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ACTIVE CONTROL OF SEISMIC RESPONSE BY VARIATION OF STRUCTURAL PARAMETERS

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ABSTRACT

The application of semi-active control for response reduction of seismic-excited civil structures is investigated. Semi-active control systems are based on the regulation of structural characteristics such as damping and stiffness, which can be done with minimal power expenditure. The application of this control strategy is demonstrated on a 10-story shear building with a controller developed by the sliding mode control approach. Numerical results show that active bracings, combined with passive dampers, can be effectively used to provide response reduction with minimal power requirement.

KEYWORDS

Structural control; sliding mode control; semi-active control.

INTRODUCTION

Seismic response control is a continuously growing field, strengthened by the development of new algorithms and actuation devices that are specifically adapted to civil engineering structures. The implementation of control systems offers a very exciting solution to the problem of response reduction due to seismic excitation. This is specially true if the control system is designed to be used in conjunction with other proven techniques for seismic mitigation, such as base isolation and energy-absorbing devices.

In active control methods the shaping of the system response is obtained by the direct application of corrective forces on the structural system. In general, the operation of these control systems is characterized by very large power requirements (Singh *et al.*, 1995). A convenient alternative is presented by semi-active control methods, which are based on the active regulation of structural parameters (Kobori *et al.*, 1993; Yang *et al.*, 1994a). In this case, the parametric changes are introduced by active dampers and/or bracings which create reactive internal forces according to the selected algorithm. The power requirements of semi-active control strategies are significantly smaller than those demanded by active control schemes based on external counteracting forces.

In this paper we consider the application of semi-active control schemes to civil structures using a control algorithm based on the sliding mode control approach (Itkis, 1976; Utkin, 1971, 1992). These type of controllers have shown remarkable robustness characteristics (DeCarlo *et al.*, 1988) and they have been applied to both linear and nonlinear systems with excellent results (Slotine, 1984; Yang *et al.*, 1993, 1994a, b). In the sequel we present a description of the sliding mode control approach and its implementation with variable stiffness devices. Numerical results are presented to evaluate the effectiveness of the proposed control strategy in the reduction of seismic response.

EQUATIONS OF MOTION

The equations of motion of an n_f-degree-of-freedom building system with m_c semi-active devices and subjected to seismic excitation $\ddot{x}_g(t)$ can be written as

$$\mathbf{M}\,\ddot{\mathbf{z}} + [\mathbf{C}+\mathbf{C_v}]\,\dot{\mathbf{z}} + [\mathbf{K}+\mathbf{K_v}]\,\mathbf{z} = -\mathbf{M}\,\mathbf{r}\,\ddot{x}_g \tag{1}$$

in which the $n_f \times 1$ vector $\mathbf{z}$ designates the relative displacements and the $n_f \times n_f$ matrices $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ represent the mass, damping and stiffness matrices, respectively. The $n_f \times 1$ vector $\mathbf{r}$ denotes the influence of the ground motion on each degree-of-freedom. The $n_f \times n_f$ matrices $\mathbf{C_v}$ and $\mathbf{K_v}$ represent the contributions of the semi-active devices, characterized by parameters c_{v_i} and k_{v_i}, respectively.

It is convenient to obtain a representation of the system that uncouples the effect of the variable stiffness and damping parameters on each degree-of-freedom. Let us define $\mathbf{z} = \mathbf{T_d}\,\mathbf{d}$, where the $n_f \times n_f$ matrix $\mathbf{T_d}$ is a transformation matrix which reduces the matrices $\mathbf{C_v}$ and $\mathbf{K_v}$ to a special diagonal form. The equations of motion can be written in terms of the coordinates $\mathbf{d}$ as follows:

$$\tilde{\mathbf{M}}\ddot{\mathbf{d}} + \left[\tilde{\mathbf{C}}+\tilde{\mathbf{C}}_\mathbf{v}\right]\dot{\mathbf{d}} + \left[\tilde{\mathbf{K}}+\tilde{\mathbf{K}}_\mathbf{v}\right]\mathbf{d} = -\mathbf{T}_\mathbf{d}^T\mathbf{M}\,\mathbf{r}\,\ddot{x}_g \tag{2}$$

where the transformed matrices are given by

$$\tilde{\mathbf{M}} = \mathbf{T}_\mathbf{d}^T\,\mathbf{M}\,\mathbf{T_d}\ ;\ \tilde{\mathbf{C}} = \mathbf{T}_\mathbf{d}^T\,\mathbf{C}\,\mathbf{T_d}\ ;\ \tilde{\mathbf{K}} = \mathbf{T}_\mathbf{d}^T\,\mathbf{K}\,\mathbf{T_d}\ ;\ \tilde{\mathbf{C}}_\mathbf{v} = \mathbf{T}_\mathbf{d}^T\,\mathbf{C_v}\,\mathbf{T_d}\ ;\ \tilde{\mathbf{K}}_\mathbf{v} = \mathbf{T}_\mathbf{d}^T\,\mathbf{K_v}\,\mathbf{T_d} \tag{3}$$

