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NONLINEAR MULTI-INTERFACE FINITE ELEMENT METHOD FOR PREDICTION OF HYSTERETIC RESPONSE OF REINFORCED CONCRETE STRUCTURES

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ABSTRACT

A finite element for nonlinear analysis with more interface elements is formulated, and a **M–φ** formulation **(MF-NONMIFE)** as well with distributed nonlinearity that is introduced on a cross-section level taking into account the interaction among bending moment and varying axial force. For that purpose for each interface element are defined: **(1)** a family of trilinear moment-curvature relations for different levels of constant axial load, which covers the range of expected variation of axial force within the element, and **(2)** the stress-strain relation for concrete as well from which, depending on current level of deformation, the axial force magnitude at each moment of time is determined. The verification of the model is demonstrated by comparing the analytical and experimental results obtained from quasi-static tests of R/C columns within a portal frame subjected to constant and variable axial force and reversed cyclic bending load. Since the model is based on the moment-curvature relations the accuracy of the predict hysteretic response of reinforced concrete structural section and reinforced concrete integral structures generally depend on the accuracy of the characteristics which are used for definition of the input parameters for these relations.

KEYWORDS

Finite element; nonlinear analysis; bending moment; variable axial force; interface element; moment-curvature relations; stress-strain concrete relation, quasi-static test; reinforced concrete; columns.

INTRODUCTION

During the past decade, various simplified models have been proposed simulating the hysteretic behaviour of isolated reinforced concrete (R/C) elements, columns or beams. Introducing certain simplification and corresponding assumptions, applicability of the models has been generally limited to the specified cases. Most of the proposed analytical models are developed based on specified force-displacement or moment-curvature hysteretic relations, in which case the assumption of constant axial force during structural vibration has been additionally introduced. However, the actual behaviour of R/C structural elements and complete frame structure under strong seismic excitation is nonlinear, with dominant effect of interaction between the bending moment and variable axial force, especially to exterior base-story columns, that are not taken into account in the computation according to regulation in our country and the regulations of more countries throughout the world. Therefore, the development of a mathematical model that will predict the nonlinear behaviour of structures with consideration of different phenomena, is a subject to many investigations that have been performed world wide for the last ten years (Bedell et al., 1983; Lai et al., 1984; Li et al., 1988; Low et al., 1987; Ristic 1988).

For this purpose, the authors proposed a nonlinear multi-interface finite element model, **M–φ** formulation **(MF-NONMIFE),** for prediction of hysteretic response of R/C members, which is able to simulate the interaction. The parameters of the model were determined on the basis of materials properties, member section geometry and expected variation of axial force of each element separately.

FORMULATION OF THE M–φ FINITE ELEMENT

The proposed formulation of the nonlinear multi-interface finite element represents a modification of the originally developed stress-strain finite element, SS-NONMIFE (Ristic, 1988), that is incorporated in the originally developed computer program **NORA** (Nonlinear Response Analysis Program). An approach, using incremental numerical solution, is applied to the hysteretic response computation for R/C cross sections, elements and structural systems, including variation of axial loads. The first and an important step in the mathematical model is definition of the corresponding nonlinear stiffness matrix of the element for presentation of its nonlinear behaviour under the effect of generalized loads. From this point of view, several main assumption in the proposed model have been introduced.

Finite element proposed in the present study is defined by two nodal points connected by a straight line, each of them having three global degrees of freedom-two translation and one rotation, Fig. 1.(a). In the local coordinate system, the element is characterized by two rotation at the ends and an axial deformation associated with two end moments and axial force, respectively. It is divided into a finite number of sub-elements, Fig. 1.(b), which are defined between corresponding interface elements **(IE)**, initially specified along the structural member. The cross-sections (IE) are used to include the induced nonlinearity and their extension along the length of the element depending on the previous loading history and the current axial force level. Each of the interface element **(IE)** has two local degrees of freedom: axial deformation ε_{as} - strain at the plastic centered of the cross-section, and φ_s -interface element curvature, Fig. 1.(e).

