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EVALUATION ON MECHANICAL CHARACTERISTICS OF LAMINATED RUBBER BEARINGS BY TRANSFER MATRIX METHOD

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ABSTRACT

In the application of seismic isolation system to the design, it is one of the most important items to estimate characteristics of laminated rubber bearing both accurately and objectively. So, we conducted evaluation for mechanical characteristics of laminated rubber bearings by transfer matrix method. The present report deals with the analytical method and its comparison with experimental results. Simplified method is given for the analysis of laminated rubber bearings considering geometrical nonlinearity, material nonlinearity and viscous damping. In order to compare the various properties of the bearing under designed loading conditions, such as stiffness and damping constants, the tests for natural rubber bearing and high damping rubber bearing were performed. The analytical results were compared with experimental results. The agreements are good qualitatively and as well quantitatively. Applicability of this method was confirmed. It is valuable for estimating horizontal characteristics of the bearing.

KEYWORDS

Seismic isolation; transfer matrix method; laminated rubber bearing; natural rubber bearing; high damping rubber bearing; geometrical nonlinearity; material nonlinearity; viscous damping.

INTRODUCTION

Seismic isolation is expected to be effective in reducing seismic load on important equipments or structures. The laminated rubber bearings are considered to compose one of the most promising type of seismic isolation device for structures. The adequate numerical methodology is needed to evaluate the stiffness and ultimate condition of the bearing for the design of the bearings. In this paper, numerical analysis of the bearings were conducted using transfer matrix method. The results were compared with the experimental ones to evaluate the applicability of the method for the design of the bearings.

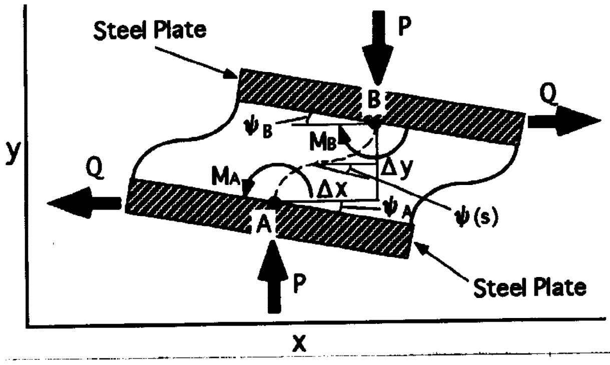
ANALYSIS METHOD

Leading of Relationship

In this method, numerical analyses of laminated rubber bearings are conducted assuming the deformation of single rubber sheet with two steel plates. The basic assumptions of deformation are as follows.

①The strain of each rubber layer is composed of shear strain, axial strain and bending strain.
②The bending of shim steel plates is assumed to be negligible.
③The material properties of rubber are assumed to be linear except for shear modulus.

The process of leading of relationship is as follows.
As in Fig.1, the balance of forces in the center points of lower and upper of rubber sheet were considered.



P : vertical force (compressive force is positive)
Q : horizontal force (right direction is positive)
M_A,M_B : bending moment at points A and B
ψ_A, ψ_B : rotation angle at points A and B
Δx : displacement along x-axis
Δy : displacement along y-axis
$\psi(s)$: rotation angle at any points

Fig. 1 Relationship between force and displacement of single rubber layer

From the relationship in Fig.1, the bending moment M_B at point B is as follows.

$$M_B = M_A - \Delta y \cdot Q - \Delta x \cdot P \tag{1}$$

assuming that the shear deformation of single rubber sheet occurs along average rotation angle ψ_m between point A and B. Therefore, the relationship between shear force T_m along ψ_m and shear displacement $\Delta\gamma$ are as follows.

$$T_m = P \cdot \sin\psi_m + Q \cdot \cos\psi_m \tag{2}$$

$$T_m = \frac{G \cdot A_r}{t_r} \cdot \Delta\gamma + \frac{\mu \cdot A_r}{t_r} \cdot \Delta\dot{\gamma} \tag{3}$$

where, G is shear modulus of rubber, t_r is thickness of rubber sheet, and A_r is an area of cross section of rubber sheet. In the same way, assuming that the axial deformation of single rubber sheet occurs along right angle of ψ_m, therefore, the relationship between axial force N_m along right angle of ψ_m and axial displacement Δs along right angle of ψ_m are as follows.

$$N_m = Q \cdot \sin\psi_m - P \cdot \cos\psi_m \tag{4}$$

$$N_m = \frac{E_c \cdot A_r}{t_r} \cdot \Delta s \tag{5}$$

where, E_c is Young's modulus of rubber concerned with compression.
In the next step, when a rotation angle ψ at any point s of single rubber sheet, axial strain ε_n of rubber sheet along right angle of ψ is as follows.

