



STRENGTH-REDUCTION FACTORS FOR STRUCTURES WITH ADDED DAMPING AND STIFFNESS DEVICE

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ABSTRACT

In this paper the strength-reduction factors for frame installed with Added Damping and Stiffness device made of triangular plates (TADAS), satisfying predetermined ductility ratio of TADAS device, are investigated using 20 earthquakes recorded in Taipei area and 58 rock-site earthquake records of Taiwan area. Assuming the load-displacement relation of both frame and TADAS device to be bilinear, a trilinear load-displacement relation for the entire system is constructed first. The strength-reduction factors are then computed for a given set of parametric values. In the analysis, the following criteria are adopted: (1) the ductility ratio of frame should not be greater than 2, and (2) the displacement amplification factor should be less than 5.5. For the structural period between 0.5 seconds to 5 seconds, the strength-reduction factors for TADAS device with ductility ratio equal to 6 varies from 8.3 and 10.7. A set of formulae, function of structural period and the ratio of initial stiffness of brace members to that of TADAS device, are also proposed.

KEYWORDS

Strength-reduction factor; nonlinear analysis; ADAS device; trilinear model; site effect; energy dissipation.

INTRODUCTION

For economic reasons the current seismic design practice allows structures to undergo controlled local excursions into inelastic behavior without collapse when subjected to strong ground motions. With this design philosophy the design lateral strength in seismic codes is lower, and in some cases much lower, than that required to maintain the structure in the elastic range, since the nonlinear behavior of structure during the earthquake dissipates the input energy, leading to the reduction in the forces exerted in the structure. Currently in designed practice the reduction in forces is determined by multiplying the linear elastic design spectrum by the strength-reduction factor, and only the elastic design is needed. As a result, the strength-reduction factor has been the topic of several studies (Riddell *et al.*, 1989, Miranda, 1993). All these studies indicated that the strength-reduction factor is mainly influenced by the natural period of the system, the maximum tolerable inelastic displacement demand, and the soil conditions at a site.

In recent years the use of energy dissipation devices on structures to reduce the structural responses during earthquakes has been the topic of many research interests, and some devices have already been implemented on real structures. The main philosophy in using energy dissipation device on structures is that the input energy is dissipated by device not by the inelastic behavior of the primary structural members. The prominent advantage of this can be seen from the investigation of damage of structures in recent earthquakes such as Northridge earthquake and Kobe earthquake, indicating that engineers face many difficult situations in repairing the damaged but not collapsed structures. Among many types of energy dissipation device proposed is the one called Added Damping and Stiffness (ADAS) device which can provide both initial elastic stiffness as well as hysteretic energy dissipation to structures (Su et al., 1990, Su, 1992). The ADAS device is a mechanical device made of steel plates assembled with bolts and/or welds to end block, the top of which is connected to girder while the end block is attached to the brace members. It can be installed at specific locations of either new or existing buildings. As the building sways, the inter-story drift causes the top of the plate to move horizontally relative to the bottom and the energy can be dissipated through the yielding of plates. It has been shown through experimental work that ADAS device can provide a great degree of reliability in the earthquake-resistant design or upgrading of building, and its use would lead to more reliable building performance during severe shaking, if properly designed and implemented. Consequently, it is imperative that the strength-reduction factors be developed for each type of energy dissipation device to facilitate the seismic design for structures installed with the same type of energy dissipation device. Therefore, in this study a one-story moment-resisting frame installed with ADAS device made of triangular plates (TADAS) will be considered to compute the corresponding strength-reduction factor.