The matrices $\tilde{\mathbf{C}}_\mathbf{v}$ and $\tilde{\mathbf{K}}_\mathbf{v}$ are $n_f \times n_f$ diagonal matrices which show the m_c coefficients c_{v_i} and k_{v_i}, respectively, as the only nonzero entries along the diagonal. For a shear building model, this representation can be obtained if the coordinates $\mathbf{d}$ correspond to the interstory drifts.

The equations of motion (2) can be written in terms of $n = 2n_f$ state equations as follows:

$$\dot{\boldsymbol{\eta}} = \mathbf{A}\,\boldsymbol{\eta} + \mathbf{B}\,\mathbf{u} + \mathbf{e}\,\ddot{x}_g \tag{4}$$

where the state vector $\boldsymbol{\eta}$ and the excitation input vector $\mathbf{e}$ are defined, respectively, as

$$\boldsymbol{\eta} = \left\{\begin{array}{c}\mathbf{d}\\ \dot{\mathbf{d}}\end{array}\right\}\ ;\ \mathbf{e} = \left\{\begin{array}{c}\mathbf{0}\\ -\tilde{\mathbf{M}}^{-1}\mathbf{T_d}\mathbf{M}\,\mathbf{r}\end{array}\right\} \tag{5}$$

and the state matrix $\mathbf{A}$ and the control input matrix $\mathbf{B}$ are given, respectively, as follows:

$$\mathbf{A} = \left[\begin{array}{cc}\mathbf{0} & \mathbf{I}_{\mathbf{n_f}}\\ -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}} & -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{C}}\end{array}\right]\ ;\ \mathbf{B} = \left[\begin{array}{c}\mathbf{0}\\ \tilde{\mathbf{M}}^{-1}\end{array}\right]\mathbf{L} \tag{6}$$

with $rank(\mathbf{B}) = m_c$. The matrix $\mathbf{L}$ is a $n_f \times m_c$ location matrix which identifies the m_c coordinates $\mathbf{h}$ associated with semi-active actions, i.e. $\mathbf{h} = \mathbf{L}^T\mathbf{d}$. Using these variables, the control $\mathbf{u}$ is defined as

$$\mathbf{u} = -\left[\begin{array}{cc}\tilde{\mathbf{K}}_\mathbf{v}^\mathbf{r} & \tilde{\mathbf{C}}_\mathbf{v}^\mathbf{r}\end{array}\right]\left\{\begin{array}{c}\mathbf{h}\\ \dot{\mathbf{h}}\end{array}\right\} \tag{7}$$

in which $\tilde{\mathbf{K}}_\mathbf{v}^\mathbf{r}$ and $\tilde{\mathbf{C}}_\mathbf{v}^\mathbf{r}$ are reduced size $(m_c \times m_c)$ matrices of the form $\tilde{\mathbf{K}}_\mathbf{v}^\mathbf{r} = diag(k_{v_i})$ and $\tilde{\mathbf{C}}_\mathbf{v}^\mathbf{r} = diag(c_{v_i})$.

SLIDING MOTION

The main idea of the sliding mode control approach consists of forcing the system to move along a pre-defined hyper surface in the state space, called the sliding surface. For simplicity, let us consider a sliding surface defined by a set of m_s linear equations of the form:

$$\mathbf{s}(\boldsymbol{\eta}) = \mathbf{C_s}\boldsymbol{\eta} = \mathbf{0} \tag{8}$$

with $m_s \leq m_c$ and where $\mathbf{C_s}$ is a $m_s \times n$ matrix to be determined such that the sliding motion - that is, the resulting motion when the system is confined to the sliding surface - shows desirable characteristics.