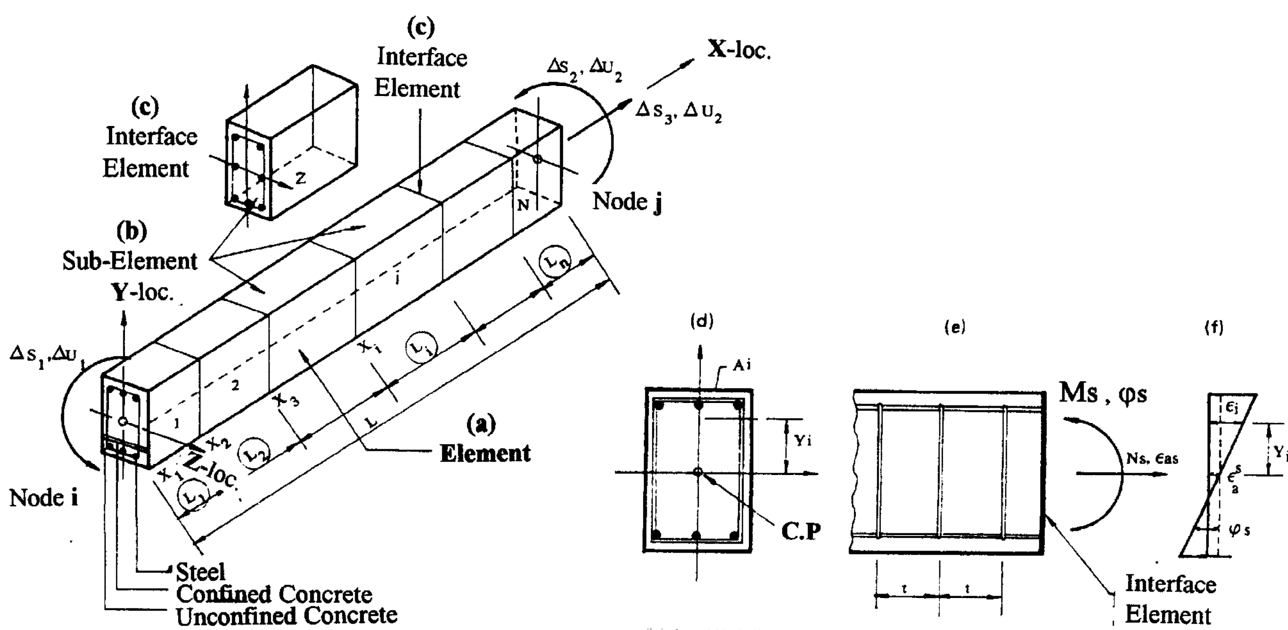

Fig. 1. Formulation of **MF-NONMIFE** Model; (a) Element; (b) Sub-Element; **(c)** Interface Element **(IE);** (d) Cross Section; (e) Interface Element Force and Degrees of Freedom; (f) Strain Distribution

For each of IE, which are independent and generally different, are defined: **(a)** Stress-strain **(σ–ε)** relationship for the concrete in which all the characteristic parameters are included, Fig. 2.; and **(b)** Input set-family of trilinear **M–φ** relations for different axial force levels depending on the expected variation of axial forces in the element, defined by previous analysis of the element cross-section, Fig. 3.

The base assumption in definition of the nonlinear stiffness matrix of the finite element is that the plane cross-section remains plane after the deformations, which leads to the linear strain variation with the depth. Nonlinear element tangent stiffness matrix is derived for each solution step by inversion of the current element flexibility matrix. Assuming linear variation of flexibility along the length to the sub-element , the element flexibility matrix can be assembled based on calculated contributions from each sub-element, along the element length, by closed-form integration, taking into account current state of the corresponding interface element flexibilities. The interface element current flexibility matrices are computed by inversion of the previously calculated interfaces element stiffness matrices. According to this the location of the cross-sections is selected that it enables a realistic presentation of the variation of flexibility along the element during its hysteretic behaviour. In general, moment and curvature distributions are know a priori, which make difficulties in the optimum spacing of the interface elements. In addition, because the element flexibility distribution changes due to the loading, unloading, load reversals, as well as the moment-axial load interaction, the spacing of larger number of interface elements along the member may be recommended. The shape functions are not determined previously, but are obtained using the current matrixes that connect the deformations of the IE and the displacements at the ends of the element, and they depend on the moment state of the individual cross-sections and are modified in each step-solution. It is important to note that the effects of shrinkage and yielding of concrete as well as shear deformations are neglected. The external loads act on the nodal points of the elements, in the direction of the global degrees of freedom of each node.