CHARACTERISTICS OF TADAS DEVICE

Figure 1 depicts a TADAS device (Su, 1992). The advantage of using triangular plate is that when the free end of triangular plate is subjected to a concentrated load, it will yield uniformly over its height, and the plastic deformation will be distributed uniformly over the height of plate. It can be shown using the theory of strength of material that the initial stiffness of a triangular plate is

$$K = \frac{EBt^3}{6h^3} \quad (1)$$

where E, t, B and h are the Young's modulus, thickness, bottom width and height of triangular plate, respectively, and the relation between the load and deflection at the free end after the yielding is

$$\frac{P}{P_y} = \frac{1}{2} \left(3 - \frac{1}{\left(\frac{\Delta}{\Delta_y} \right)^2} \right) \quad (2)$$

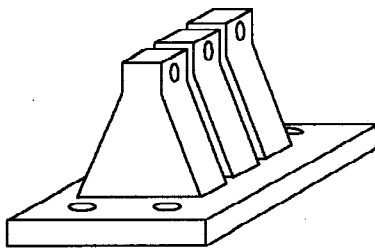


Fig. 1 TADAS device

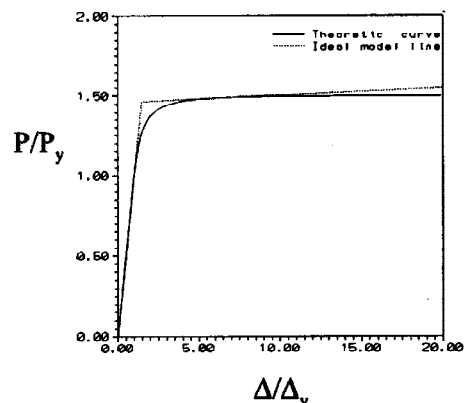


Fig. 2 Load-deflection relation for triangular plate

where P is the load acting at the free end, Δ is the deflection of free end under load P , P_y is the yield load that produces the first yielding of plate, and Δ_y , as shown below, is the deflection of free end when the first yielding of plate occurs .

$$\Delta_y = \frac{6P_y h^3}{EBt^3} \tag{3}$$

The solid line in Fig. 2 represents the theoretical load-deflection diagram of triangular plate when its bottom is fixed and the free end is subjected to a concentrated load; the straight line is for the elastic behavior, and the curve is plotted using Eq. (2), the slope of which is $1/(\Delta/\Delta_y)^3$. The dash line in Fig. 2 is the bilinear model used in this study, and the slope of straight line after yielding is taken as 0.0045 which is very near the slope of curve at $\Delta=6\Delta_y$. It should also be noted that for the bilinear model adopted in this study, the yield load and yield deflection are 1.465 times those of theoretical values. Note that if n plates are used to form the TADAS device, the total stiffness of the device is n times that given in Eq. (1).

SINGLE-DEGREE-OF-FREEDOM SYSTEM WITH TADAS DEVICE

Shown in Fig. 3 is a braced single-story moment-resisting frame; the free end of TADAS device is connected to the center of girder, while the bottom is attached to the top of brace members. It is assumed that the girder is infinitely rigid, and the system can undergo horizontal motion only. The brace members are assumed to remain elastic during vibration, while the load-displacement relation for both frame and TADAS device follows bilinear behavior. In the subsequent discussions, K_b is the stiffness of brace members, K_d , R_d , P_d , δ_y represent the initial stiffness, the ratio of post-yielding stiffness to initial stiffness, the yield strength and the yield displacement of TADAS device, respectively, K_f , R_f , P_f , δ_f are the initial stiffness, the ratio of post-yielding stiffness to initial stiffness, the yield strength and the yield displacement of frame, respectively, R_1 is the ratio of combined initial stiffness of brace members and TADAS device to the initial stiffness of frame, and R_2 is the ratio of initial stiffness of brace members to that of TADAS device.