To obtain the set of equations describing this motion, first we seek a system representation characterized by a special structure of the matrix $\mathbf{B}$. Let us consider the singular value decomposition of $\mathbf{B}$, given by

$$\mathbf{B} = \mathbf{V_1}\,\mathbf{R}\,\mathbf{V_2}^T \tag{9}$$

where $\mathbf{V_1}$ $(n \times n)$ and $\mathbf{V_2}$ $(m_c \times m_c)$ are orthogonal matrices. The $n \times m_c$ matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{\Sigma} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{\Sigma} = diag(\sigma_i)$ with $\sigma_i > 0$, $i = 1, 2, ..., m_c$. Using this factorization of the matrix $\mathbf{B}$, we define the following state transformation (Matheu *et al.*, 1996):

$$\eta = \mathbf{T}\,\mathbf{y} \;\; ; \;\; \mathbf{T} = \mathbf{V_1}\mathbf{E_p} \tag{11}$$

where $\mathbf{E_p}$ is a $n \times n$ permutation matrix. Using (11), the state equations (4) can be written as

$$\dot{\mathbf{y}} = \bar{\mathbf{A}}\,\mathbf{y} \; + \; \bar{\mathbf{B}}\,\mathbf{u} \; + \; \bar{\mathbf{e}}\,\ddot{x}_g \tag{12}$$

where

$$\bar{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{A}\,\mathbf{T} \;\; ; \;\; \bar{\mathbf{B}} = \mathbf{T}^{-1}\mathbf{B} \;\; ; \;\; \bar{\mathbf{e}} = \mathbf{T}^{-1}\mathbf{e} \tag{13}$$

The transformed state equations (12) can be partitioned in the following form:

$$\begin{Bmatrix} \dot{\mathbf{y}}_1 \\ \dot{\mathbf{y}}_2 \end{Bmatrix} = \begin{bmatrix} \bar{\mathbf{A}}_{11} & \bar{\mathbf{A}}_{12} \\ \bar{\mathbf{A}}_{21} & \bar{\mathbf{A}}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{Bmatrix} + \begin{bmatrix} \bar{\mathbf{B}}_1 \\ \bar{\mathbf{B}}_2 \end{bmatrix} \mathbf{u} + \begin{Bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{Bmatrix} \ddot{x}_g \tag{14}$$

where the state variables $\mathbf{y}$ have been separated into a set of $n - m_s$ variables, arranged in the vector $\mathbf{y_1}$, and the remaining m_s variables collected in the vector $\mathbf{y_2}$. It is easy to see that the partition $\bar{\mathbf{B}}_1$ has the following structure, with a $(m_c - m_s) \times m_c$ lower block $\bar{\mathbf{B}}_{1b}$ which is not identically zero:

$$\bar{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{B}}_{1b} \end{bmatrix} \tag{15}$$

Let us assume now that the sliding surface (8) has been defined with respect to the variables $\mathbf{y}$ as follows:

$$\mathbf{s}(\mathbf{y}) = \bar{\mathbf{C}}_s\,\mathbf{y} = \mathbf{0} \tag{16}$$

where $\bar{\mathbf{C}}_s = \begin{bmatrix} \bar{\mathbf{C}}_{s1} & \mathbf{I}_{m_s} \end{bmatrix}$. Consider that the system reaches this surface at some time t_h and it is forced to stay there by some control action $\hat{\mathbf{u}}$; that is, the system satisfies the following conditions:

$$\mathbf{s}(\mathbf{y}) = \bar{\mathbf{C}}_s\,\mathbf{y} = \mathbf{0} \quad \forall t \geq t_h \tag{17}$$

$$\dot{\mathbf{s}}(\mathbf{y})\,|_{\mathbf{u}=\hat{\mathbf{u}}} = \bar{\mathbf{C}}_s\,\dot{\mathbf{y}} = \mathbf{0} \;\; \forall t > t_h \tag{18}$$

It can be shown that the control $\hat{\mathbf{u}}$ must be a solution to the following system of equations:

$$\bar{\mathbf{C}}_s\,\bar{\mathbf{B}}\,\hat{\mathbf{u}} = -\bar{\mathbf{C}}_s\,\bar{\mathbf{A}}\,\mathbf{y} - \bar{\mathbf{C}}_s\,\bar{\mathbf{e}}\,\ddot{x}_g \tag{19}$$

where the coefficient matrix $\bar{\mathbf{C}}_s\,\bar{\mathbf{B}}$ has dimension $m_s \times m_c$. To obtain a unique solution for the control action $\hat{\mathbf{u}}$ we add $m_c - m_s$ extra conditions of the form:

$$\bar{\mathbf{B}}_{1b}\hat{\mathbf{u}} = \mathbf{0} \tag{20}$$

It can be shown that the control $\hat{\mathbf{u}}$ satisfying (19) and (20) is given by

$$\hat{\mathbf{u}} = -\hat{\mathbf{B}}_1^{-1} \begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{C}}_s \bar{\mathbf{A}} \end{bmatrix} \mathbf{y} - \hat{\mathbf{B}}_1^{-1} \begin{Bmatrix} \mathbf{0} \\ \bar{\mathbf{C}}_s \bar{\mathbf{e}} \end{Bmatrix} \ddot{x}_g \tag{21}$$

where the $m_c \times m_c$ matrix $\hat{\mathbf{B}}_1$ is defined as follows:

$$\hat{\mathbf{B}}_1 = \begin{bmatrix} \bar{\mathbf{B}}_{1b} \\ \bar{\mathbf{B}}_2 \end{bmatrix} \tag{22}$$