$$\varepsilon_n = \frac{\Delta s}{t_r} - \frac{d\psi}{ds} \cdot n \tag{6}$$

where, s is a coordinates along axis of rubber sheet and n is the length from the center point along ψ. From these relationships, bending moment M and axial force N in any section of rubber sheet are as follows.

$$M = -\int n \cdot E_b \cdot \varepsilon_n dA \tag{7}$$

$$N = \int E_c \cdot \varepsilon_n dA \tag{8}$$

where, E_b is Young's modulus of rubber concerned with bending. Therefore, when the change of bending moment in single rubber sheet is linear, the rate of change of rotation angle ψ at any point s is as follows.

$$\frac{d\psi}{ds} = a + b \cdot s \ (0 \le s \le t_r) \tag{9}$$

From these relationships, bending moment M_A at point A and bending moment M_B at point B are as follows.

$$M_A = E_b \cdot I \cdot a \tag{10}$$

$$M_B = E_b \cdot I \cdot (a + b \cdot t_r) \tag{11}$$

where, I is geometrical moment of inertia. The change of rotation angle $\Delta\psi(s)$ at any point is as follows.

$$\Delta\psi(s) = a \cdot s + \frac{b}{2} \cdot s^2 \tag{12}$$

So, the change of a rotation angle $\Delta\psi(t_r)$ in single rubber sheet is decided as follows.

$$\Delta\psi(t_r) = \frac{t_r}{2E_b \cdot I} \cdot (M_A + M_B) \tag{13}$$

Using the above relationships,the displacement Δx along x axis and the displacement Δy along y axis are expressed as follows.

$$\Delta x = \Delta\gamma \cdot \cos\psi_m + \frac{t_r + \Delta s}{t_r} \int_0^{t_r} \sin\psi ds \tag{14}$$

$$\Delta y = -\Delta\gamma \cdot \sin\psi_m + \frac{t_r + \Delta s}{t_r} \int_0^{t_r} \cos\psi ds \tag{15}$$

$$\psi_m = \psi_A + \frac{1}{2} \Delta\psi(t_r) \tag{16}$$

All relationships can be expressed in matrix form as follows.

$$\begin{Bmatrix} x_B \\ y_B \\ \psi_B \\ M_B \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \alpha_1 \\ 0 & 1 & 0 & \alpha_2 \\ 0 & 0 & 1 & \alpha_3 \\ 0 & 0 & 0 & 1+\alpha_4 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 1 & 0 & 0 & -2\alpha_1 \\ 0 & 1 & 0 & -2\alpha_2 \\ 0 & 0 & 1 & -\alpha_3 \\ 0 & 0 & 0 & 1-2\alpha_4 \end{bmatrix} \begin{Bmatrix} x_A \\ y_A \\ \psi_A \\ M_A \end{Bmatrix} + \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \\ \beta_3 \end{Bmatrix} \right\} \tag{17}$$

$$\alpha_1 = \frac{-(t_r+\Delta s)t_r \cdot \cos\psi_A}{6E_b \cdot I} \quad \alpha_2 = \frac{(t_r+\Delta s)t_r \cdot \sin\psi_A}{6E_b \cdot I} \quad \alpha_3 = \frac{t_r}{2E_b \cdot I} \quad \alpha_4 = \frac{-(t_r+\Delta s)t_r}{6E_b \cdot I}(Q \cdot \sin\psi_A - P \cdot \cos\psi_A)$$

$$\beta_1 = \Delta\gamma\cos\psi_m + (t_r+\Delta s)\sin\psi_m \quad \beta_2 = -\Delta\gamma\sin\psi_m + (t_r+\Delta s)\cos\psi_m$$

$$\beta_3 = \Delta\gamma(Q\sin\psi_m - P\cos\psi_m) - (t_r+\Delta s)(P\sin\psi_m - Q\cos\psi_m)$$

where, x_A is the displacement at point A along x axis, x_B is the displacement at point B along x axis which is equal to $x_A + \Delta x$, y_A is the displacement at point A along y axis, and y_B is the displacement at point B along y axis which is equal to $y_A + \Delta y$.