Initial stiffness of system

If the mass of the system is assumed to be unity, then the initial stiffness of system, K_1 , and the initial stiffness of frame, brace members, TADAS device, respectively, can be expressed in terms of the natural period of system, T , as (Chang, 1993)

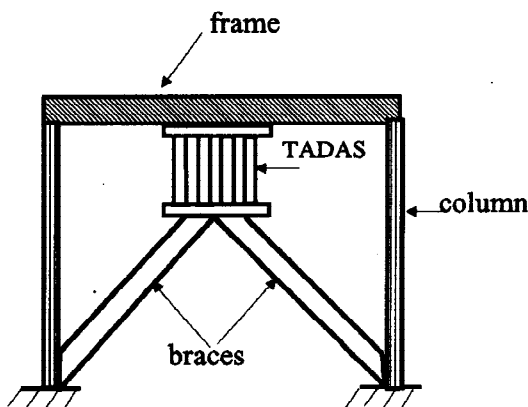


Fig. 3 Installation of TADAS on one-story frame

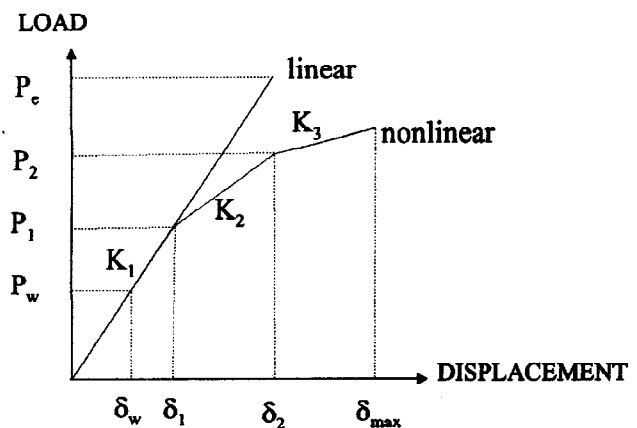


Fig. 4 Trilinear load-displacement relation

$$K_1 = \frac{4\pi^2}{T^2} \quad (4)$$

$$K_f = \frac{4\pi^2}{T^2(1+R_1)} \quad (5)$$

$$K_b = \frac{4\pi^2 R_1(1+R_2)}{T^2(1+R_1)} \quad (6)$$

$$K_d = \frac{4\pi^2(1+R_2)R_1}{T^2(1+R_1)R_2} \quad (7)$$

Trilinear load-displacement relation of system

Since both frame and TADAS device follow bilinear load-displacement relation, the load-displacement relation of the system considered is trilinear as depicted in Fig. 4. When all the components remain elastic, the initial stiffness of the system is as described in Eq. (4). When the yielding of TADAS device occurs and the frame still remains elastic, which is the usual case, the stiffness of the system at this stage, K_2 , is

$$K_2 = K_f + K_{cp} \quad (8)$$

where K_{cp} is the combined stiffness of brace members and TADAS device after TADAS device yields, and is expressed as

$$K_{cp} = \frac{1}{\frac{1}{K_b} + \frac{1}{R_d K_d}} \quad (9)$$

If the frame also yields, the system stiffness at this stage, K_3 , becomes

$$K_3 = R_f K_f + K_{cp} \quad (10)$$

However, to completely define the trilinear load displacement relation, we still have to define P_1 , δ_1 , P_2 and δ_2 as shown in Fig. 4, where P_1 and δ_1 correspond to the load and the displacement of system, respectively, when the first yielding of TADAS device occurs, and P_2 and δ_2 correspond to the load and the displacement of system, respectively, when the first yielding of frame also occurs.

For the displacement of the system corresponding to the first yielding of TADAS device, it can be expressed in terms of the yield displacement of TADAS device as (Chang, 1993)

$$\delta_1 = \left(1 + \frac{1}{R_2}\right)\delta_y \quad (11)$$

This indicates that unless the brace members are infinitely rigid, the displacement of system at the moment when the first yielding of TADAS device occurs will not be equal to the yield displacement of TADAS device. The corresponding load of system, P_1 , is simply

$$P_1 = \left(1 + \frac{1}{R_2}\right)K_1\delta_y \quad (12)$$

When the frame also yields, the displacement of system, δ_2 , equals to the yield displacement of frame. If α is defined as the ratio of yield strength of TADAS device to that of frame, that is,

$$\alpha = \frac{P_d}{K_f \delta_f} \tag{13}$$

then, when the first yielding of frame occurs, knowing the value of α , initial stiffness of frame and force taken by TADAS device, P_d , the displacement of system, δ_2 , can be computed using Eq. (13)

$$\delta_2 = \delta_f = \frac{P_d}{\alpha K_f} \tag{14}$$

Finally the corresponding load P_2 of system can be determined easily from Fig. 4.