Substituting (21) into (14) and taking condition (17) into account, we have that the behavior of the system under sliding condition can be described by the following reduced order set of equations:

$$\dot{\mathbf{y}}_1 = \left[\bar{\mathbf{A}}_{11} - \bar{\mathbf{A}}_{12}\,\bar{\mathbf{C}}_{s1}\right]\, \mathbf{y}_1 + \bar{\mathbf{e}}_1 \ddot{x}_g \tag{23}$$

SLIDING SURFACE DESIGN

We consider here a method for sliding surface design based on the minimization of a performance index of the form (Utkin and Young,1978):

$$J_1 = \int_0^\infty \left(\boldsymbol{\eta}^T \mathbf{Q}\, \boldsymbol{\eta}\right) dt \tag{24}$$

where the matrix $\mathbf{Q}$ is symmetric and positive semi-definite. Considering (11), we can write

$$J_1 = \int_0^\infty \left(\mathbf{y}_1^T\, \bar{\mathbf{Q}}_{11}\, \mathbf{y}_1 + 2\, \mathbf{y}_1^T\, \bar{\mathbf{Q}}_{12}\, \mathbf{y}_2 + \mathbf{y}_2^T\, \bar{\mathbf{Q}}_{22}\, \mathbf{y}_2\right) dt \tag{25}$$

where the matrices $\bar{\mathbf{Q}}_{11}$, $\bar{\mathbf{Q}}_{12}$ and $\bar{\mathbf{Q}}_{22}$ are appropriate partitions of the matrix $\bar{\mathbf{Q}} = \mathbf{T}^T \mathbf{Q}\, \mathbf{T}$. Assuming that the matrix $\bar{\mathbf{Q}}_{22}$ is positive-definite and neglecting the seismic excitation, the optimal solution is given by $\mathbf{y}_2 = -\bar{\mathbf{C}}_{s1}\mathbf{y}_1$, where

$$\bar{\mathbf{C}}_{s1} = \bar{\mathbf{Q}}_{22}^{-1} \left[\bar{\mathbf{A}}_{12}^T\, \bar{\mathbf{P}} + \bar{\mathbf{Q}}_{12}^T\right] \tag{26}$$

in which the matrix $\bar{\mathbf{P}}$ is the solution of the algebraic Ricatti equation:

$$\bar{\mathbf{P}}\, \left[\bar{\mathbf{A}}_1 + \bar{\mathbf{A}}_c\right] + \left[\bar{\mathbf{A}}_1 + \bar{\mathbf{A}}_c\right]^T\, \bar{\mathbf{P}} + \bar{\mathbf{P}}\, \bar{\mathbf{A}}_{12}\, \bar{\mathbf{Q}}_{22}^{-1}\, \bar{\mathbf{A}}_{12}^T\, \bar{\mathbf{P}} = -\bar{\mathbf{Q}}_{11} + \bar{\mathbf{Q}}_{12}\bar{\mathbf{Q}}_{22}^{-1}\bar{\mathbf{Q}}_{12}^T \tag{27}$$

where $\bar{\mathbf{A}}_c = \bar{\mathbf{A}}_{12}\bar{\mathbf{Q}}_{22}^{-1}\bar{\mathbf{Q}}_{12}^T$. Finally, the matrix $\mathbf{C}_s$ is obtained as follows:

$$\mathbf{C}_s = \bar{\mathbf{C}}_s\, \mathbf{T}^{-1} = \begin{bmatrix} \bar{\mathbf{C}}_{s1} & \mathbf{I}_{m_s} \end{bmatrix} \mathbf{T}^{-1} \tag{28}$$

CONTROLLER DESIGN

Having established the sliding surface, it is necessary now to define the control actions required to force the system state to reach this surface, and then maintain it there. In the case of semi-active control, we note that the control actions cannot achieve any arbitrary value. They are constrained by the fact that the semi-active devices can only provide non-negative stiffness and damping values k_{v_i} and c_{v_i}.