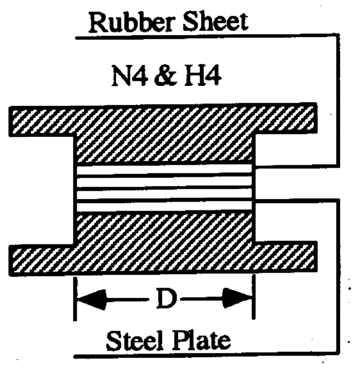
Procedure of Analysis

Analysis of the Rubber Layer at the End. Δx of the rubber layer at the end is decided from taget horizontal displacement δ of laminated rubber bearing. Δx is equal to δ /N, where, N is number of rubber sheet. In the next step, initial value of bending moment M_0 at the point A of the rubber layer at the end is assumed from (P δ +Qh)/2, where, initial value of Q is assumed from the design formula of horizontal stiffness of laminated rubber bearing. Initial value of Δy is assumed t_r . Initial value of $\Delta \psi$ is zero. By using these values, calclulations of Q, M_B and Δy are iterated up to settlement.

Analysis of the Upper Rubber Rayers. From caluculated M_B and ψ_B of the end rubber layer, M_A and ψ_A of the upper rubber layer are decided. Q is constant. Other initial values are assumed same as the rubber layer at the end. By using theses values, calclulations of Δx, Δy and M_B are iterated up to settlement.

Handle at the Center Point. Bending moment M_m of center layer of laminated rubber bearing is zero from symmetry of geometry and loading condition. So, when N is an odd number, thickness of center layer is assumed tr /2, and M_B of center layer is equal to M_m. When N is even number, M_m is evaluated at the center of center steel plate.

NUMERICAL ANALYSIS



Specimen		N4 & H4
Diameter	D(mm)	209
Thickness of a Rubber Sheet	tr(mm)	10.5
Number of Rubber Sheets	n	4
Thickness of a Steel Plate	ts(mm)	4.4
Shape Facter	S1*	4.98
Aspect Ratio	S2**	4.98

*S1=D/(4 • tr), **S2=D/(n • tr)

Fig. 2 Detail of specimens

Specimens

Natural rubber bearing N4 and high damping rubber bearing H4 with same shape factor and aspect ratio were prepared for the specimens. Fig. 2 shows the detail of the specimens.

Loading Condition

In order to investigate various properties of the test specimen under designed loading conditions, such as the stiffness and damping constants, the tests of natural rubber bearing was performed under low frequency cyclic loading condition(0.01Hz). But, the mechanical properties of a high damping rubber bearing depend on the frequency of the load. So, the high damping rubber bearing was tested under the conditions of both high frequency cyclic loading(0.5Hz) and low frequency(0.01Hz). Four cycles of sinusoidal horizontal displacement were applied to every specimen under constant vertical load. The amplitude of displacement was 200% in shear strain of rubber. And the axial stress, which was kept constant during each test, was varied from 0kgf/cm2 to -50kgf/cm2, and the records of the third cycle were adopted as the data to evaluate the characteristics.

Table 1 Mecanical properties of rubber

Rubber		Natural	High Damping	
Young´s Modulus	E_0(kgf/cm^2)	22.4	78.77	
Shear Modulus	G(kgf/cm^2)	4.548	*	
Bulk Modulus	E_∞(kgf/cm^2)	10890	15000	
Correction Coefficient	κ	0.85	0.50	
Fixed Value of Equivalent Viscous Damping h_H		1.187	0.01Hz	0.140
			0.5Hz	0.166

