Paper No. 791. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

COMPENSATION OF TIME-DELAY IN SCALED-TIME PSEUDODYNAMIC TESTING

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ABSTRACT

One application of servo-controlled piston systems is in the field of structural dynamics. An area where such systems are used is in pseudodynamic testing. The version of the pseudodynamic method presented in this paper is run in a continuous manner. As in any control process a time delay is inevitable, but it introduces errors in the imposed displacements which are said to undershoot, or phase-lag, the target displacements; as a result the forces measured during the pseudodynamic test are not synchronised with the target displacements, and therefore, are incorrect. This paper presents an analysis of the effect of these delays on the performance of the pseudodynamic method.

KEYWORDS

Pseudodynamic test; control time-delay; phase-lag; negative damping; modal decomposition; extrapolation of delayed signal

INTRODUCTION

Pseudodynamic testing over the past twenty years has evolved as a versatile testing method in structural dynamics. The quality and quantity of tests has increased considerably and this is reflected in the number of papers published on the subject. It is interesting to browse through the literature and discover that the key concepts were laid down, and have remained unchanged, since the initial tentative tests were conducted nearly twenty years ago. One particular example, and the subject of this paper, is the continuous, as opposed to stepped, application of the target displacement (Takanashi et al., 1986).

It must be remembered that, the pseudodynamic method is, in essence, a hybrid: a mathematical model in the guise of an experiment. Research work on this subject has been divided into two broad areas: on the one hand numerical integration problems; on the other, experimental implementation, in particular, control problems. A major problem for the experimenters when performing the standard, step-wise, form was implementation accuracy, for, as it was pointed out (Shing et al. 1991), the success of the experiment was often conditioned by a phenomenon usually described as experimental error propagation. This type of error tends to introduce unwanted damping terms in the form of a false hysteresis mechanism; when the imposed displacement overshoots the target a positive damping term is created; when displacement undershoot occurs it is negative, and hence, unstable; how unstable depends on the structure being tested, generally speaking: the stiffer it is the worse things get.

Two other problems exist in the step-wise implementation, namely; stress relaxation and strain rate effects. To overcome these aspects some researchers (Nakashima et al.1992) suggested trying out continuous pseudodynamic testing.

CONTINUOUS PSEUDODYNAMIC TESTING

One of the first references to continuous testing, under displacement controlled form, is described (Kaneta et al. 1983a, Kaneta et al. 1983b) as an analogue integration device which, through adjustment of the amplifier gains, can regulate the time scale. Other formulations were proposed under force control (Thewalt et al. 1987) but whose implementation, up to now, has not been achieved. A real-time loading digital displacement-controlled concept was developed (Nakashima et al. 1992), the goal being to test rate-sensitive materials. However, it was pointed out that in order to do so, the real inertial mass was excluded during the test, whereas the remaining structure was modelled with substructuring techniques. The subject of the stability analysis was not specifically addressed, although perhaps this aspect would not become apparent given that the element tested was a viscous energy-absorbing device. Recently, a continuous pseudodynamic formulation which permits the selection of any desired time scale (Molina et al. 1994) but which can include the effects of real inertial masses was shown to provide satisfactory results, however, it was pointed out that the procedure is unstable for testing undamped elastic systems, unless some damping is introduced; the reason for the instability was attributed to the negative damping associated to the phase-lag between imposed and target displacements. The technique was found to be stable if the testing speed was slow enough to balance the negative damping with the, small, but positive, damping produced by the internal (hysteretic) structural damping.

INSTABILITY ANALYSIS

If it were possible to quantify accurately the amount of negative damping, and thereafter compensate it precisely, on-line, then such a form of a continuous method would result in a much simpler implementation and higher test speeds. It will be shown how, by correlating the structural vibration modes to the control characteristics, the negative damping can be expressed as a form of proportional damping. First, the negative damping will be described for the case of an elastic single-degree-of-freedom system (SDoF). Secondly, it will be shown how this comes about during a continuous pseudodynamic test and how to compensate it. Through modal decomposition, the analysis is extended to the case of multi-degree-of-freedom systems (MDoF). Finally, for a generally non-linear pseudodynamic test, the correction is expressed as an extrapolation of the delayed force history.

