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ANALYSIS OF LAYERED SOLID-FLUID MEDIA IN A GRAVITY FIELD

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ABSTRACT

Layered solid-fluid media in a gravity field were analyzed to investigate effects of solid-fluid interactions as well as gravity on wave propagation. A numerical method introduced here was a normal mode superposition method, in which a transient response of layered solid-fluid media in a gravity field was represented in terms of the normal modes. Numerical results showed that propagation of the gravity perturbated wave could be seen after the propagation of body waves and the Rayleigh wave. The gravity perturbated wave was characterized as the wave having a very low phase velocity and very low frequency.

KEYWORDS

layered solid-fluid media, effects of gravity, normal modes, solid-fluid interactions

INTRODUCTION

The effects of fluid layer need to be taken into consideration in the analysis of oceanic structure and/or liquefied area due to strong ground motion. In the analysis, the gravity term in the governing equation is sometimes nesessary to investigate *tsunamis* in an oceanic structure or gravity perturbated waves in a liquefied layer.

Some research for examining effects of gravity on wave propagation has been already carried out. Gilbert (1967) proposed that there is a gradual transition between Rayleigh waves and gravity perturbated waves for a very soft incompressible sediment. Chávez-Garcia and Bard (1993a,b) discussed the possibility of gravity perturbated waves in Mexico City during the September 1985 earthquakes using Aki and Larner method.

In this paper, numerical calculations are performed to investigate effects of solid-fluid interactions and gravity on wave propagation in a thin fluid layer. A normal mode superposition method (Touhei, 1995) is applied to the numerical analysis to examine the properties of normal modes for the gravity perturbated wave as well as the Rayleigh wave.

FORMULATION

Basic Equations

Following Lamb (1945), the governing equation for compressible fluid in a gravity field is expressed by,

$$\rho_f \partial_t^2 \varphi = \kappa \nabla^2 \varphi - \rho_f g \partial_z \varphi \tag{1}$$

where ρ_f denotes the mass density of fluid, ∂ the paritial diffrerential operator whose subscript refers to the parameter for the differentiation, t time, φ the velocity potential, κ the bulk modulus of the fluid, g the gravity acceleration, z the vertical coordinate with positive direction upwards. The governing equation of solid is as follows,

$$(\lambda + \mu)\nabla\nabla \cdot \boldsymbol{u} + \mu \nabla^2 \boldsymbol{u} - \rho_s \partial_t^2 \boldsymbol{u} = -\boldsymbol{f} \tag{2}$$

where λ and μ denotes the Lamé constants, $\boldsymbol{u}$ the displacement field, ρ_s the mass density of the solid, $\boldsymbol{f}$ the force density acting on the solid.

It is possible to couple the governing equations for fluid and solid using the solid-fluid interaction equations, which are the equilibrium of traction vectors and continuity of vertical displacements for both regions at the solid-fluid interface boundary. In case that the solid-fluid interface boundary lies horizontally, those solid-fluid interaction equations are as follows,

$$\boldsymbol{\sigma} + \rho \boldsymbol{n} g(\boldsymbol{n} \cdot \boldsymbol{u}) = -\boldsymbol{n} p \tag{3}$$

$$\boldsymbol{n} \cdot \nabla \varphi = \boldsymbol{n} \cdot \boldsymbol{u} \tag{4}$$

where $\boldsymbol{\sigma}$ is the traction vector of the solid due to the elastic deformation, $\boldsymbol{n}$ the normal vector outward from the solid boundary, whose direction agrees with the vertical coordinate in the present analysis and p the pressure of fluid. Note that the 2nd term of the left hand side of Eq. (3) refers to the increment of the traction vector due to a gravity effect. The relationship between the fluid pressure and displacement potential for fluid is as follows,

$$p = -\rho_f \partial_t^2 \varphi - \rho_f g \partial_z \varphi \tag{5}$$

Normal Mode Superposition Method

To formulate a normal mode superposition method, let us introduce the discrete wavenumber representations for $\boldsymbol{u}$, $\boldsymbol{f}$ and φ as follwos,

$$\boldsymbol{u} = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} [U_{zk_n}^m(z,t)\boldsymbol{R}_{k_n}^m(r,\phi) + U_{rk_n}^m(z,t)\boldsymbol{S}_{k_n}^m(r,\phi) + U_{\phi k_n}^m(z,t)\boldsymbol{T}_{k_n}^m(r,\phi)] \tag{6}$$

$$\boldsymbol{f} = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} [F_{zk_n}^m(z,t)\boldsymbol{R}_{k_n}^m(r,\phi) + F_{rk_n}^m(z,t)\boldsymbol{S}_{k_n}^m(r,\phi) + F_{\phi k_n}^m(z,t)\boldsymbol{T}_{k_n}^m(r,\phi)] \tag{7}$$

$$\varphi = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \Phi_{k_n}^m(z,t) J_m(k_n r) \exp(im\phi) \tag{8}$$

where J_m is the first kind of the Bessel function of order m, k_n the discrete wavenumber for the radial direction, (r, ϕ) the cylindrical horizontal coordinate, m the azimuthal order number and $\boldsymbol{R}$, $\boldsymbol{S}$ and $\boldsymbol{T}$ are the surface vector harmonics (Olson *et al.*. 1984).