Apparent Young´s Modulus for Compression E_{apc}

$$E_{apc}=E_0(1+2\cdot\kappa\cdot S_1^2)$$

Young´s Modulus for Compression considering Compressibility E_c

$$E_c=\frac{E_{apc}\cdot E_\infty}{E_{apc}+E_\infty}$$

Apparent Young´s Modulus for Bending E_{apb}

$$E_{apb}=E_0(1+0.667\cdot\kappa\cdot S_1^2)$$

Young´s Modulus for Bending considering Compressibility E_b

$$E_b=\frac{E_{apb}\cdot E_\infty}{E_{apb}+E_\infty}$$

$$*G(\gamma,0.01Hz)=1.11\gamma^4-4.47\gamma^3+6.95\gamma^2-4.93\gamma+4.41$$

$$G(\gamma,f)=G(\gamma,0.01Hz)\cdot(0.99+0.67f)$$

Analitical Results

Numerical analysis of the natural rubber bearing N4 was conducted using transfer matrix method considering viscous damping of rubber. Table 2 shows the mechanical properties of each rubber material. Caluculated and experimntal results were compared to evaluate the applicability of the code. Fig. 3 shows the horizontal hysteresis loops of natural rubber bearing N4 under varied axial stress which were obtained from the test and analysis. Fig.4 shows the shear strain-vertical strain curve of natural rubber bearing N4. Horizontal hysteresis loops of N4 are approximately linear irrespective of axial stress, whereas the horizontal hysteresis loop has remarkable dependency on the axial stress of rubber. The solid line shows experimental result and dotted line shows calculated result by transfer matrix method.With dependency on axial stress, horizontal stiffness of natural rubber bearing derived from tests agreed approximately with calculated results.On the other hand, numerical analysis of high damping rubber bearing H4 was conducted considering viscous damping and material nonlinearity of shear modulus G of rubber. Fig. 5 shows the horizontal hysteresis loops of high damping rubber bearing H4 under varied loading frequency, which were obtained from the test and analysis. Fig. 6 shows the shear strain-vertical strain curve of high damping rubber bearing H4. Stiffness of high damping rubber bearings which depends on shear strain and loading frequency decreases as shear strain increases and increases as loading frequency increases. Viscous damping of high damping rubber bearing is about 15% and almost constant. The agreement between experimental result and analytical one is good for large shear strain.