The nonlinear **σ–ε** model for the concrete is presented by a polygonal envelope curve with 6 lows of loading and unloading, Fig. 2. The unloading stiffness equals the initial stiffness, with an excluded positive part, not allowing the occurrence of the tensile axial forces. This relationship enables definition of axial stiffness and axial force level at each moment of time. For generalization of the procedure of simulation of **M–φ** relationships, the complete Takeda's trilinear hysteretic model was used (Takeda et al., 1970). Each curve of these relationships is defined by eight parameters (6+2). The first six representing data on the curvature and moment at the three characteristic points C, Y, U, and an additional two data defining the lower and the upper limit of axial force for which the corresponding relationship is valid.

σ_u(MPa)	σ_M	σ_R	ϵ_U	ϵ_M	ϵ_R	ϵ_L
21.5	28.5	28,5	0.001	0.002	0.0035	0.01

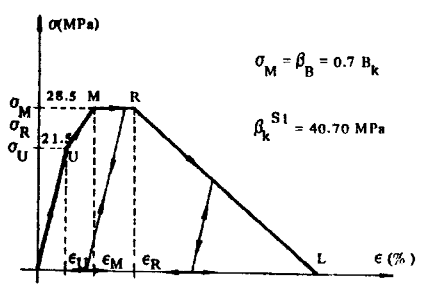

Fig. 2. **σ–ε** relationship for concrete which defined current axial stiffness

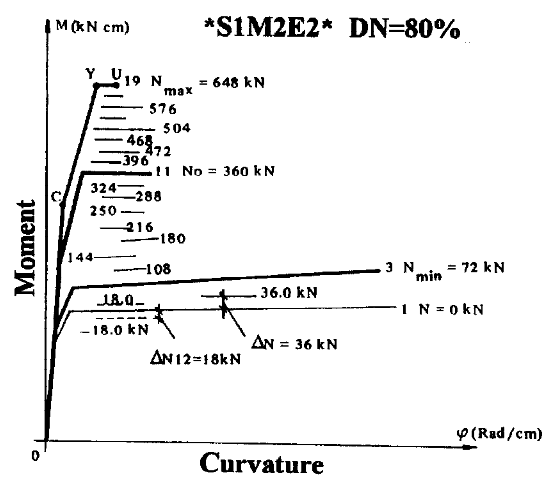

Fig. 3. Input set-family of **M–φ** relationships for different axial force levels

The proposed algorithm for mathematical modeling enables definition of the axial force value at each step of the solution, and hence selection of a corresponding **M–φ** curve from the family of the relationships, defining in this way the current bending stiffness of the element. The condition for a continuous variation of axial load is sufficient enough to provide a continuous selection of the corresponding moment-curvature relation.

The presented procedure for computation of the current element tangent stiffness matrix, depending on the degree of involved nonlinearity, is based on the approximation that is constant during the discrete time interval of integration and that change in stiffness due to the nonlinear behaviour of the elements is realized at the transition from one interval into another. The relation between the tangential stiffness matrix and time-dependent external forces is established by means of hysteretic model.

VERIFICATION OF THE PROPOSED MATHEMATICAL MODEL

In order to obtain necessary experimental results for actual hysteretic behaviour of R/C elements , a number of quasi-static tests have been conducted, in which case R/C columns within a portal frame were subjected to constant and variable axial force and to reversed cyclic lateral load (Oncevska, 1992, Oncevska et al. , 1994a, b).

A total of ten R/C specimens in scale 1:2 have been tested. Four elements were subjected to constant axial forces, whereas six of them were tested under variable axial forces. The column cross section of a typical model was designed to be **20/25 cm, 130 cm** high. The distance between columns in the portal frame was **150 cm** and they were connected with horizontal rigid beams of size **40/60/250 (270) cm** at both ends. Two types of models have been constructed each having five identical elements differing only in the total reinforcement percentage as follows: μ = **1.08% (6φ10mm)** for columns of **No.1-M1** model and μ = **2.14% (6φ14mm)** for columns of **No.2-M2** model. The transverse reinforcement is constructed with closed stirrups (ties) φ6mm placed at a distance of 7.5 cm, that is 5 cm close to the joints. Smooth reinforcement of **C240/360** (σ_y=240 MPa) was used. The designed crushing strength of the concrete was **MB30** (30 MPa).

