong motions records including records from the 1979 Imperial Valley, California, earthquake. *Bull. Seism. Soc. Am.*, **71**, 2011-2038.

Joyner W.B., Boore D.M. (1988). Measurement, characterization and prediction of strong ground motion. *Proceedings, Earthquake Engineering and Soil Dynamics II, ASCE/GT Div., Park City, Utah*, June 27-30, 431032.