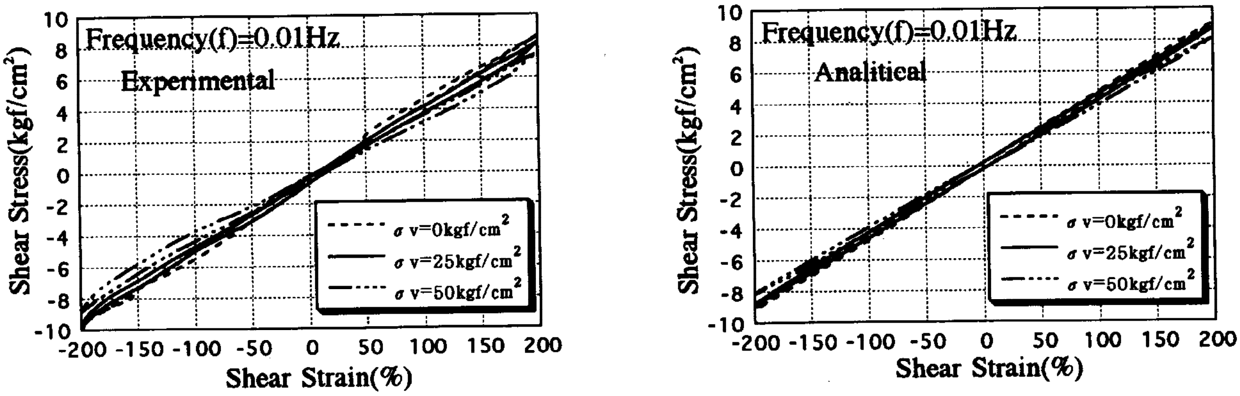


Fig. 3 Shear strain - stress hysteresis loops of N4

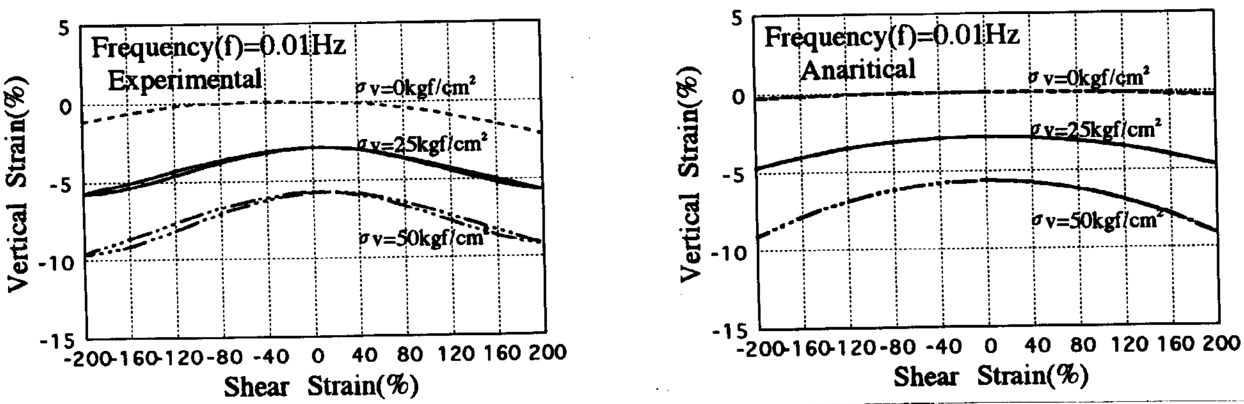


Fig. 4 Shear strain - vertical strain loops of N4

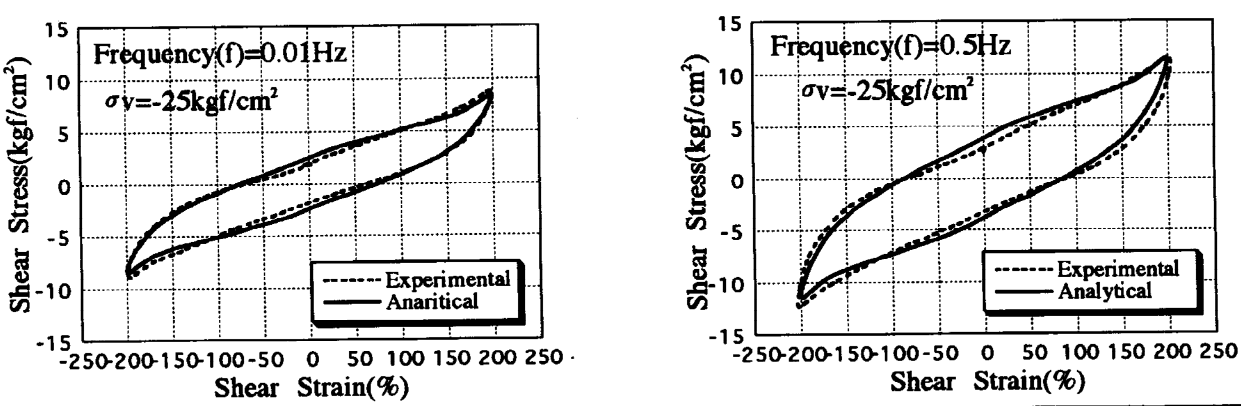


Fig. 5 Shear strain - stress hysteresis loops of H4

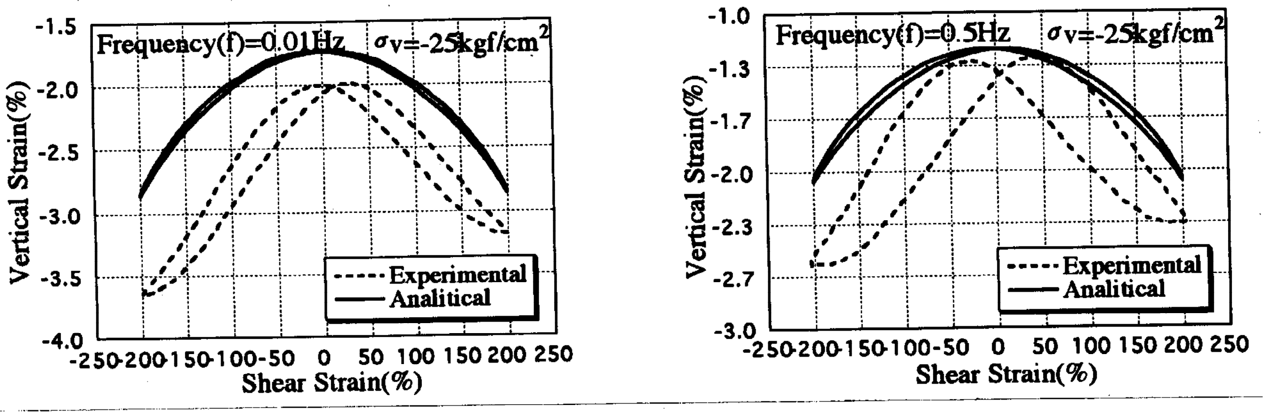


Fig.6 Shear strain - vertical strain loops of H4

CONCLUSION

The analytical results may be summarized as follows.

(1) The evaluation on mechanical properties of natural rubber bearing with small shape factor was conducted by transfer matrix method considering viscous damping. The dependency of horizontal stiffness on axial stress was simulated by this method. Horizontal hysteresis loops obtained from experiment approximately agreed with the calculated result.

(2) The evaluation on mechanical properties of high damping rubber bearings with small shape factor was conducted by transfer matrix method considering viscous damping and material nonlinearity of shear modelus G of rubber. The dependency of horizontal stiffness on shear strain and frequency was simulated by this method. Horizontal hysteresis loops obtained from experiment approximately agreed with the calculated result.

(3) Applicability of transfer matrix method was confirmed. It is valuable for estimating horizontal characteristics of the bearing.

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