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SEISMIC UPLIFTING OF FLEXIBLY SUPPORTED LIQUID-STORAGE TANKS

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ABSTRACT

A summary is presented of a systematic study on the seismic response of vertical, cylindrical, liquid-storage tanks that are supported directly on flexible soil foundations. First, a realistic yet highly efficient solution is presented for the partially uplifted base plate of the tank, taking into consideration the effects of nonlinearities arising from variations in base contact area, spatial and temporal variation of hydrodynamic base pressures, membrane action, and plastic yielding. Next, a rational model for dynamic response analysis is formulated. Finally, a study is presented of the effects of various parameters, such as soil flexibility, intensity of shaking, and the thicknesses of base plate and tank wall, on the various responses of the tank. It is shown that the flexibility of the foundation reduces the overturning base moment, and reduces significantly the axial compressive stresses in the tank wall, but these reductions may be accompanied by increased values of base uplifting and plastic rotations.

KEYWORDS

Liquid-storage; tank; steel; soil-supported; unanchored; uplift; nonlinear.

INTRODUCTION

A large number of unanchored tanks are supported directly on flexible soil foundations. During strong ground shaking, these tanks have a tendency to uplift on one side and penetrate their flexible foundation on the opposite side; the resulting response is therefore highly nonlinear. The earthquake damage to such tanks has been in the form of: (1) The failure of the piping connections to the wall, caused by large base uplifting; (2) rupture at the plate-shell junction, caused by excessive joint stresses; (3) buckling of the tank wall, caused by large axial compressive stresses; and (4) failure of the soils underneath, caused by excessive foundation penetrations [e.g., Smoots 1973, Gates 1980, Manos and Clough 1985, "Northridge" 1994].

In order to understand the behavior of flexibly supported unanchored tanks, Manos and Clough (1982), Cambra (1982), Sakai *et al.* (1988), and Akiyama and Yamaguchi (1988) conducted static-tilt and shake table tests on scaled models of these tanks. The flexibility of the foundation soils was simulated by placing a 1 in. (2.54 cm) thick rubber pad under the base plate. It was shown that the foundation flexibility reduces significantly the axial compressive stresses, but increases base uplifting, foundation penetration, and hoop compressive stresses in the tank wall.

In this paper, a summary of a systematic analytical study on the seismic response analysis of flexibly supported, unanchored tanks is presented.

SYSTEM CONSIDERED

Shown in Fig. 1a is a cylindrical tank of radius R, filled to a height H with a liquid of mass density ρ_l. Resting on a flexible base of subgrade modulus κ, the tank is excited by a unidirectional component of horizontal ground motion $\ddot{x}_g(t)$, the intensity of which is sufficient to induce rocking of its wall and partial uplifting of its flexible base plate, as shown in Fig. 1b. After uplifting, the contact between the wall and the foundation is confined to an arc of central angle δ. The maximum width of the uplifted portion of the base plate is denoted by L. Points in the tank are specified with a cylindrical coordinate system (r, ϕ, z), the origin of which is taken at the center of the base plate.

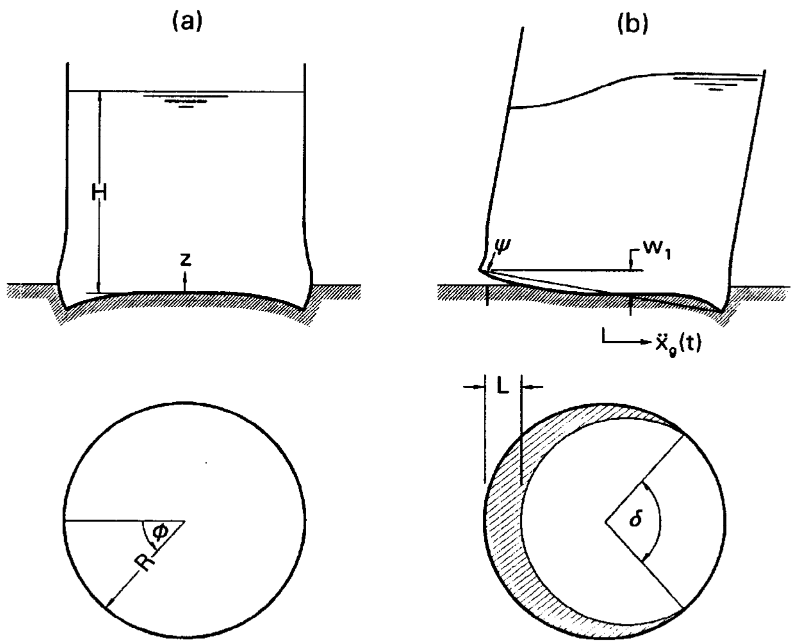

Fig. 1. Tank-liquid system considered.