Apparent Hysteresis

When the load-deflection characteristics of an elastic spring are measured correctly, a linear relationship is obtained, the slope of which is the stiffness k. If the measurement is carried out so that the measured force corresponds to some prior imposed displacement an apparent, sometimes referred to as numerical, hysteresis would result. The phenomenon may be considered as force and displacement vectors following a circular path measured by φ, but where the force measurement is lagging the displacement by the angle ϕ. The sense of the resultant force-displacement loops would run anti-clockwise: the spring would not appear to be conservative;

it would act as a source of energy. If this spring is fixed to a mass, m, and no energy-absorbing device is attached to it then the system will become unstable. The rate of growth of the unbounded oscillations can be expressed by a logarithmic increment term (Clough et al. 1975) via a negative damping factor, ζ, as follows:

$$\zeta = \frac{W_d}{4 \cdot \pi \cdot Se_{max}} \tag{2}$$

Where W_d is the work done per cycle and Se_{max} is maximum strain energy reached during the same cycle. It has been shown (Den Hartog 1956) that the work done by a harmonically varying force upon a harmonic motion of the same frequency, may be expressed in terms of the phase-lag, ϕ, such that

$$W_d = \oint_{2\pi} F \cdot d\vec{\delta} d\varphi \tag{33}$$

$$Se = \frac{1}{2} F_r \delta_r Sin\varphi Sin(\varphi - \phi) \tag{4}$$

where $F = F_r Sin\phi$ and $d\vec{\delta} = \delta_r Cos(\varphi - \phi)$, and F_r and δ_r are the amplitudes of the harmonic forces and displacements respectively.

Given that Se_{max} given is found for $dSe/d\varphi = 0$, it can be shown that :

$$\zeta = -\tan(\phi / 2) \tag{5}$$

where ϕ is expressed in radians. This procedure (Bousias et al. 1995) was applied for real hysteretic structural members. Equivalently, if the delay, τ, between the force measurement and the imposed displacement is expressed in seconds then, ϕ, is simply:

$$\phi = \omega \cdot \tau \tag{6}$$

where $\omega = \sqrt{k/m}$ is the natural frequency of the spring-mass system.

Equation (6) can be applied directly to the familiar logarithmic decrement (in this case increment) equation for a damped single-degree-of-freedom (SDoF) system so that the ratio R between two consecutive peaks for small ϕ is given by:

$$R = e^{\pi\omega^2\tau} \tag{7}$$

It should be noticed that the instability is dependent on the square of the natural frequency.

Instability for numerical integration procedures with phase lag

The following analysis is concerned with solving the linear second-order differential equation which models a a linear spring-mass system using the pseudodynamic method.

$$m\ddot{x} + f_r = f_{in} \tag{8}$$

where m is the mass; f_{in} the input force; $f_r = kx$ is the restoring force and k and x are the spring stiffness and displacement respectively.

The equation can solved by an approximate integration procedure which may be numerical or analogue. However, for the purpose of this analysis, it will be solved using a discrete-time integration procedure, namely; the central differences method.

The solution can now be expressed recursively in a discrete-time form as:

$$x^{n+1} = 2x^n + \left(\frac{f_{in}^n - kx^n}{m}\right)\Delta t^2 - x^{n-1} \tag{9}$$

where Δt is the time increment.

During a pseudodynamic test, the solution for the next target displacement, x^{n+1}, is obtained by measuring the restoring force at a particular displacement at time $n\Delta t$, and then substituting for it in (8) to obtain the acceleration and using that in (9).

Between two integration time steps the load must be measured, the new displacement is evaluated and then the pistons must reach that target displacement. Because this does not happen instantaneously, a time delay, τ, is incurred. Thus the measured restoring force introduced in (9) is not kx^n but $kx^{n-\tau/\Delta t}$, this sets up the energy-creating mechanism described above, and hence why the continuous form of the pseudodynamic method is inherently unstable.

If the delay is assumed to be independent of the displacement amplitude then it would be sufficient to modify (8) by including the damping ratio given by (6), hence the value of the damping coefficient, c, would be :

$$c = 2 \cdot \zeta \cdot m \cdot \omega \tag{10a}$$

or simply;

$$c = \tau \cdot K \tag{10b}$$

By including the positive damping term in (9) in equation (8) the imlanace due to the phase lag would be cancelled:

$$m\ddot{x} + \tau \cdot k\dot{x} + f_r = f_{in} \tag{11}$$

<u>Phase lag Correction for MDoF systems.</u> For MDoF lumped-parameter systems the extension of (11) must be a damping matrix which absorbs the equivalent energy which each vibration mode is receiving from the phase-lag mechanism. The equation of motion, initially, is of the form:

$$\boldsymbol{M\ddot{X}} + \boldsymbol{KX} = \boldsymbol{F}_{in} \tag{12}$$

It is recalled (Bathe 1982) that a proportional damping matrix **C** may be constructed such that

$$\Psi_i^T C \Psi_j = 2 \cdot \omega_i \cdot \zeta_i \cdot \delta_{ij} \tag{13}$$

