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TWO TECHNIQUES FOR TREATMENT OF FRICTION FORCES - EXPERIMENTAL VERIFICATION AND ANALYSIS

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ABSTRACT

The paper deals with a comparison of the commonly used hysteretic model of friction forces with the recently proposed velocity model. The numerical results are verified with experimental data in order to ascertain how adequately these two methods describe the dynamic behaviour of systems with friction devices. When employing hysteretic model of friction force, "oscillations" of relative velocity difference can appear at times when the sliding ceases. Introduction of a correctness condition is proposed to avoid this numerical inaccuracy. Similar and well coinciding with experimental data results are calculated by hysteretic and velocity models, when dealing with correct solutions. The velocity model, because of its simplicity, could be proposed for further development and implementation.

KEYWORDS

Earthquake engineering; modelling of friction forces; friction devices; numerical simulations.

INTRODUCTION

The main concept of incorporating energy absorbing devices in a structure is to concentrate the absorption of seismic input energy in devices designed especially for this purpose, thus avoiding or minimizing inelastic response of the main bearing elements of the structure. In this connection the use of friction devices for construction and retrofitting of buildings has attracted considerable attention in the last decade, as reviewed by Dowdell and Cherry, 1995, Dimova, Krätzig and Meskouris, 1995. The recent achievements in this area deal with a simplified yet reliable procedure for seismic design of damped steel braced frames (Vulcano, 1995), evaluation the feasibility of using the transfer function approach to establish the optimum slip load of friction damped structure (Dowdell and Cherry, 1995), determination of the shape of the hysteresis loops and the magnitude and distribution of contact pressure of two friction devices (Dorka, 1995), experimental testing of slotted bolted connections (Grigorian and Popov, 1994).

The study of the dynamic behaviour and the resulting correct estimate of the effectiveness of isolation systems with friction devices under seismic excitations relies upon a numerical solution of pertinent nonlinear differential equations. The nonlinear terms in these equations are connected with the Coulomb representation of the friction forces, which employs a sign function. The paper deals with the problem of the adequate numerical simulation of the dynamic behaviour of systems with friction devices.

The ad-hoc numerical methods for treatment of the friction forces, when studying the dynamic behaviour of systems with friction elements, could be related to the following four groups:

(i) equivalent linearization methods, in which the friction force is substituted by a viscous damping force with an appropriate damping coefficient. Following this concept , the linearity of the problem is preserved and the solution process is completely standard. Despite the simplicity of the solution, the friction force linearization leads to inaccuracies in the system response time history, since the introduction of a viscous damping force instead of a friction force implies relatively large displacements during the stick phase. Several methods for the determination of equivalent viscous coefficients were discussed by Beucke and Kelly, 1985, Kelly and Beucke, 1983, Dimova and Georgiev, 1992.

(ii) controlling the stick - sliding conditions in accordance with the Coulomb representation of friction force. The approach is complicated in cases where many vibrating parts of the structure are connected by friction devices, since many different phase transition conditions must be taken into account. Also, regarding the numerical solution, Feldstein and Goodman, 1973, have proved that if any algorithm of order bigger than 1 is applied to a differential equation with discontinuities, its order of convergence collapses to 1 after only one discontinuity. In the case of base-isolated structures with friction devices, Dimova and Georgiev, 1992, when dealing with the numerical solution, showed that the transitions between sticking and sliding may be accompanied by a jump of structure displacement or by a jump of ground acceleration (if the ground acceleration function is not preliminary assumed to be continuous), or by high-frequency "oscillation" of the velocity near the points of zero velocity. Further, Dimova, Meskouris and Kraetzig, 1995, proved that in case of friction devices, distributed in a high of the structure, the direct implementation of this method leads to high frequency oscillation of the relative velocity difference in the following two cases:

(j) immediately after the time instances when the relative velocity equals zero if the exact time of the transition 'sticking - sliding' is not determined precisely;

(jj) during the sliding phase, if an inappropriate size of the integration step is chosen.

(iii) representation of the friction force - displacement relationship by an approximate rigid-perfectly-plastic hysteretic model. This technique is very widespread. It is not complicated to introduce such a relationship into existing structural models or to employ existing non-linear structural analysis programs. Most important, some world-wide spread programs offer the possibility to investigate systems with friction devices by employing an approximate rigid-perfectly-plastic hysteretic model of friction force (DRAIN-2DX, as described by Allachabadi and Powell, 1988), or a modified viscoplasticity model (3D-BASIS, represented by Nagarajaiach, Reinchorn and Constantinou , 1989, 1991).