The performed specimens have been tested in a horizontal position. The experiment was conducted by controlling the cyclic displacement after the constant axial force had been applied. The horizontal displacement history was preliminary prescribed for each of the specimens, considering the expected response. The amplitude of the exited displacement was increased in steps, after every 3 cycles, and the applied variable axial load for the experimental test was considered to be proportional to the applied cyclic lateral load as follows: $\mathbf{N_{1,2}(t) = N_0 \pm \alpha F(t)}$, where : $\alpha = \alpha_1 - \alpha_0$; $\alpha_1 = (\mathbf{N_{min/max} - N_0)/F_{max}}$; $\mathbf{N_{min}}$ and $\mathbf{N_{max}}$ are the extreme values of the axial forces for corresponding variation DN(%) expressed as a percentage of the gravity load $\mathbf{N_0}$; α **=0.72** and it is a coefficient which defines the relation between the axial forces and the lateral cyclic force for tests in which constant axial force is applied; F_{max} is a sum of the ultimate values of the horizontal forces for the two side rigid column loaded with constant axial forces N_{max}/N_{min}. Coefficient α is taken different for each test which enables that prescribed axial force variation in columns (**DN**, $\mathbf{N_{max}}$, $\mathbf{N_{min}}$) is achieved when the ultimate strength of the model is reached. Using the collected experimental data actual force-displacement hysteretic loops have been plotted for each of the 10 tested specimen separately.

Generally, the experimental results for the tested models show that the columns loaded with variable axial force of higher intensity show larger initial secant stiffness and larger load carrying capacity. On the contrary, in the region of post ultimate displacements, as it is the case in structures subjected to earthquake effects, the degrading level of ultimate load carrying capacity is much larger in high compressive columns. This points out the importance of taking into consideration the current level of the axial forces in nonlinear response analysis of reinforced concrete structures subjected to earthquakes.

MATHEMATICAL MODEL AND ANALYTICAL RESULTS

The mathematical model of the tested specimens, Fig. 4., represents a nonlinear model with a local nonlinearity, discretized by a total number of 5 two-dimensional linear finite elements. The horizontal rigid beam on which the system for application of the horizontal cyclic bending force is installed and the rigid zones at its connection with the columns, are discretized by linear finite elements (3 in total). The two columns of the tested model are discretized by nonlinear MF-NONMIFE finite elements divided into 9 sub-elements interconnected by a 10 interface elements in total, including also the cross-sections at the ends of the element. Created for each interface element is a library matrix containing the input parameters of the σ–ε relationship for concrete (a total of 1 relation) and the family of trilinear idealized moment-curvature relationships (19 in total) for a constant value of axial force $N_i = N_0 \pm i \cdot \Delta N$ for $i = 1, 2, 3, \ldots n$, (ΔN- increase in axial force). In this way, inclusion of expected range of variation of axial force in the structural element is made possible. In this study, it is defined by the extreme values of axial forces N_{min} and N_{max}. This means that, for each interface element, at each time step of computation there exist 20 histeretic relationships or a total number of 200 at the level of the finite element.

In the mathematical model, the loading conditions are simulated by two loading functions. They define the history of the cyclically variable horizontal bending force (load function 1) and axial force due to gravity load with a certain number of input parameters (in this case, most frequently 18 parameters) which define its value and sign at the beginning and at the end of each of its time steps (DTF = 0.3 sec.). By combining of these two loading functions, five loading cases are defined . They thoroughly simulate the original loading state during experimental testing of each tested model taken separately.