BASE PLATE ANALYSIS

Model and Solution Method

The base plate is represented by n flexibly supported, semiinfinite beams that are connected at their ends to the cylindrical wall of the tank, as shown in Fig. 2. Beams are loaded by the hydrostatic plus hydrodynamic base pressures. For clarity, only few beams are shown in Fig. 2. There is clearly an overlap between the neighboring beams, but, for the extremely small widths of the uplifted region, the overlap is quite negligible. Based on experimental studies by Sakai *et al.* (1988) and Akiyama and Yamaguchi (1988), it is assumed that the plate boundary remains in one plane at all times.

The relationship between the overturning moment M_T and base rotation ψ is established by considering small increments of base rotation $\Delta\psi$. For each increment, the vertical displacement at the end of each beam is obtained first, the uplifting force at the end of each beam is computed next, and the moment resultant of the uplifting forces is computed in the end to yield a value of M_T.

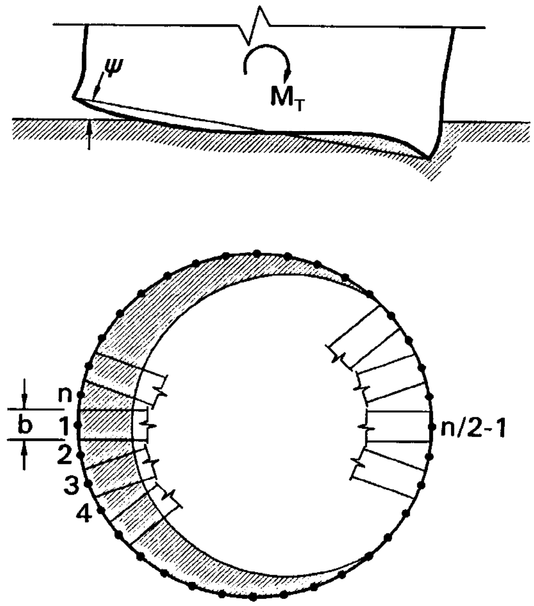

Fig. 2. Base plate model.

Numerical Results

Shown in Fig. 3a is a representative plot of the M_T--ψ relationship obtained for three different load cycles of size $\psi = 0.2°$, $0.35°$ and $0.5°$. The moment M_T in this plot is expressed as a percentage of $W_l R$, where W_l is the total liquid weight. For the results shown, the base plate thickness $h = R/2000$, the normalized yield stress $\sigma_y/p = 1800$ (p = hydrostatic base pressure), and the normalized Young's modulus of elasticity $E/p = 1.5\times10^6$. The plate is constrained at its boundary by the wall of the tank of uniform thickness $h_s = R/1000$. The weight of the tank wall W is $0.015W_l$. The normalized subgrade modulus $\kappa h/p = 2$.