(iiii) representation of the friction force - velocity relationship by an approximation of the Coulomb model. Such approach is proposed by Dimova and Georgiev, 1992, for base-isolated structures and by Dimova, Meskouris and Krätzig, 1995, for systems with friction devices distributed at different levels of the structure. The introduction of this relationship into the mathematical models is not complicated, because it involves only the damping matrix and the load vector. In the following, when dealing with the approximate Coulomb representation of the friction force - velocity relationship, it will be referred to as the "velocity model" and the technique based on the representation of the friction force - displacement relationship by an approximate rigid-perfectly-plastic hysteresis will be denoted as the "hysteretic model".

DESCRIPTION OF EXPERIMENTAL FACILITY AND MODELS FOR NUMERICAL SIMULATION

Experimental Facility

The experimental model was created in the Structural Testing Laboratory at the Civil Engineering Department of the Ruhr-University Bochum in order to validate the proposed by Dimova, Meskouris and Krätzig, 1995, numerical technique for dynamic analysis of structures with friction devices. The model consists of two frictionally coupled masses connected to the free ends of round cantilever bars, as shown in Fig. 1.

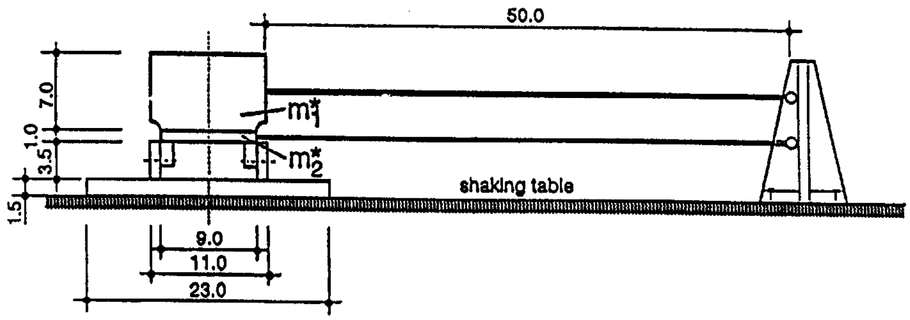

Fig. 1. Scheme (measurements in cm) of the experimental model

Friction occurs between the two masses at their interface, which was roughened in order to increase the friction force. In order to minimize energy dissipation in the contact area between the lower mass and the shaking table, the lower mass is equipped with three special wheels with roller bearings, which roll on a polished steel plate, rigidly connected to the shaking table. More details about the model construction, instrumentation and experimental determination of the system parameters are given by Dimova, Meskouris and Kraetzig , 1995. Further, the following denotations are accepted:

x_0 describes the shacking table horizontal motion with respect to a fixed coordinate system;

x_1 and x_2 are the displacements of the upper and of the lower steel blocks measured relative to the shacking table;

P is the normal force acting on the friction interface;

F_{fr} is the friction force.

μ is the friction coefficient under the assumption that the static μ_s and the sliding μ_d friction coefficients are the same ($\mu_s = \mu_d = \mu$);

m_1 and m_2 are the masses of the upper and of the lower steel blocks, corrected with the half of the masses of the corresponding supporting bars and with the masses of the accelerometers;

k_1 and k_2 are the lateral stiffnesses of the bars, supporting the upper and the lower steel blocks, respectively;

The experimental model features the following characteristics: $m_1 = 6.301$ kg, $m_2 = 2.530$ kg, $k_1 = 1084$ N/m, $k_2 = 2895$ N/m, $\mu = 0.22$. The natural period of the coupled system is $T = 0.296$ s. The viscous damping coefficient was obtained from the records of free vibrations of the coupled system as $\xi = 0.015$. In the following numerical simulations the viscous damping is accounted by introducing a Rayleigh proportional damping matrix $[C] = \psi [K]$, where $[K]$ is the stiffness matrix, $\psi = T\xi / \pi$ and T is the first natural period of the coupled system. The dynamic testing of the steady-state response of the model comprised records of absolute acceleration time histories of the steel blocks for harmonic excitations with a shaking table displacement amplitude of $a_0 \approx 4$ mm and frequencies in the range of 2 to 5 Hz, as shown in Table 1., where by Δt_r is marked the step of discretization.