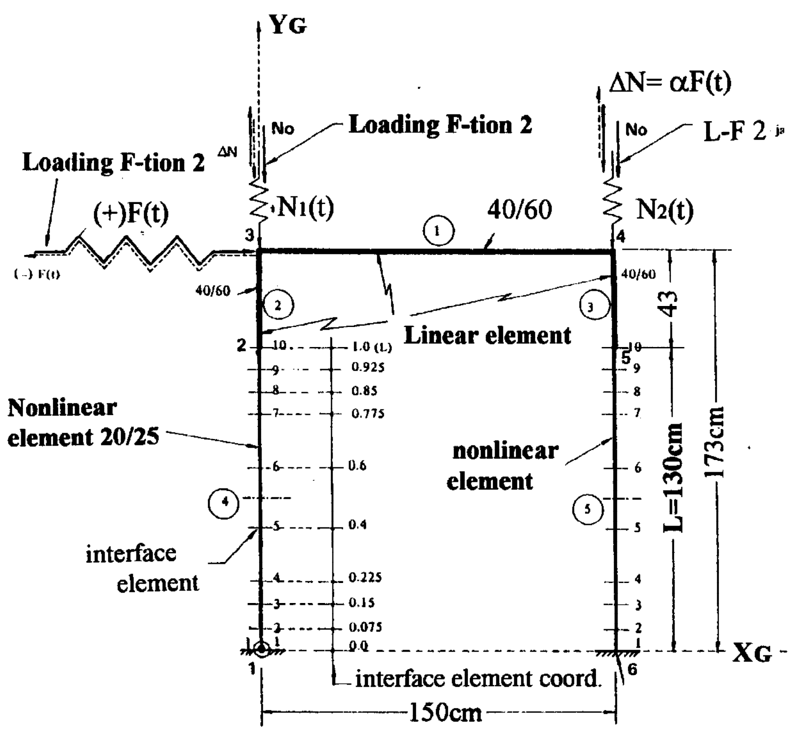

Fig. 4. Mathematical model of the tested portal frame

Presented further are the analytical results from the nonlinear analysis of the response of the portal frame model S1M2E2 under variation of axial force for 80% of the gravity load N_0 = 360kN. Based on these, the following can be generally noted:

(1) The proposed nonlinear analytical model, (MF-NONMIFE formulation) and the algorithm applied in the numerical procedure quite successfully simulate the loading conditions in quasi-static tests of elements and thoroughly provide the prescribed variation of axial forces in the frame columns, Fig. 5 (b), (c), experimentally defined by the coefficient of the linear relationship between the cyclic load and the increase in axial force;

(2) Due to variable axial forces in the columns, a nonuniform redistribution of internal shear forces takes place, Fig. 6. (b). The proposed model performs this distribution very successfully, the more compressed column sustaining a large percentage of the current level of horizontal loads than the less compressed one. This conditions asymmetry of the analytically obtained hysteretic shear force-displacement relationships that correlated quite well with the experimentally obtained envelope curves F-Δ at the left and the right column, Fig. 7.(a), (b);

(3) The distribution of the analytically obtained moment at the critical cross-section of the fixed ends of the left and the right column follows the distribution of theirs bearing capacity, Fig. 6. (c). Hence, it is provide that it is directly dependent on the distance of the inflection point from the fixed ends and is proportional to the intensity of the axial force. A higher level of axial force conditions a higher moment capacity of columns but decreases the deformability capability of the cross-section for rotation.

The practical application and the possibilities provided by the proposed concept for analysis of dynamic response of actual R/Cstructures are confirmed on the basis of the results on the linear and nonlinear response of the mathematical model of a three-span six story R/C frame (Oncevska, 1992)