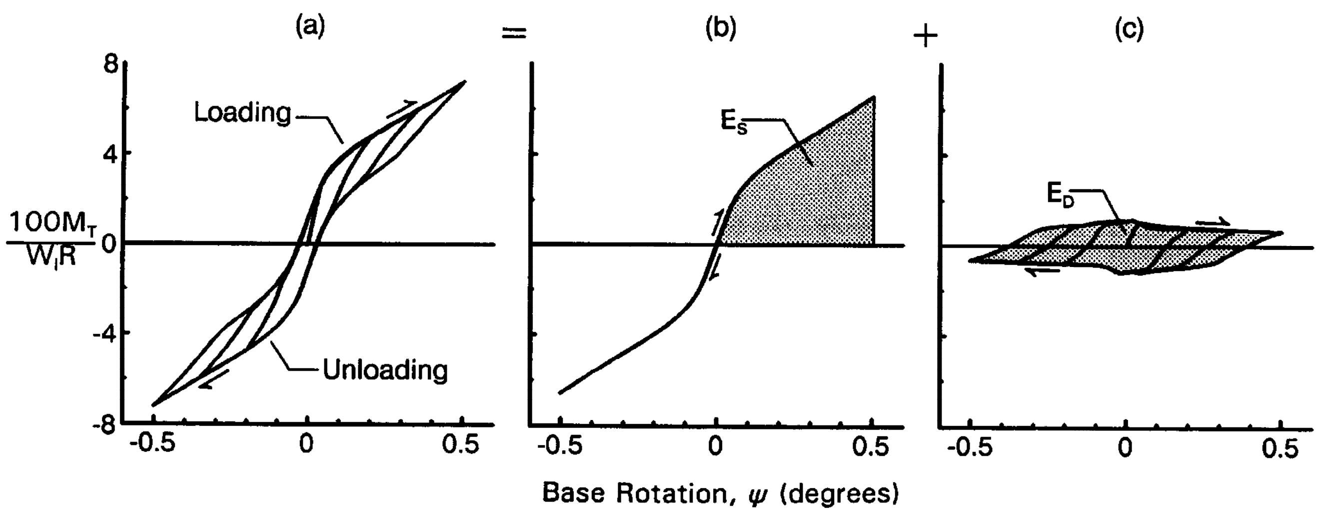

Fig. 3. Plots of: (a) Moment-rotation diagram; (b) Skeleton stiffness curve; and (c) Hysteresis loops for the flexibly supported base plate.

The slope of the M_T-- ψ plot represents the uplifting stiffness of the base plate, whereas the area enclosed within a load cycle represents the hysteretic energy loss due to plastic yielding at the plate boundary. It is desired to separate these two effects. Note that the loading and unloading paths are parallel to each other, and therefore represent the same stiffness. The area of the hysteresis loop, hence the amount of damping, is determined by the distance between the two paths. The distance reduces as the yield moment M_y reduces and in the limiting case when $M_y = 0$ the loading and unloading paths become identical and the hysteresis loop disappears completely, as shown in Fig. 3b. The plot of Fig. 3b is therefore the skeleton stiffness of the base plate, which is obtained by assuming a pin condition (zero moment) at the plate boundary. The damping curve (hysteresis loop) is obtained by subtracting the skeleton stiffness from the M_T--ψ plot in Fig. 3a, and is shown in Fig. 3c.

The area E_S under the skeleton stiffness curve (Fig. 3b) denotes the 'recoverable' strain energy, and the area E_D within the hysteresis loop (Fig. 3c) denotes the dissipated energy in a load cycle. Values of E_S and E_D for the three load cycles are shown in Table 1. The values of $\zeta_h = 100E_D/4\pi E_S$, the percentage effective viscous damping, are also shown in Table 1.

$\lvert\psi\rvert$ (deg)	$\frac{10^6 E_S}{W_l R}$	$\frac{10^6 E_D}{W_l R}$	ς_h (%)
0.2	94	89	7.5
0.35	216	167	6.2
0.5	363	236	5.2

Table 1. Values of E_S, E_D and ς_h for three load cycles shown in Fig. 3.

SEISMIC RESPONSE ANALYSIS

Model

The hydrodynamic pressures in a seismically excited tank are generated by the impulsive action of the liquid moving rigidly with the tank wall and the convective action of the liquid moving in long-period sloshing modes near the free surface. Of the two, the impulsive action usually dominates the response and in most cases fundamental impulsive mode alone provides satisfactory results. The convective response, which is required for freeboard purposes, may be computed independently of the impulsive response in an approximate analysis.

The hydrodynamic pressures in a flat bottom tank, responding in its fundamental impulsive mode of vibration, are induced by the translational and rocking motion of the tank wall, as well as, by the rocking motion of the tank base plate. For an uplifting tank, the latter contribution to the hydrodynamic pressures is expected to be negligible because in this case only a small portion of the total area of the base plate actually participates in the rocking motion (Malhotra and Veletsos 1994). For the purpose of this study, it is therefore assumed that: (1) The tank-liquid system responds in its fundamental impulsive mode, and (2) the hydrodynamic pressures are generated by the translational and rocking motions of the tank wall only. The system under these assumptions may be represented by the single-degree-of-freedom (SDOF) model shown in Fig. 4, in which the mass m represents the portion of the liquid associated with the fundamental impulsive mode, and $\bar{h}$ the height of the resultant of the hydrodynamic wall pressures. The rotational base spring represents the rocking resistance of the base plate. The moment-rotation relationship for the spring is established by the method of base plate analysis discussed earlier.