Table 1. Characteristics of experimental excitations

test No	freq. of excitation	amplitude of excitation	Δt_r
-	Hz	m	s
1	2.104	0.003994	0.01
2	2.659	0.004008	0.008
3	3.102	0.004062	0.00667
4	4.104	0.004052	0.005
5	5.123	0.004112	0.004

The recorded absolute acceleration time histories showed that the two masses were in adherence during tests No 1, 2 and 5 while periodic transitions between sticking and sliding were observed during tests No 3 and 4.

Hysteretic Model

The relationship friction force - displacement is represented by an approximate rigid-perfectly-plastic hysteresis, as shown in Fig. 2. Transition from sliding to sticking is considered when

$$\dot{x}_1 - \dot{x}_2 = 0 \tag{1}$$

thus obtaining the extrema $(x_1 - x_2)_{max}$ and $(x_1 - x_2)_{min}$ in the relative displacements difference $x_1 - x_2$. Since the exact fulfillment of (1) is practically unattainable in the numerical applications, the condition (1) is usually substituted by

$$|\dot{x}_1 - \dot{x}_2| < \varepsilon_1 \tag{2}$$

where ε_1 is a positive constant, chosen sufficiently small to describe properly the moment, when the sliding sceases.

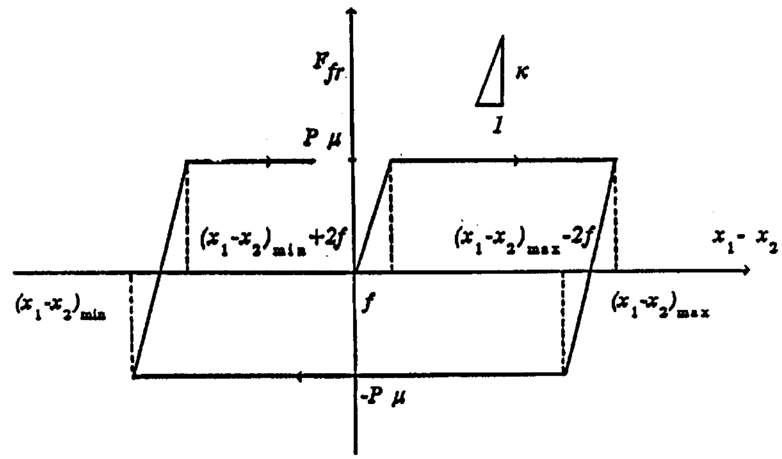

Fig. 2. Hysteretic model

During the sticking phase the friction force is linearly proportional to the relative displacements difference $x_1 - x_2$ by a coefficient k. Sliding occurs when friction force reaches the value $|F_{fr}| = P\mu$. The transition from sticking to sliding is described by

$$|x_1 - x_2| = f \quad \text{for the first sliding} \tag{3}$$

and

$$x_1 - x_2 = (x_1 - x_2)_{max} - 2f \quad \text{for } x_1 - x_2 > 0 \tag{4}$$

$$x_1 - x_2 = (x_1 - x_2)_{min} + 2f \quad \text{for } x_1 - x_2 < 0$$

where $f = P\mu / k$ will be denoted further as "sticking displacement".

The hysteretic model of the friction force implies the following mathematical description of the experimental model

$$m_1 \ddot{x}_1 + F_{fr} + k_1 x_1 = -m_1 \ddot{x}_0 \tag{5}$$

$$m_2 \ddot{x}_2 - F_{fr} + k_2 x_2 = -m_2 \ddot{x}_0$$

where F_{fr} is described according to the rigid-perfectly-plastic hysteretic relationship friction force - relative displacements difference (shown in Fig. 2) and the transitions between sticking and sliding are accounted according to eqs. (2), (3), (4).

Velocity Model

Velocity model was proposed in order to overcome the disadvantages of the method of controlling the stick - sliding conditions in accordance with the Coulomb representation of friction force, as proved by Dimova, Meskouris and Krätzig, 1995. The demonstrated oscillations of the relative velocity difference immediately after the time instances when the relative velocity equals zero or during the sliding phase naturally call for the necessity

(i) to introduce some appropriate approximation of the Coulomb model in order to provide a reliable estimation of the times of transition between sticking and sliding;

(ii) to control the relative velocity difference during the sliding phase when processing a numerical simulation.