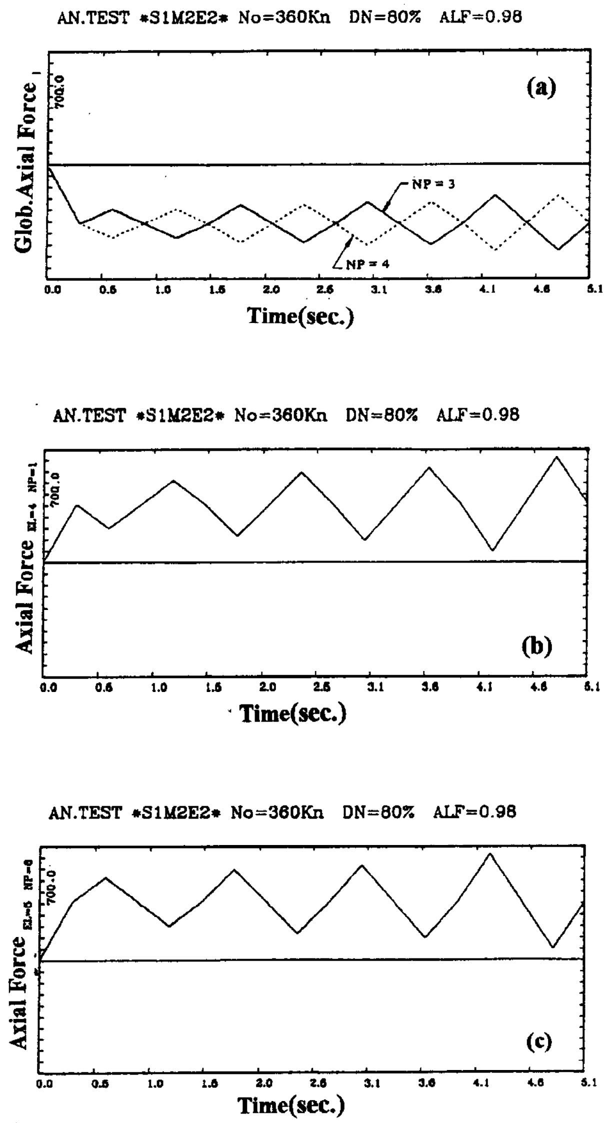

Fig. 5. History of: (a) applied axial force; the axial forces in the columns:(b) left; (c) right

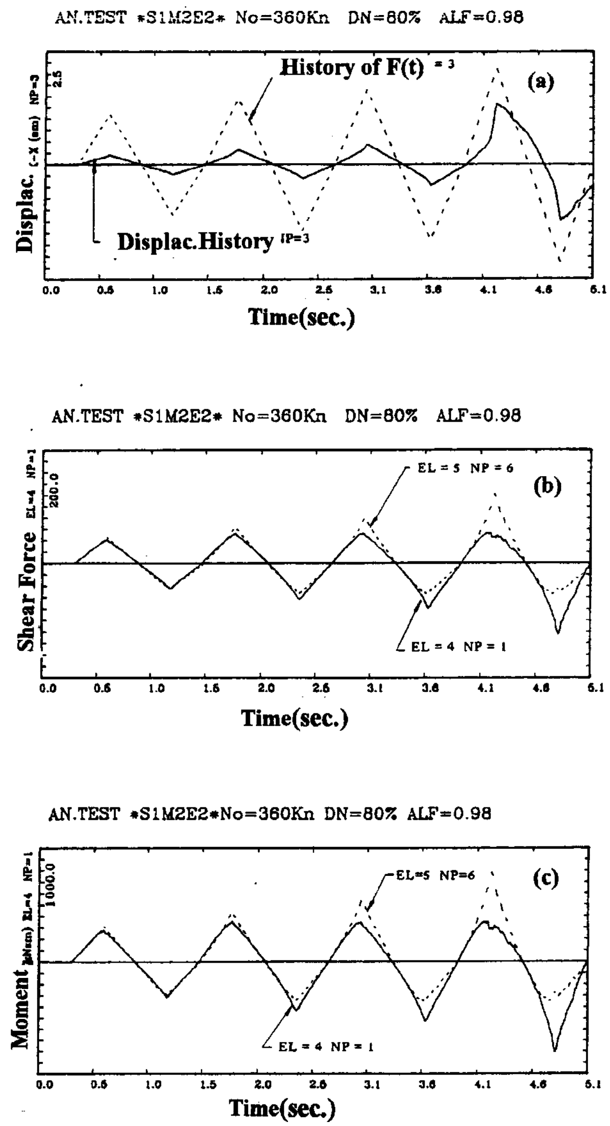

Fig. 6. History of:(a) applied lateral force; internal column's shear force(b); (c) moment