Solution Method

The equations governing the motion of the mass in Fig. 4 are:

$$m\ddot{u}_o + c(\dot{u}_o - \bar{h}\,\dot{\psi}) + k(u_o - \bar{h}\,\psi) = -m\ddot{x}_g(t) \tag{1a}$$

$$[c(\dot{u}_o - \bar{h}\,\dot{\psi}) + k(u_o - \bar{h}\,\psi)]\bar{h} = M_T(\psi) + c_\psi\dot{\psi} \tag{1b}$$

where u_o = overall horizontal displacement of the mass relative to the moving base; ψ = rotation of the base; a dot superscript denotes differentiation with respect to time t; c = damping coefficient for the tank in its fixed-base condition; c_ψ = rotational damping coefficient that accounts for soil damping; k = stiffness of the superstructure; and $M_T(\psi)$ = moment in the base spring, as a function of the time-dependent rotation ψ.

Because the M_T--ψ relationship is nonlinear, Eqs. (1a) and (1b) are solved incrementally, using the linear acceleration method (Clough and Penzien 1993). For each value of base rotation ψ computed from the solution of (1a) and (1b), important responses, such as uplifting of plate boundary, radial separation between the base plate and foundation, plastic rotations at plate boundary, axial and hoop compressive stresses in tank wall, and shear force in tank wall, are obtained concurrently from the analysis of uplifted plate.

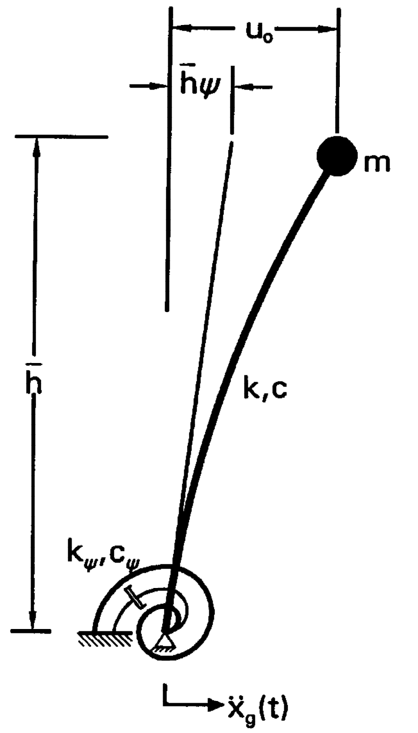

Fig. 4. Model of tank-liquid system (Fig. 1) for seismic response analysis.

Numerical Results

The tank examined is a steel tank of 50 ft (15.2 m) radius, filled with water to a height of 40 ft (12.2 m). The shell of the tank has a maximum thickness h_s = 0.54 in. (1.37 cm) at the base; and the base plate is of uniform thickness h = 0.41 in. (1.04 cm). The material properties for the tank are: Young's modulus of elasticity $E = 29\times10^3$ ksi (200 GPa), yield stress σ_y = 36 ksi (248 MPa), and Poisson's ratio ν = 0.3. The unit weights of water and the tank material are: $\rho_l g$ = 62.4 pcf (9.81 kN/m^3) and ρ_t = 490 pcf (77 kN/ m^3), respectively. The total weight of tank wall and roof W = 316 kips (1.41 MN). The system parameters for the first impulsive mode of vibration are obtained from Veletsos and Tang (1990) for an assumed value of the equivalent uniform thickness of the shell h_e = 0.45 in. (1.14 cm). These parameters are: mg = 8,855 kips (39.4 MN), $\bar{h}$ = 16.2 ft (4.9 m), and period of the fixed-base system T = 0.22 sec. The damping factor c is such that the system exhibits 2% critical damping in its fixed-base condition. The rotational damping factor $c_\psi = 2.1\times10^6$ kip-in-sec (237 MN-m-sec), which gives an effective damping ratio of ζ_e = 5% critical.

The tank is examined for these three conditions of support: (1) Fully anchored on a rigid foundation, (2) unanchored on a rigid foundation, and (3) unanchored on a flexible foundation. The subgrade modulus for the flexible foundation is $\kappa = 200$ pci (54.3 N/cm^3), which is representative of a compacted gravel fill. A value 100 times larger, $\kappa = 20{,}000$ pci (5.43 kN/ cm^3), is assumed for the rigid foundation. The ground motion considered was the first 6.3 seconds of the 1940 El Centro earthquake record. The record is scaled to a peak value of 0.4g. Although more strong-motion records are now available, the El Centro record was used to facilitate comparison with other studies using this record.