As a reponse to these requirements the relationship friction force - relative velocity difference during the phase of adherence is considered to be linear over a small value band of the relative velocity difference as follows:

$$F_{fr} = c_1(\dot{x}_1 - \dot{x}_2) \qquad \text{for } /\dot{x}_1 - \dot{x}_2/ < \varepsilon \tag{6}$$

$$F_{fr} = P\mu sgn(\dot{x}_1 - \dot{x}_2) \qquad \text{for } /\dot{x}_1 - \dot{x}_2/ \geq \varepsilon \tag{7}$$

where $c_1 = P\mu\varepsilon^{-1}$ and ε is a positive constant chosen sufficiently small in order to describe as better as possible the sticking phase. This way, according to eq. (6), the friction interfaces are coupled during the phases of adherence by forces which are proportional to the relative velocity difference. Assuming the validity of the approximation eqs. (6) and (7), the difference of the relative velocity does not change sign during a sliding phase (for $/\dot{x}_1 - \dot{x}_2/ \geq \varepsilon$), therefore

$$sgn[\dot{x}_1(t) - \dot{x}_2(t)] = sgn[\dot{x}_1(t - \Delta t) - \dot{x}_2(t - \Delta t)] \tag{8}$$

where Δt is the integration step. Solutions obtained under the assumption eq. (8) are considered to be correct at any time instant t during the sliding phase if the condition

$$[\dot{x}_1(t) - \dot{x}_2(t)][\dot{x}_1(t - \Delta t) - \dot{x}_2(t - \Delta t)] > \varepsilon^2(1 - 1/n) \tag{9}$$

is satisfied. Herein, $n > 1$ is introduced to control the exactness of the solution when a transition between sticking and sliding occurs. Generally, the condition (9) implies no changes of the relative velocity difference sign during the sliding phase. Also, when a transition occurs, the absolute value of the relative velocity difference should exceed the value $(\varepsilon - \varepsilon/n)$. To conclude, utilizing the approximations (6), (7) and the assumption eq. (8), the mathematical description of the experimental model is

$$m_1\ddot{x}_1(t) + c_1[\dot{x}_1(t) - \dot{x}_2(t)] + k_1 x_1(t) = -m_1\ddot{x}_0(t) \qquad \text{for } /\dot{x}_1 - \dot{x}_2/ < \varepsilon \tag{10}$$

$$m_2\ddot{x}_2(t) - c_1[\dot{x}_1(t) - \dot{x}_2(t)] + k_2 x_2(t) = -m_2\ddot{x}_0(t)$$

and

$$m_1\ddot{x}_1(t) + k_1 x_1(t) = -m_1\ddot{x}_0(t) - P\mu\, sgn[\dot{x}_1(t - \Delta t) - \dot{x}_2(t - \Delta t)] \qquad \text{for } /\dot{x}_1 - \dot{x}_2/ \geq \varepsilon$$

$$m_2\ddot{x}_2(t) + k_2 x_2(t) = -m_2\ddot{x}_0(t) + P\mu\, sgn[\dot{x}_1(t - \Delta t) - \dot{x}_2(t - \Delta t)] \tag{11}$$

NUMERICAL RESULTS

The unconditionally stable Newmark method with two different integration time steps was employed for the numerical integration. A time increment of $\Delta t_1 = \Delta t_r / n_1$ was used when no transitions between sticking and sliding occurred. Here by Δt_r is marked the step of discretization of the record of the shacking table motion. Whenever a transition occurred, a smaller step $\Delta t_2 = \Delta t_1 / n_2$ was introduced.

"Oscillations" of the Relative Velocity Difference in Case of Hysteretic Model

Initially, when dealing with the hysteretic model, values of $f = 0.00001$ m, $\varepsilon_1 = 0.001$ m/s, $n_1 = 2$, and $n_2 = 2$ were assumed and base excitation according to test No 4 was considered. The resulting absolute acceleration time histories differed significantly from the experimental recorded ones, as depicted in Fig. 3a for the upper mass. This phenomenon can be explained by the large number of changes of the friction force magnitude due to the change of the relative velocity difference sign in a frame of one integration step, as depicted in Fig. 3b.