$$P_2 = K_1 \delta_1 + K_2 (\delta_2 - \delta_1) \tag{15}$$

The trilinear model shown in Fig. 4 forms the backbone curve of the hysteresis loop considered in this study. Note that since in the elastic stage the brace members take the same load as TADAS device and the combined TADAS and brace system undergoes the same displacement as frame, the value of α can also be expressed as (Chang, 1993)

$$\alpha = \frac{1+R_1}{R_1} \tag{16}$$

COMPUTATION OF STRENGTH-REDUCTION FACTOR

In this study the strength-reduction factor, R_a , is defined as the ratio of elastic strength demand, P_e , to the inelastic demand associated with the theoretical yielding point of TADAS device, P_w , as shown in Fig. 4.

$$R_a = \frac{P_e}{P_w} \tag{17}$$

However, when the bilinear model is adopted to simulate the nonlinear behavior of TADAS device, as shown in Fig. 2, the yield stress, denoted as P_1 in Fig. 4, has been set at 1.465 times the theoretical yielding stress; therefore, using Eqs. (4), (12) and (17), the strength-reduction factor can be rewritten as

$$R_a = \frac{1.465 T^2 R_2 P_e}{4\pi^2 \delta_y (1+R_2)} \tag{18}$$

From Eq. (18), for given values of T and R_2 , The strength-reduction factor can be easily obtained providing the associated values of P_e and δ_y satisfying the predetermined ductility ratio of TADAS device are computed. As for δ_y , it can be determined using the following equation of motion where the mass is set to unity.

$$\ddot{X}(t) + 2\xi\omega\dot{X}(t) + R(t) = -\ddot{X}_g(t) \tag{19}$$

where $X(t)$ is the relative displacement response, the dot denotes the time derivative, ω is the system's natural circular frequency, $X_g(t)$ is the input base acceleration, $R(t)$ is the nonlinear resistance function which is characterized by the relation shown in Fig. 4, and ξ is the fraction of critical damping ratio. This equation is solved using the constant average acceleration method. In order to model the transition from one stage to another stage in the trilinear load-displacement relation accurately, the procedure proposed by Lamb et al. (1986) is adopted, which requires the calculation of correction factor for time. It should be pointed out that in this study the strength-reduction factor is computed under the following three conditions: (1) the predetermined ductility of TADAS device is satisfied, (2) the ductility of frame, μ_p , should

not exceed 2.0, and (3) the displacement amplification factor, C_d , should not be greater than 5.5, where, referring to Fig. 2, μ_f , the ratio of δ_{\max} to δ_2 , and C_d , ratio of δ_{\max} to δ_1 , are as follows (Hwang, 1994).

$$\mu_f = \frac{r[B(1+R_2)-C](R_1+R_2)}{R_1R_d(1+R_2)} \quad (20)$$

$$C_d = \frac{-[B(1+R_2)-C](R_1+R_2)}{R_d(1+R_2)} \quad (21)$$

where

$$r = R_1 \frac{\delta_1}{\delta_2} \quad (22)$$

$$B = \frac{1}{1+R_2} - \frac{R_d}{R_2+R_d} \quad (23)$$

$$C = 1 + R_d(\mu_d - 1) \quad (24)$$

and μ_d is the desired ductility ratio of TADAS device.