The finite element formulation (Lysmer and Drake, 1972) is used to obtain solutions $U_{zk_n}^m$, $U_{rk_n}^m$, $U_{\phi k_n}^m$ and $\Phi_{k_n}^m$. The finite element equation for obtaining the solutions are as follows,

$$\left[[M] \frac{d^2}{dt^2} + \left[K_{k_n}^m \right] \right] \left\{ \begin{array}{c} U_{k_n}^m \\ \Phi_{k_n}^m \end{array} \right\} = \left\{ \begin{array}{c} F_{k_n}^m \\ G_{k_n}^m \end{array} \right\} \tag{9}$$

where $[M]$ is the mass matrix, $[K_{k_n}^m]$ is the stiffness matrix depending on k_n and m. These matrices are not symmetry due to the solid-fluid interaction equations (Touhei, 1995). Note that $U_{k_n}^m$ represent the nodal displacement, $\Phi_{k_n}^m$ the nodal displacment potential for fluid, $F_{k_n}^m$ the nodal force acting on the solid region, and $G_{k_n}^m$ the nodal force acting on the fluid region, which corresponds to the gradient of displacement potential for fluid, respectively.

Based on the modal analysis procedure(Touhei, 1994 and 1995), the solution of Eq. (9) is represented by,

$$\left\{ \begin{array}{c} U_{k_n}^m(t) \\ \Phi_{k_n}^m(t) \end{array} \right\} = \int_0^t \left[V_{k_n}^m\right] \left[\Lambda_{k_n}^m(t-\tau)\right] \left[V_{k_n}^m\right]^{-1} [M]^{-1} \left\{ \begin{array}{c} F_{k_n}^m(t) \\ G_{k_n}^m(t) \end{array} \right\} d\tau \tag{10}$$

where $\left[V_{k_n}^m\right]$ denotes the the modal matrix for $[M]^{-1}\left[K_{k_n}^m\right]$ and $\left[\Lambda_{k_n}^m(t-\tau)\right]$ is the diagonal matrix whose components are

$$\left[\Lambda_{k_n}^m(t)\right] = \text{diag.}\ \left[\zeta_{(1)k_n}^m(t), \ldots, \zeta_{(j)k_n}^m(t), \ldots\right] \tag{11}$$

$$\zeta_{(j)k_n}^m(t) = \frac{\sin \omega_{(j)k_n}^m\, t}{\omega_{(j)k_n}^m}\theta(t) \tag{12}$$

Note that the subscript inside the parenthesis in Eqs. (11) and (12) denotes the order of the eigenvalue for matrix $[M]^{-1}\left[K_{k_n}^m\right]$, $\omega_{k_n}^m$ the square root of the eigenvalue and $\theta(t)$ the unit step function.

Using Eqs. (6), (8) and (10), the complete solution for the layered solid-fluid media in a gravity field is represented by,

$$\left\{ \begin{array}{c} u(r,\phi,t) \\ \varphi(r,\phi,t) \end{array} \right\} = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \left[C_{k_n}^m(r,\phi)\right] \int_0^t \left[V_{k_n}^m\right] \left[\Lambda(t-\tau)\right] \left[V_{k_n}^m\right]^{-1} [M]^{-1} \left\{ \begin{array}{c} F_{k_n}^m(\tau) \\ G_{k_n}^m(\tau) \end{array} \right\} d\tau \tag{13}$$

where $\left[C_{k_n}^m(r,\phi)\right]$ denotes a matrix whose components are the surface vector harmonics and $J_m(k_n r)\exp(im\phi)$, which transform the solution in the wavenumber domain into the space domain.

NUMERICAL EXAMPLES

Analyzed Model

Numerical calculations are performed to the analyzed model shown in Fig. 1, in which a thin fluid layer overlies an elastic homogeneous half space. The analyzed model is set to investigate gravity perturbated waves in a thin fluid layer such as a liquefied layer. For this model, the depth of the fluid layer is 30 m, mass density of the fluid ρ_f 1.2 g/cm^3, sound velocity of the fluid α_f 1.5 km/s, S wave velocity of the solid β_s is 0.5 km/s, P wave velocity of the solid α_s 1.5 km/s, and mass density of the solid ρ_s 2.0 g/cm^3. A rigid boundary is imposed at a depth of 60 km from the surface of the elastic solid to obtain the normal modes as well as to define time window for which the responses are not affected by a rigid boundary. The thickness of the finite elements in the fluid region is set to 1.0 m, while that in the solid region is shown in Table 1.