Response Time Histories

Shown in Fig. 5a are the plots of the moment M_T in the rotational spring for three different conditions of support described above. A comparison between the top and the middle plot shows that, for a rigidly supported tank, a change from fully anchored to unanchored condition causes the response period to elongate from 0.22 sec to about 0.4 sec, and the peak base moment to reduce by nearly 25%. A comparison between the middle and the bottom plots in Fig. 4a shows that a change from rigid to flexible foundation causes the response period to elongate further to about 0.55 sec and the overturning base moment to reduce by an additional 10%.

In Fig. 5b are shown the plots of the axial stress at the base of the tank wall computed at $\phi = 180°$ (Fig. 1a); negative values imply compression. A comparison between the top and middle plots shows that, for a rigidly supported tank, a change from fully anchored to unanchored condition causes the axial compressive stress to increase to nearly four times. This dramatic increase is due to considerably small contact between the wall and the foundation of a rigidly supported uplifting tank. A comparison between the middle and the bottom plots in Fig. 5b shows that, for an unanchored tank, a change from rigid to flexible foundation causes the axial compressive stress to reduce to less than one third. This reduction is due to significantly increased contact between the wall and the foundation for a flexibly supported uplifting tank.

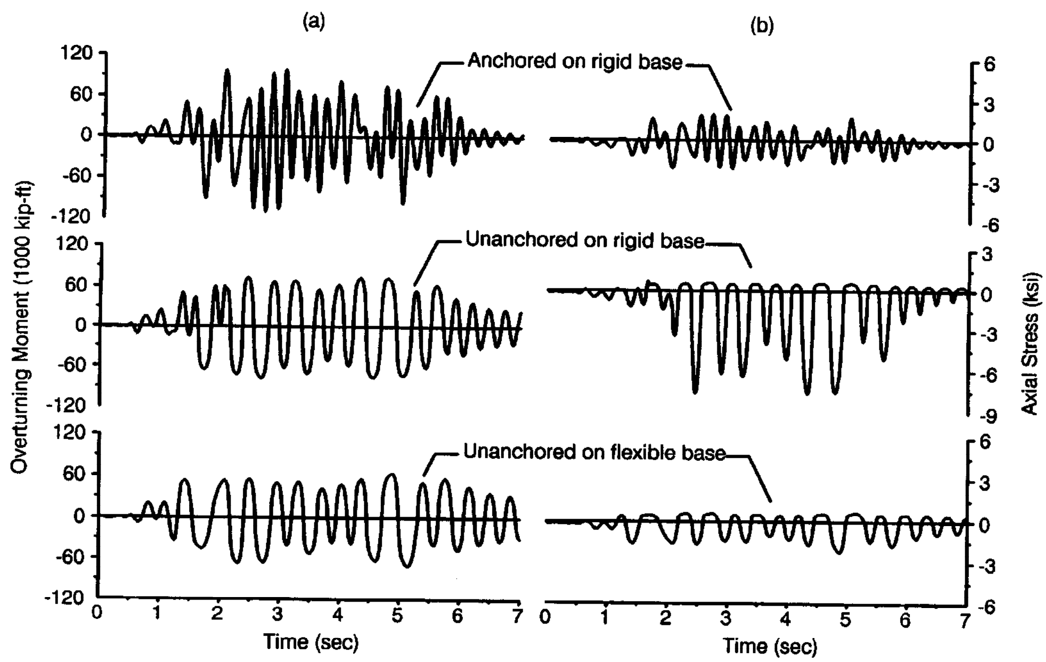

Fig. 5. Histories of overturning base moment and axial stress in tank wall for three different conditions of support.

Circumferential Distribution of Responses

The plots of circumferential distribution of selected responses at the base of rigidly and flexibly supported unanchored tank are shown in Fig. 6. The plots are at an instant of time when the response of the tank is maximum. The dashed line represents the footprint of the tank wall and the solid line represents the magnitude of the response; positive values are plotted radially inward. The rigidly supported tank experiences an uplift of 5.2 in. (13.2 cm) at $\phi = 180°$ and practically no foundation penetration at $\phi = 0°$.