RESULTS AND DISCUSSIONS

In this section, the strength-reduction factors are computed using a given set of parametric values: $R_f=0.6$, $R_d=0.0045$, $\alpha=0.4$ (i.e. $R_1=2/3$), $\mu_d=6$, $r=0.25$. In this set of values, μ_d and r are determined from the requirement that the ductility ratio of frame, μ_f , and the displacement amplification factor, C_d , should not be greater than 2 and 5.5, respectively. Figure 5 shows the variation of C_d with respect to μ_d and R_2 . It can be seen that the C_d values are less than 5.5 for $\mu_d=6$ and R_1 ranging from 2 to 6. Shown in Fig. 6 is the plot for variation of μ_f with respect to R_2 and r , which indicates that the appropriate value for r is

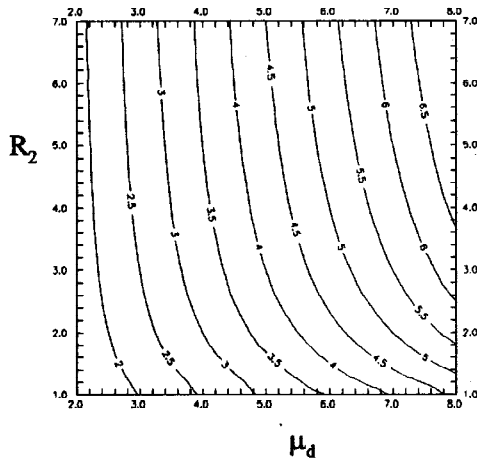


Fig. 5 Variation of C_d with respect to R_2 and μ_d

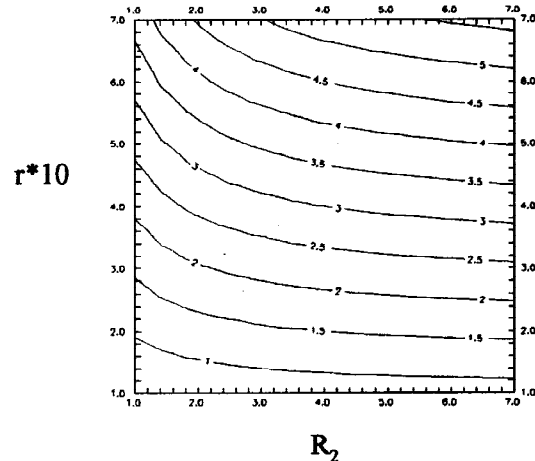


Fig. 6 Variation of μ_f with respect to R_2 and r

0.25 so that μ_f is less than 2. As for the period, 188 unequal intervals between 0.1 second and 10 seconds are selected. Finally the strength-reduction factors are computed using 20 Earthquakes recorded in Taipei basin and 58 rock-site earthquake records of Taiwan area.

Shown in Fig. 7 and Fig. 8 are the strength-reduction factors for Taiwan rock-site earthquake records and Taipei basin earthquakes with $R_2=2, 3, 4, 5, 6$, respectively. For the Taiwan rock-site earthquakes, the