Dispersion Curves

Dispersion curves for the layered solid-fluid structure with the imposed rigid boundary are presented in Fig. 2. Dispersion curves give the relationship between the phase velocity of the normal modes and frequency. In Fig. 2, the ordinate denotes the dimensionless phase velocity as the ratio of actual phase velocity β to S wave velocity of the solid β_s.

Most of the dispersion curves tend to flatten out near the P and S wave velocities, which show that undispersed body waves can be synthesized from the dispersed body waves. Among them, there are

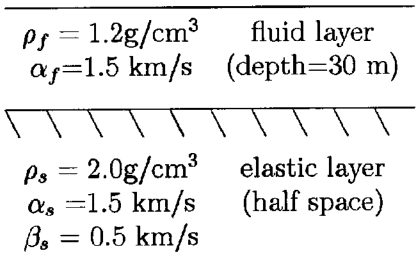


Fig. 1. Layered solid-fluid model.

Table 1 Thickness of the finite elements in the solid

depth (km)	thickness of the elements (km)
0.0-0.1	0.01
0.1-0.2	0.02
0.2-0.4	0.05
0.4-1.0	0.1
1.0-2.0	0.2
2.0-5.0	0.25
5.0-10.0	1.0
10.0-20.0	2.5
20.0-40.0	5.0
40.0-60.0	10.0

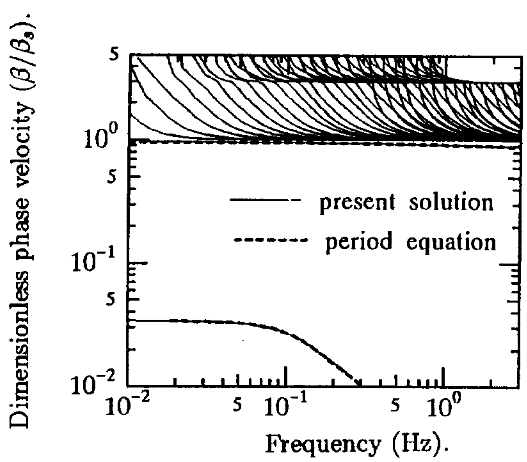


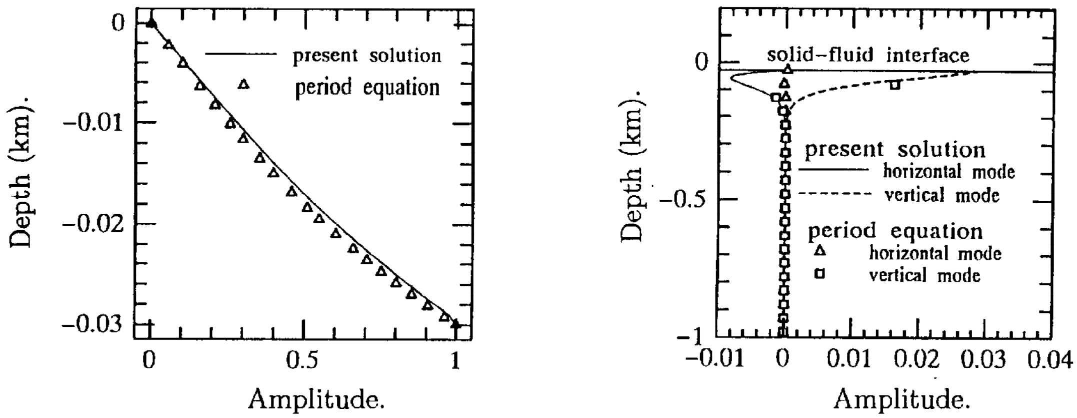
Fig. 2 Dipsersion property of normal modes

two dispersion curves which do not synthesize the body waves. The phase velocity for one of the two dispersion curves is almost equal to the S wave velocity, but always less than that velocity. This dispersion curve is for the Rayleigh wave mode. The other mode is located in the region where both the frequency and phase velocity are very low. This mode is for the gravity perturbated wave.