The flexibly supported tank, on the other hand, experiences an uplift of 6.1 in. (15.5 cm) and a foundation penetration of 1.6 in. (4.1 cm), as shown in Fig. 6b. The maximum separation between the base plate and the foundation is 41 in. (104 cm) for the rigidly supported, and 37 in. (94 cm) for the flexibly supported tank. These values are significantly small compared to the radius of the tank which is 50 ft (15.2 m). The net area of the uplifted portion of the base plate is about 5% of the total base area for both tanks.

The axial force generated by the moment M_T and the weight of tank W is distributed over an arc of central angle $\delta = 40°$ for the rigidly supported, and $\delta = 113°$ for the flexibly supported tank. The maximum axial compressive stress for the rigidly supported tank is 8.3 ksi (57.2 MPa), which is more than thrice the value of 2.6 ksi (17.9 MPa) for the flexibly supported tank (Fig. 6b).

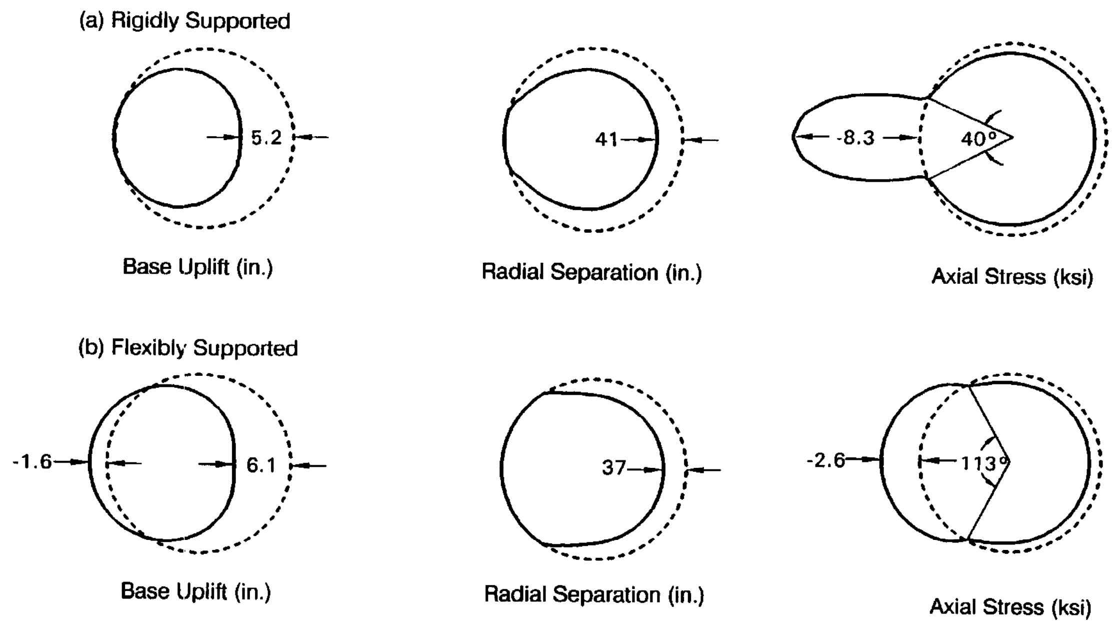

Fig. 6. Circumferential distribution of responses at the base of rigidly and flexibly supported unanchored tank.

CONCLUSIONS

An approximate method has been formulated for seismic response analysis of unanchored liquid-containing cylindrical tanks that are supported directly on flexible soil foundations. The method is highly efficient as it requires less than an hour of computing time on a personal computer. The trends observed from the detailed parametric studies are in agreement with those observed experimentally by others. Following conclusions are based on the numerical results obtained in this study.

1. Uplifting at the base of tanks reduces significantly the magnitude of the hydrodynamic forces generated by ground shaking. The flexibility of the supporting foundation further reduces the hydrodynamic forces.

2. For rigidly supported unanchored tanks, base uplifting is associated with significantly increased axial compressive stresses in the tank wall. For flexibly supported tanks, on the other hand, base uplifting usually does not lead to a significant increase in the axial compressive stresses. Unanchored tanks on flexible soils are, therefore, less likely to experience 'elephant-foot' type buckling of their walls as compared to similar tanks on rigid concrete mat type foundations.

3. The flexibility of the foundation may allow significant foundation penetration and may lead to greater plastic rotations at the plate boundary and, therefore, greater dissipation of energy due to hysteretic action. For the same reason, the likelihood of rupture at the plate-shell junction is greater for a flexibly

supported tank. The radial separation between the base plate and the foundation is not affected significantly by changes in foundation flexibility.

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