Ideally we would like the system to stay on the sliding surface. However, because of the limited control action available in the semi-active case, $\mathbf{s}(\boldsymbol{\eta})$ may not be equal to zero. This nonzero value is the extent of separation of the system from the desired sliding surface. The distance from $\mathbf{s} = \mathbf{0}$ can be represented by the following function

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s} \tag{29}$$

The objective of the semi-active control is to minimize the value of this function, in order to reduce any tendency of the system state moving away from $\mathbf{s} = \mathbf{0}$. That is, the control actions should be such that they make the time rate of change of the function (29) as small as possible, preferably less than zero. Considering Eqs. (4) and (8), we can write

$$\frac{d}{dt}(V) = \mathbf{s}^T \mathbf{C_s} \left\{ \mathbf{A}\,\boldsymbol{\eta} + \mathbf{e}\,\ddot{\mathbf{x}}_\mathbf{g} \right\} + \mathbf{s}^T \mathbf{C_s}\,\mathbf{B}\,\mathbf{u} \tag{30}$$

The last term of Eq. (30) can be expressed as

$$\mathbf{s}^T \mathbf{C_s}\,\mathbf{B}\,\mathbf{u} = -\sum_{i=1}^{m_c}(\beta_i h_i)\,k_{v_i} - \sum_{i=1}^{m_c}(\beta_i \dot{h}_i)\,c_{v_i} \tag{31}$$

where the coefficient β_i represent the i^{th} component of the vector $\boldsymbol{\beta} = \mathbf{s}^T\mathbf{C_s}\mathbf{B}$. From this equation it is immediately apparent that whenever the terms $(\beta_i h_i)$ and $(\beta_i \dot{h}_i)$ are negative, we should adopt the smallest values for the corresponding coefficients k_{v_i} and c_{v_i}, and whenever these terms are positive we should choose the largest values of the coefficients. This can be achieved by the following control law, assuming that each device can only provide two values of stiffness $\{0; k_{v_i}^{\max}\}$ and damping $\{0; c_{v_i}^{\max}\}$:

$$k_{v_i} = \frac{k_{v_i}^{\max}}{2}\left(1 + sgn(\beta_i h_i)\right) \;\; ; \;\; c_{v_i} = \frac{c_{v_i}^{\max}}{2}\left(1 + sgn(\beta_i \dot{h}_i)\right) \tag{32}$$

NUMERICAL RESULTS

We will consider a 10-story shear building model for the numerical simulations. Each story has the same mass, stiffness and damping parameters. The mechanical properties and the frequencies of this model are given in Figure 1. The resulting proportional damping matrix for the structure provided a modal damping ratio of 3.1% of the critical in the fundamental mode. The m_c semi-active devices were modelled as variable stiffness mechanisms operating in a on/off regime as indicated by Eq. (32). In addition to the variable stiffness characteristics, each device is assumed to have damping elements which passively provide additional damping.

Two different cases were considered to investigate the effect of the number and position of the semi-active devices on the performance of the controlled system. In the first model, it is assumed that there are active bracings installed in the first four floors of the structure. The first two stories have devices with a parameters $\{0.3k_{ref}, 0.25c_{ref}\}$. The third and fourth stories have devices with parameters equal to $\{0.2k_{ref}, 0.15c_{ref}\}$ and $\{0.1k_{ref}, 0.1c_{ref}\}$, respectively. In the second model, we assume that there are active bracings installed in all floors but the top one. The values of the stiffness and damping parameters for the first seven stories are $\{0.3k_{ref}, 0.25c_{ref}\}$. The devices in the eighth and ninth stories are characterized by $\{0.2k_{ref}, 0.15c_{ref}\}$ and $\{0.1k_{ref}, 0.1c_{ref}\}$, respectively. For both models considered, the reference value k_{ref} is the story stiffness, that is 654.98 [MN/m], and the damping reference value c_{ref} is 6.15 [MN.sec/m]. The numerical results have been obtained using El Centro ground acceleration record, normalized to a maximum ground acceleration of $0.3g$.

Figure 2 shows the time histories of the top floor displacements for the uncontrolled and semi-actively controlled structure. Figure 2 (a) corresponds to the case $m_c = 4$ and it shows that the maximum controlled response is reduced to 85% with respect to the uncontrolled case. The reduction of the response is more effective for the case $m_c = 9$, shown by Figure 2 (b), in which the maximum displacement is reduced to 64% of the uncontrolled peak value.