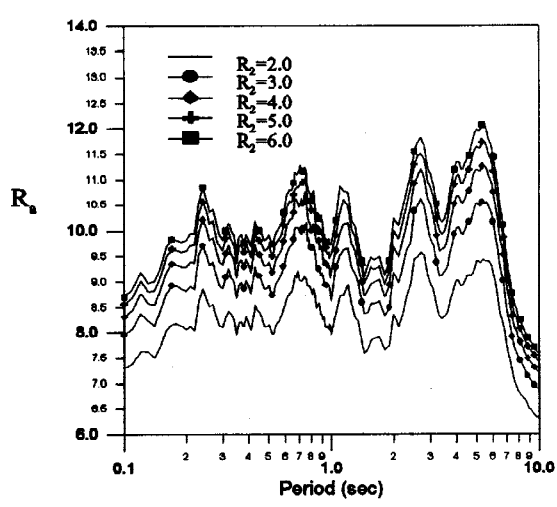


Fig. 7 Strength-reduction factors for Taiwan rock-site earthquakes

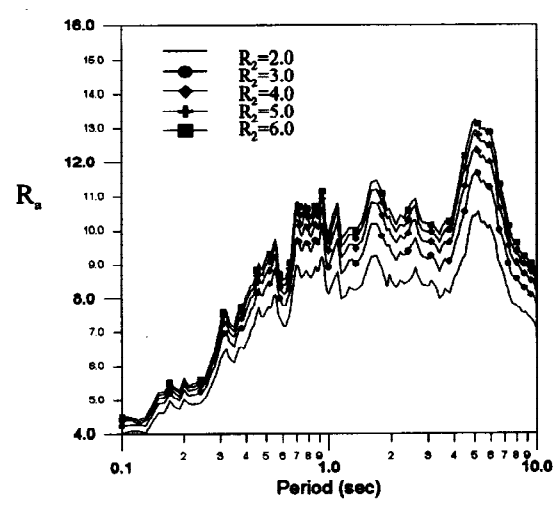


Fig. 8 Strength-reduction factors for Taipei basin earthquakes

values more or less fluctuate with respect to constant values for period less than 6 seconds, while as period is larger than 6 seconds, the values decrease with increasing period. On the other hand, when the Taipei basin earthquakes are used, the values of strength-reduction factors increase with period for period below 0.7 seconds; over the range of period between 0.7 seconds and 5 seconds, the values remain more or less fluctuate with respect to constant values; and as period is larger than 5 seconds, the values then decrease with increasing period. However, no matter what site condition is, it can also be seen that the values of strength-reduction factor increase with increasing R_2 . Significant increase can be observed as R_2 increases from 2 to 4; however, further increase in R_2 from 5 to 6 does not lead to considerable increase. In general, for the structural period between 0.5 seconds to 5 seconds, the strength-reduction factors for TADAS device with ductility ratio equal to 6 varies from 8.3 and 10.7.

To facilitate the computation of strength-reduction factor, a set of formulae are proposed using the results in Figs. 7 and 8. Shown below are the proposed formulae which consider the variation of strength-reduction factor with T and R_2 .

For Taiwan rock-site earthquakes ($\mu_d=6; \alpha=0.4$):

$R_s = 11T + 5.1(R_2)^{0.24}$	$0.1 < T < 0.23$ seconds	(25.a)
$R_s = 2.53 + 5.1(R_2)^{0.24}$	$0.23 < T < 6.5$ seconds	(25.b)
$R_s = 7.08 - 0.7T + 5.1(R_2)^{0.24}$	$6.5 < T < 10$ seconds	(25.c)

For Taipei basin earthquakes ($\mu_d=6; \alpha=0.4$):

$R_s = 9T + 1.95(R_2)^{0.47}$	$0.1 < T < 0.67$ seconds	(26.a)
$R_s = 6.03 + 1.95(R_2)^{0.47}$	$0.67 < T < 7.0$ seconds	(26.b)
$R_s = 10.23 - 0.6T + 1.95(R_2)^{0.47}$	$7.0 < T < 10$ seconds	(26.c)

CONCLUSIONS

In this paper assuming the load-displacement relation of both the structure and the TADAS device to be bilinear, a trilinear load-displacement relation for the entire system is constructed first. Then, the strength-reduction factors, satisfying predetermined ductility ratio of TADAS device, are computed for a given set of parametric values using 20 earthquakes recorded in Taipei basin and 58 rock-site earthquake records of Taiwan area. In the analysis, the following criteria are adopted: (1) the ductility ratio of frame should not be greater than 2, and the (2) displacement amplification factor should be less than 5.5. The results showed that the strength-reduction factor increase with R_2 , no matter what site condition is; however, the values of strength-reduction factors depend on the site conditions. In general, for the structural period between 0.5 seconds to 5 seconds, the strength-reduction factors for TADAS device with ductility ratio equal to 6 varies from 8.3 and 10.7. A set of formulae, function of structural period and the ration of initial stiffness of brace members to that of TADAS device, are also proposed.

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