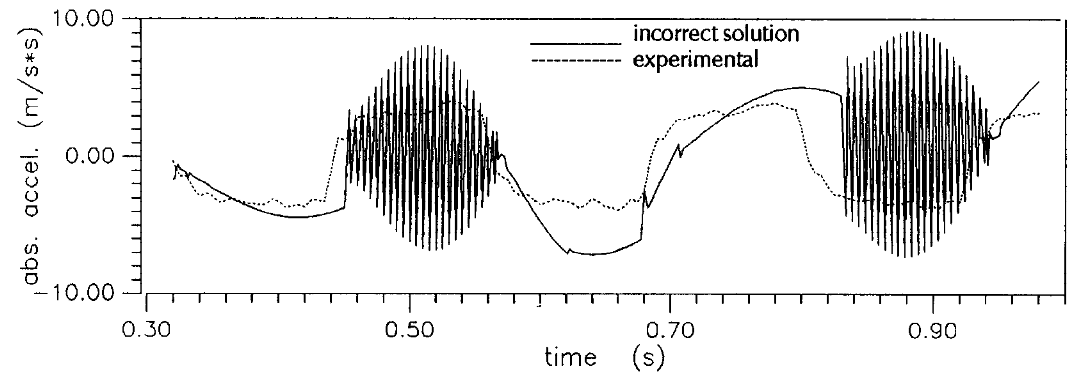

Fig. 3a. Absolute acceleration time history for the upper mass

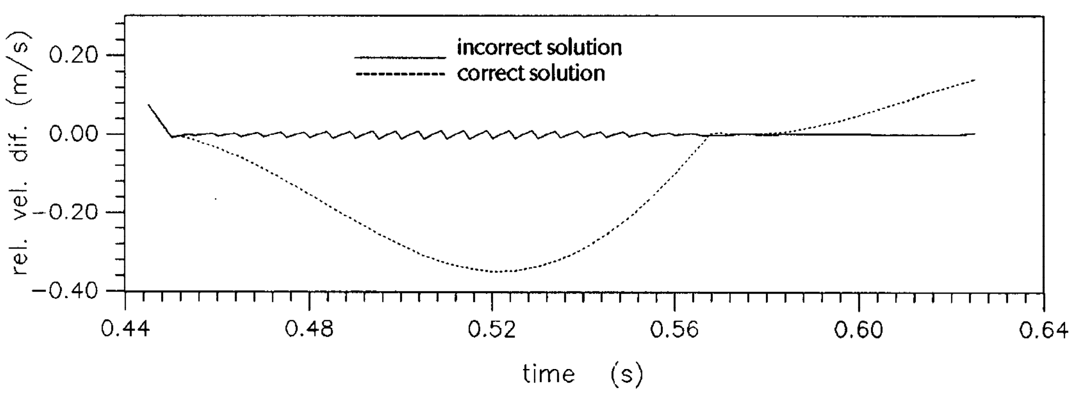

Fig. 3b. Relative velocity difference time history

The change of the sign of the relative velocity difference contradicts the constant sign of the friction force, preliminary accepted for the considered integration step, i.e. the obtained solution is not correct. Starting the solution in the next integration step by use of incorrect initial conditions, another incorrect solution is obtained and so on. A similar phenomenon was demonstrated by Dimova, Meskouris and Krätzig, 1995, when dealing with the Coulomb model and with the velocity model, if the correctness condition is not satisfied. In case of hysteretic model these "oscillations" appear when a phase transition from sliding to sticking occurs. In order to avoid these disaccuracies of the hysteretic model, the following correctness condition is imposed at every moment t during the sliding phase

$$[\dot{x}_1(t) - \dot{x}_2(t)]\,[\dot{x}_1(t - \Delta t) - \dot{x}_2(t - \Delta t)] > \varepsilon_1^{\,2}\,(1 - 1/n) \qquad (12)$$

where Δt is the step of integration ($\Delta t = \Delta t_1$ or $\Delta t = \Delta t_2$) and $n > 1$ has the same meaning as in (9). Thus, the condition (12) implies no changes of the relative velocity difference sign in a frame of one integration step during the sliding phase and allows to obtain the phase transition from sliding to sticking for an absolute value of the relative velocity difference, exceeding $(\varepsilon_1 - \varepsilon_1/n)$. During the solution process by the hysteretic model the validity of condition (12) should be checked and if (12) is not satisfied, one should increase the value of ε_1 or decrease the value of the integration step Δt. The less the ε_1 the higher the accuracy in finding the right moment of transition between sliding and sticking, but the computational losses increase (Δt should be less) and vice versa.

For these same initial values of ε_1 and f the correct solution satisfying (12) was obtained by introducing n = 10 and by decreasing the integration time step through choosing n_1 = 20 and n_2 = 50. The relative velocity difference for this correct solution is shown in Fig. 3b. The resulting absolute acceleration time histories show a very good coincidence with the experimental recorded ones, as demonstrated in Fig. 4a for the upper mass and in Fig. 4b for the lower mass.

This good correlation between the experimental and calculated results shows that the numerical simulations using the hysteretic model can describe adequately the dynamic behaviour of systems with friction devices, when the correctness condition (12) is introduced. Therefore, the implementation of condition (12) should be recommended for implementation in numerical studies by use of hysteretic model.