In Figure 3 we investigate the effectiveness of the control system in reducing the peak responses in different stories. The response quantities are normalized with respect to the uncontrolled response and they are shown for both cases $m_c = 4$ and $m_c = 9$. The response of the controlled system is compared with the response obtained by considering that the devices act passively. In Figure 3 (a) we observe that additional stiffness can increase some response quantities and that the semi-active operation of the bracings provides a reduction of the response with respect to the passive case. Figure 3 (b), which corresponds to the case $m_c = 9$, shows how the increased control authority generates more important reductions of the response quantities for most of the floors.

The reduction of the response of the semi-actively controlled system with respect to that with passive additional bracings is due to the dissipation of energy associated with the active regulation of the stiffness parameters. This effect can be appreciated in Figure 4. The device force, normalized with respect to the floor weight, is plotted versus the corresponding story drift for the semi-active device installed in the first story. Figures 4 (a) and (b) correspond to cases $m_c = 4$ and $m_c = 9$, respectively, and they show that the behavior of the device is characterized by the formation of hysteresis loops.

The effect of the device stiffness on the maximum responses is shown in Figure 5 for the case $m_c = 9$. Figures 5 (a) and (b) show normalized peak values of absolute acceleration and relative displacement for two different stories, respectively. The peak responses corresponding to the passively stiffened structure are also shown in these figures. Note that for zero device stiffness the reduction in the response is caused by the additional damping introduced by the devices. Increasing the active stiffness has the effect of improving the performance of the controlled structure. From the responses corresponding to the passive stiffness case, we clearly observe that additional stiffness may or may not increase some response quantities. This effect depends on the relative position of the resulting fundamental frequencies with respect to the input motion response spectrum.

CONCLUDING REMARKS

In this paper, the performance of semi-active control approaches was numerically evaluated. A 10-story building with mass and frequency characteristics similar to those found in practice was selected as the example problem. Two different cases were considered to examine the influence of the number of semi-active devices and their location on the resulting performance. The passive stiffening of a structure may reduce or increase certain response quantities, but the active regulation of the additional stiffness was found to be effective in reducing the structural responses.

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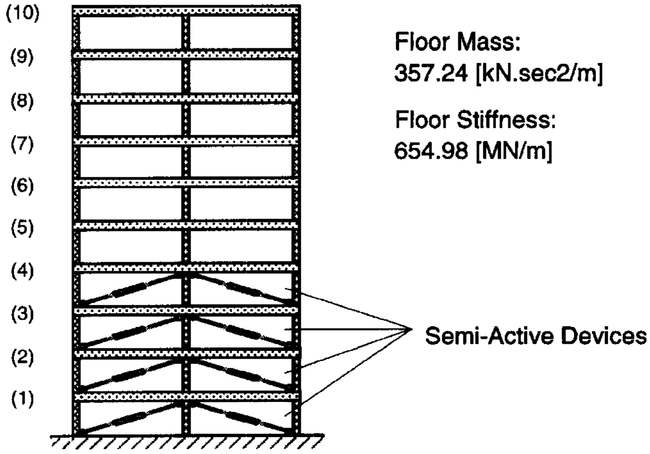
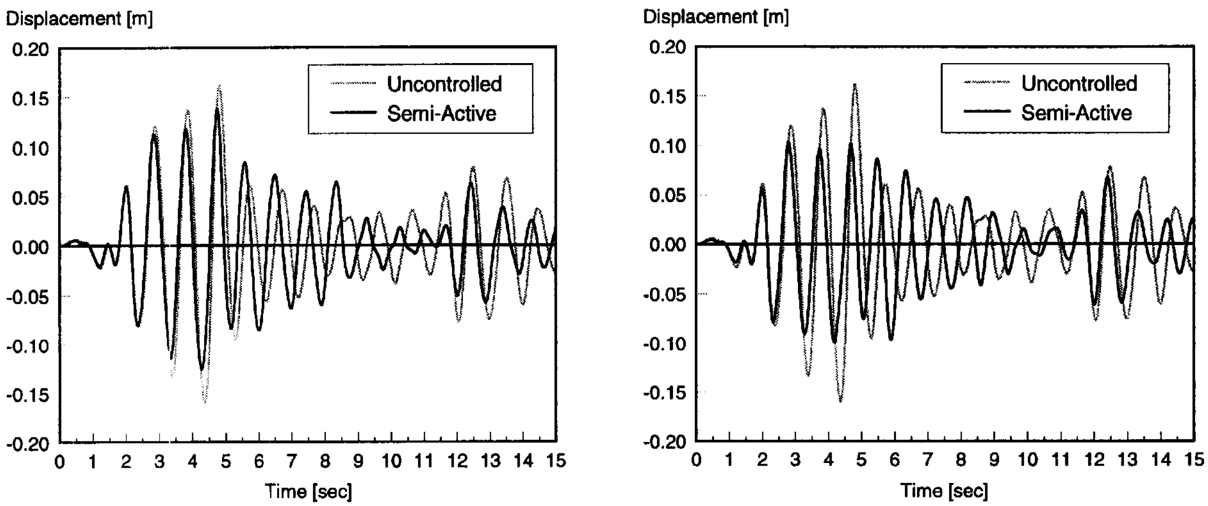