where δ_{ij} is the Kroneker delta; ω_i, ζ_i, Ψ_i are the i^{th} natural frequencies, damping coefficients and mode shapes respectively. Through orthonormality properties (13) simplifies to:

$$\Psi_i^T C \Psi_i = 2 \cdot \omega_i \cdot \zeta_i \tag{14a}$$

also, for small φ values, the tangent term simplifies to $\zeta_i = {}^{\tau \cdot \omega_i}\!/_2$ then:

$$\Psi_i^T C \Psi_i = \tau \cdot \omega_i^2 \tag{14b}$$

also orthonormal rule gives;

$$\Psi_i^T K \Psi_i = \omega_i^2 \tag{15}$$

where $\mathbf{K}$ is the stiffness matrix, so we have;

$$C = \tau \cdot K \tag{16}$$

Thus the correction for MDoF systems is nominally identical to (16).

Corollary of phase compensation to Numerical integration techniques. An application of this compensation is its application as a pure numerical integration technique. If the time delay is purposely included so that the equilibrium force used in (12) is that corresponding to the previous time step, then the original, discrete model equation is transformed into:

$$M\ddot{X}_n + K\left(\frac{X_{n+1} - X_{n-1}}{2}\right) = F_{in} \tag{17}$$

which is simply arrived at by putting $\tau = \Delta t$ into (16) and substituting into (12). The central difference operator can still be applied, for the expression is still explicit if the stiffness matrix is known at every time step. Equation (17) can also be understood as an equivalent form of mass-penalty technique algorithms (Macek et al.1995) which are nearly-explicit but more stable than the standard central difference operator.

MAGNITUDE AND NATURE OF PHASE-LAG

The phase lag does not originate from a single source, but is made up of two principal components. The, usually, smaller one is the time needed to measure the force signal and perform the digital conversion and numerical integration, which, for a SDoF system will be quite small. The other time delay arises from the piston control loop; for as it was pointed out, a finite time must elapse between the moment when the new target displacement is known and when it is applied. This will be heavily dependent on the type of structure and pistons used for the test. Typically, it will be at least an order of magnitude greater than for the numerical part, so that if the measurement, signal conditioning and computation time is of he order of 0.5 mSec then the piston-controller system of a large-scale test will add on a further 50 mSec.

The qualitative behaviour of these delays is substantially different. Whereas the conditioning-computational delay is nearly constant, the piston-controller delay is characterised by being non-linear. A typical delay-time

function for most structural servo-hydraulic systems looks like a step function. The low frequency band, typically in the range 0.1 to 1Hz display delay times of a few milliseconds. As the frequency increases, the delay jumps to a second level, usually one order of magnitude higher, but thereafter remains substantially constant. The compensation for a SDoF system is trivial since all that is required is to obtain the delay time of the scaled-down natural frequency of the structural element under test and apply the correction in (11). However, for MDoF systems some of the scaled-down frequencies may lie in the low-band range, whereas it would be necessary to use a time compensation corresponding to the highest (and most unstable) eigenvalues. This implies that the lower modes may be over damped, while keeping the highest ones just stable. This undoubtedly affects the accuracy of the integration algorithm which is no longer, nominally, second order accurate. However, because pseudodynamic tests are performed on non-linear structures, the small over damping term will be considerably smaller than the hysteretic damping associated to those smaller modes where the error is greatest.

Correction of phase lag for pseudodynamic testing of non-linear systems

The concepts for phase-lag compensation apply as they stand to elastic systems or to non-linear ones if the stiffness matrix is known, as may be the case for numerical integration using finite elements. During pseudodynamic tests, the stiffness matrix is not known on-line so an approximation must be made. The most simple one is to evaluate the real force by a first order extrapolation of the phase-lagging force measurement thus the equilibrium restoring force at step n, f_r^n, may be approximated by:

$$f_r^n = \mu \cdot f_r^{n+1-\mu} - (\mu - 1) \cdot f_r^{n-\mu} \tag{18}$$

where $\mu = \tau/_{\Delta t}$ is simply the ratio of the delay time to the force-sampling period. Higher order approximations are under review which, produce optimised damping function, that is, low damping compensation for the low frequency range but more powerful at the highest structural frequencies. This should result in near second order accuracy in the low modes whereas the higher modes may be kept under control or substantially attenuated.

CONCLUSION

A methodology for the analysis, in the elastic regime, of the energy-producing mechanisms which can affect the performance of continuous, scaled-time pseudodynamic has been presented. It has been shown how the phase-lag between the target and imposed displacements results in a negative damping phenomenon which can be compensated by including an exact but opposite damping term. The concept has been extended to non-linear cases in the sense that the stiffness terms are unknown, by adopting extrapolation techniques on the delayed force measurement.

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