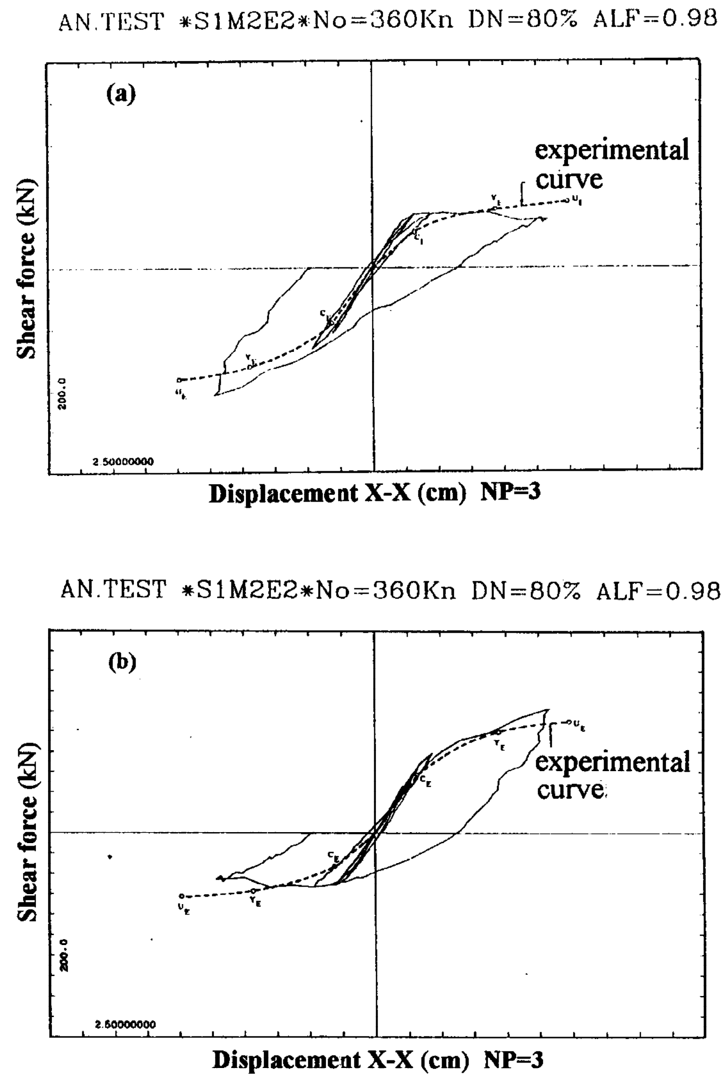

Fig. 7. Compare of analytical and experimental envelope curves of the left (a) and right (b) column: specimen S1M2E2

CONCLUSIONS

Considering the achieved satisfying level of agreement between the experimentally obtained envelope curves and the corresponding analytical hysteretic relationships, it was concluded that the proposed MF-NONMIFE finite element could be successfully applied for discretization and simulation of the nonlinear behaviour of elements exposed to interactive effects of the bending moment and variable axial force during severe seismic excitation. Since the model is based on the moment-curvature relations the accuracy of the predict hysteretic response of reinforced concrete structural section and reinforced concrete integral structures generally depend on the accuracy of the characteristics which are used for definition of the input parameters for this relations.

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REFERENCES

Bedell, R. and D.P. Abrams (1983). Scale Relationships of Concrete Columns. *Structural Research Series* **No.8302,** Civil Engineering Department, University of Colorado, Boudler, 132pp.

Lai, S.-S. G.T. Will and S. Otani (1984). Model for Inelastic Biaxial Bending of Concrete Members. *Journal of the Structural Devision, ASCE* **Vol. 110,** pp 2563-84.

Li, K.N., H. Aoyama and S. Otani (1988). Reinforced Concrete Columns under Varying Axial Load and Bi-Directional Lateral Load Reversals. *Proceedings of Ninth World Conference on Earthquake Engineering*, **Vol. VIII,** Tokyo-Kyoto, Japan, pp. 537-542.

Low, S. and J.P. Moehle (1987). Experimental Study of Reinforced Concrete Columns Subjected to Multi-Axial Cyclic loading. *EERC Report* **No.UCB/EERC-87/14,** University of California, Berkeley.

Oncevska, S.P. (1992). Stiffness and Strength Characteristics of Reinforced Concrete Columns under Variable Axial Force. *Ph.D. Dissertation,* Faculty of Civil Engineering, University "Sts. Kiril and Metodij", Skopje, Macedonia.

Oncevska, S.P., M. Gugulovski and Lj. Lazarov (1994a). Experimental Study on Nonlinear Behaviour of Reinforced Concrete Columns under Variable Axial Force. *Proceedings of the 10 Th. European Conference on Earthquake Engineering,* **Vol. 3**, Vienna, Austria , pp.2399-2404.

Oncevska, S.P. and M. Gugulovski (1994b). Stiffness and Strength Characteristics of Reinforced Concrete Columns Loaded with Variable Axial Force. *Proceedings of the Ninth Japan Earthquake Engineering Symposium,* **Vol. 3**, Tokyo, Japan, E39 pp. E-229-234.

Ristic, D. (1988). Nonlinear Behaviour and Stress-Strain Based Modeling of Reinforced Concrete Structures under Earthquake Induced Bending and Varying Axial Loads. *Ph.D. Dissertation,* Kyoto, Japan.

Takeda, T., M.A. Sozen and N.N. Nelsen (1970). Reinforced Concrete Response to Simulated Earthquake. *Journal of the Structural Devision, ASCE* **Vol. 96,** pp 2557-73.