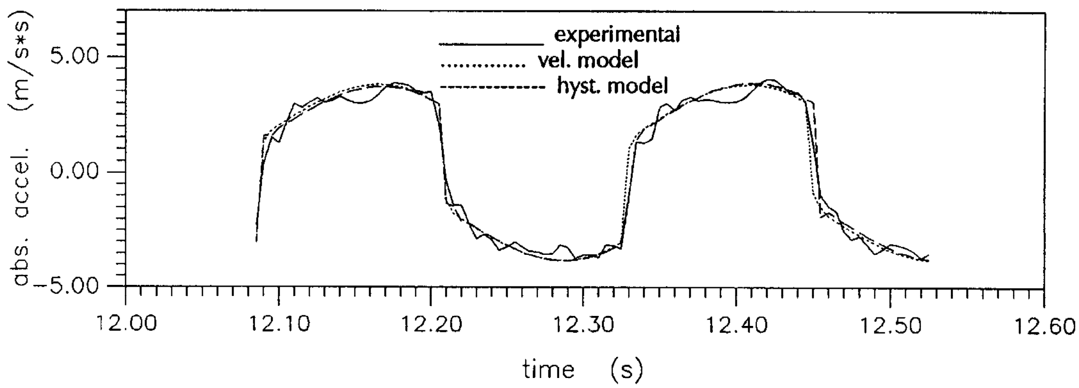

Fig. 4a . Absolute acceleration time histories for the upper mass

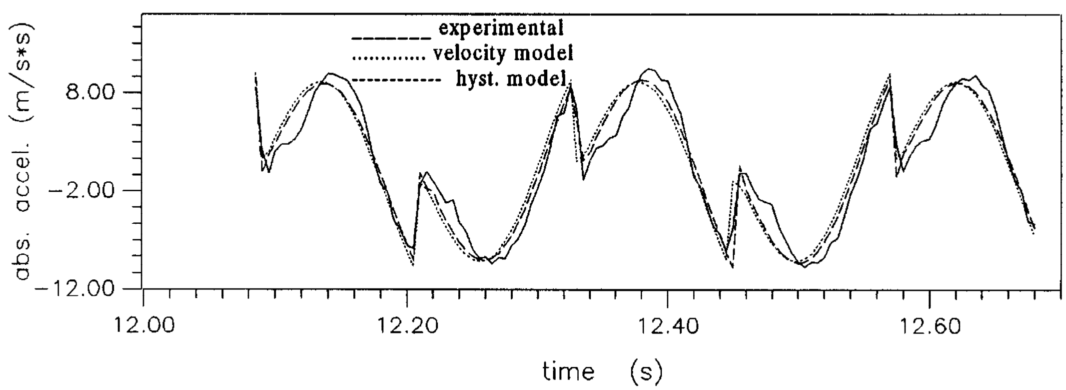

Fig. 4b. Absolute acceleration time histories for the lower mass

Comparison of Hysteretic and Velocity Models

Correct solutions are computed by hysteretic model for ε_1 = 0.001 m/s , f = 0.00001 m and n = 10 and by velocity model for ε = 0.001 m/s and n = 10. The periodic transitions from sticking to sliding, recorded during test No 4 are compared with computed response in Fig. 4a and Fig. 4b for the upper and the lower mass, respectively. Both the solutions by hysteretic model and by velocity model coincide well with experimental results. We thus conclude that the two considered models can describe adequately the dynamic behaviour of system with friction devices.

The implementation of hysteretic model requires to chose a couple of parameters (ε_1 and f) to approximate friction force with a given magnitude in contrast to velocity model, which employs only one parameter (ε_1). The considered velocity model employs a relatively simple approximation of the Coulomb representation of the friction force - velocity relationship. The introduction of this relationship to the mathematical model of the structure is not complicated, because it involves only the damping matrix and the load vector. Furthermore, if experimental data of the relationship between the sliding friction coefficient and the relative velocity for the special interfaces are available, the direct introduction of the relationship between friction force and relative velocity allows a relatively easy consideration of this nonlinearity.

CONCLUDING REMARKS

1. "Oscillations" of relative velocity difference can appear at times when the sliding ceases, when employing hysteretic model of friction force. To avoid this inaccuracy a correctness condition (12) should be introduced and checked during the numerical simulations.

2. When dealing with correct solutions, similar and well coinciding with experimental data results are calculated by hysteretic and velocity models. Thus the velocity model, because of its simplicity, could be proposed for further development and implementation.

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