	Natural Freqs. [rad/sec]
1	6.40
2	19.06
3	31.29
4	42.82
5	53.39
6	62.78
7	70.76
8	77.16
9	81.83
10	84.68

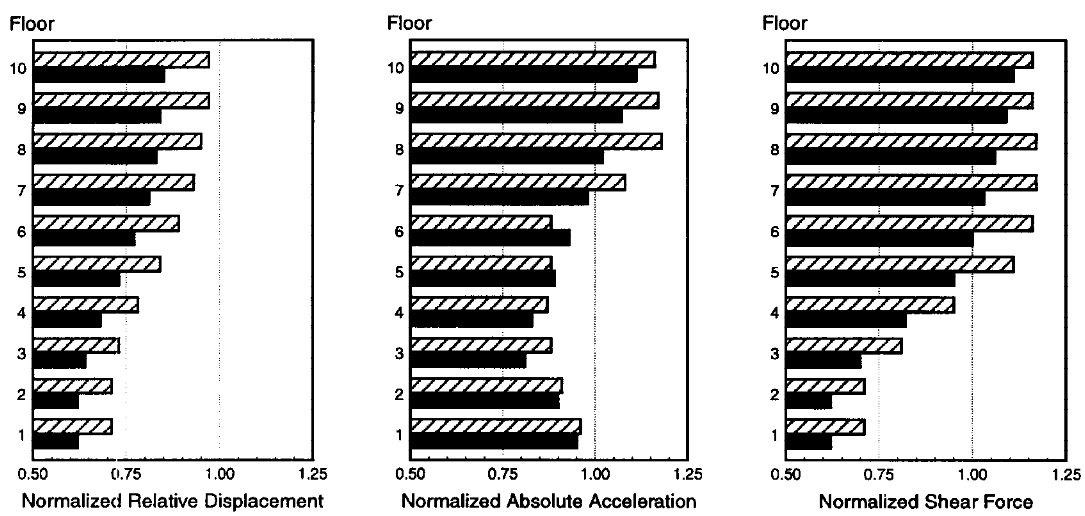
FIGURE 1: 10-STORY BUILDING MODEL USED FOR NUMERICAL SIMULATIONS.



a) Number of semi-active devices = 4

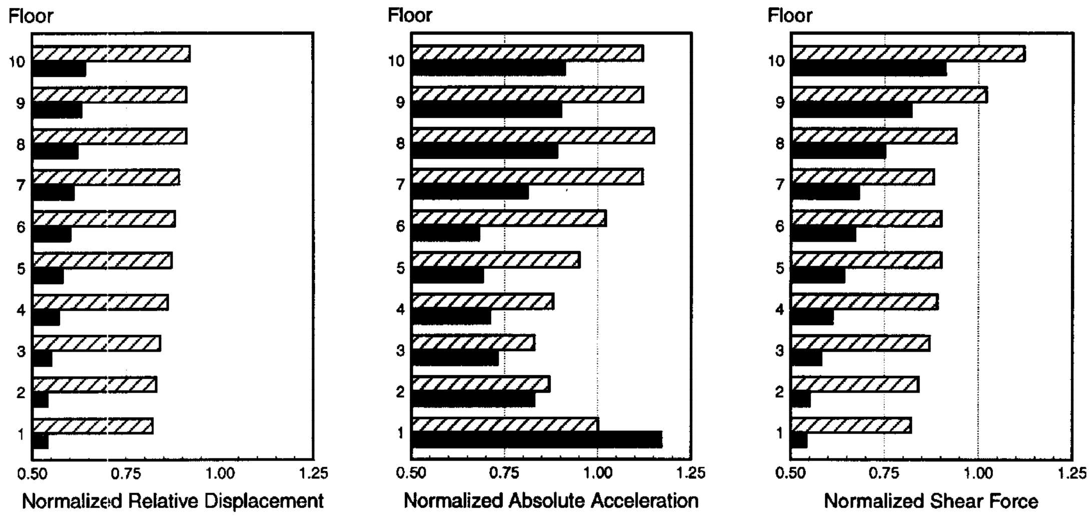
b) Number of semi-active devices = 9

FIGURE 2: UNCONTROLLED AND CONTROLLED TOP FLOOR DISPLACEMENT.