The dimensionless phase velocity of the gravity perturbated wave near 0 Hz is about 0.034 which agrees with the result from the following equation,

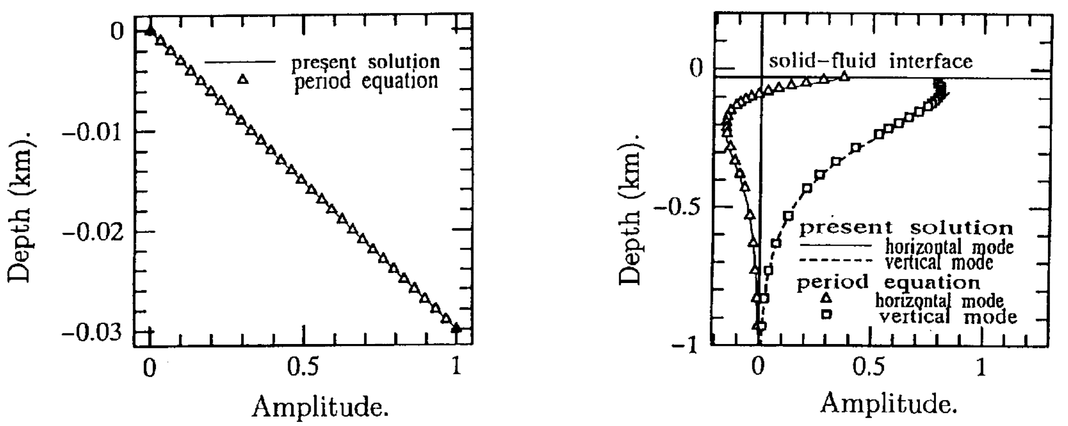
$$c_g = \sqrt{gH}/\beta_s \tag{14}$$

where H denotes the depth of the fluid layer. Equation (14) gives the phase velocity of gravity perturbated wave of incompressible fluid on a rigid bottom at the infinite wavelength. We can predict from the agreement that compressibility as well as the effect of solid-fluid interaction are small for the present gravity perturbated wave mode.



(a) Fluid pressure. (b) Solid displacement.

Fig. 3 The gravity perturbated wave mode (0.1 Hz).



(a) Fluid pressure. (b) Solid displacement.

Fig. 4 The fundamental Rayleigh wave mode (1.0 Hz).

Dispersion properties obtained from the period equation (Ewing, 1957) are added to Fig. 2. Figure 2 shows that the present solution for the Rayleigh and gravity perturbated waves are in almost complete agreement with solutions from the period equation. These results give the validity of present eigenvalue analyses.

Modal Shapes

Modal shapes for the Rayleigh and gravity perturbated wave modes are shown in Figs. 3 and 4. In these figures, modal shapes for a fluid are expressed in terms of fluid pressure. Modal shapes are normalized so that fluid pressure has a unity amplitude. To examine the accuracy of the present modal shapes, the modal shapes obtained from the period equation are added to Figs. 3 and 4. Figures 3 and 4 shows that the present modal shapes are almost in complete agreement with the solution from the period equation.

According to the modal shapes for the gravity perturbated wave, the effects of solid-fluid interactions are found to be very small. In other words, the displacement amplitude of the gravity perturbated

CONCLUSIONS

Effects of solid-fluid interactions and gravity on wave propagation in a thin fluid layer were examind using the normal mode superposition method. A transient response of layered solid-fluid media was represented in terms of normal modes. According to the numerical calculations, effects of the solid-fluid interactions on the gravity perturbated wave were very small. On the other hand, the effects of soid-fluid interactions were not small in the Rayleigh wave mode. The porpagation of the gravity perturbated wave showed that the gravity perturbated wave had a very low frequency and phase velocity, which agreed with the propertis of the dispersion curve.

REFERENCES

Chávez-Garcia, F. J. and Bard, P. (1993a). Gravity waves in Mexico City?-I. Gravity perturbated waves in an elastic solid, *Bull. Seism. Soc. Am.* **83**, 1637-1655.

Chávez-Garcia, F. J. and Bard, P. (1993b). Gravity waves in Mexico City?-II. Coupling between an anelastic solid and a fluid layer, *Bull. Seism. Soc. Am.* **83**, 1656-1675.

Ewing, W. M., Jardetzky, W. S. and Press, F. (1957). *Elastic Waves in Layered Media*, McGraw-Hill.

Gilbert, F. (1967). Gravitational perturbated elastic waves, *Bull. Seism. Soc. Am.* **57**, 783-794.

Lamb, H. (1945). *Hydrodynamics, 6th ed.*, Dover Publications, New York.

Lysmer, J. and L. A. Drake (1972). A finite element method for seismology, in *Methods of Computational Physics*, **11**, B. A. Bolt, Editor, Academic Press, New York, 181-216.

Olson, A. H., J. A. Orcutt and G. A. Frazier (1984). The discrete wavenumber/finite element method for synthetic seismograms, *Geophys. J. R. astr. Soc.* **77**, 421-460.

Touhei, T. (1994). A discrete wavenumber and normal mode superposition method for synthetic seismograms, *Bull. Seism. Soc. Am.* **84**, 1473-1483.

Touhei, T. (1995) Analysis of layered solid-fluid media using a discrete wavenumber and normal mode superposition method, *Bull. Seism. Soc. Am.* **85**, (in press).