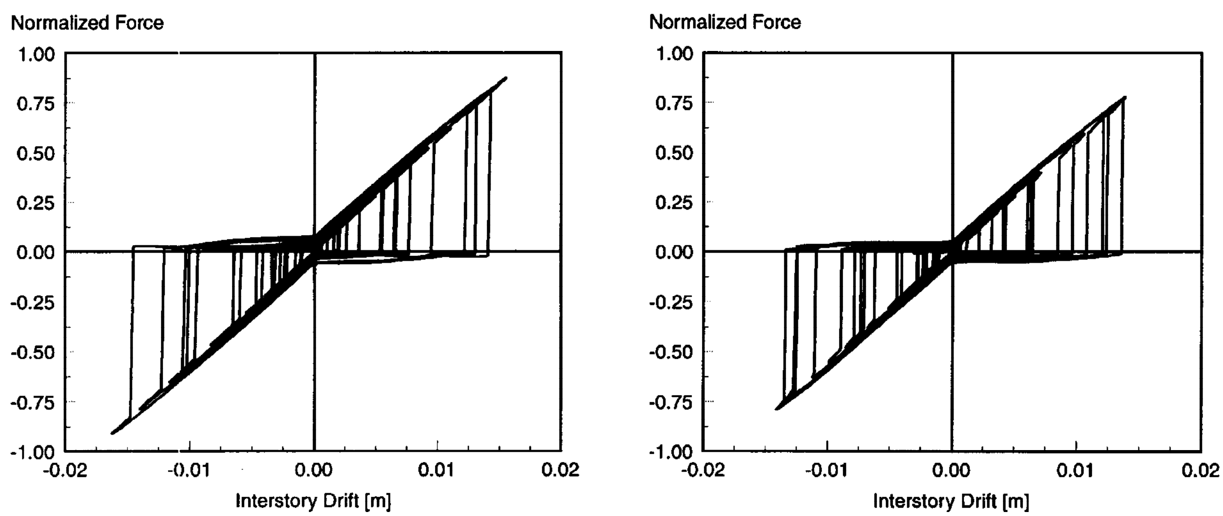
a) Number of semi-active devices = 4 Passive Semi-active

FIGURE 3: COMPARISON OF RESPONSE REDUCTIONS FOR PASSIVE AND SEMI-ACTIVE CONTROL.



b) Number of semi-active devices = 9 Passive Semi-active

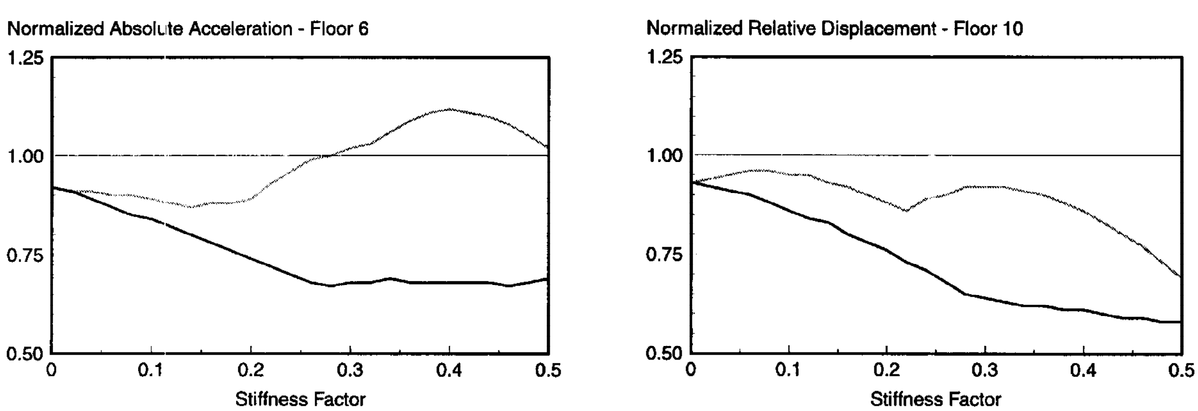
FIGURE 3 (Cont.): COMPARISON OF RESPONSE REDUCTIONS FOR PASSIVE AND SEMI-ACTIVE CONTROL.



a) Number of semi-active devices = 4

b) Number of semi-active devices = 9

FIGURE 4: FORCE-DISPLACEMENT RELATION (SEMI-ACTIVE DEVICE: FLOOR No 1).



Passive Semi-active (Number of semi-active devices = 9)

FIGURE 5: EFFECT OF ADDITIONAL STIFFNESS ON BUILDING RESPONSES.