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# RESPONSE OF BASE ISOLATED SYSTEMS EQUIPPED WITH HYBRID MASS DAMPERS TO RANDOM EXCITATIONS

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## ABSTRACT

Random response of Base Isolated Systems equipped with Hybrid Mass Dampers, subject to horizontal random excitations, is analyzed. Considering the superstructure motion described by its first modal contribution, a three-degree-of-freedom equivalent linear model under stationary Gaussian excitations, modelled by the modified Kanai-Tajimi power density spectrum, was used in the analysis. The performance of the new system was tested under random excitations in order to evaluate the sensitivity of the response to the input motion in comparison with the one of Base Isolated Systems with and without passive Mass Damping. Comparisons have shown significant reductions of the hybrid system response in respect to the passive one.

## KEYWORDS

Hybrid Control; Hybrid Mass Damping; Base Isolation; Random Vibrations.

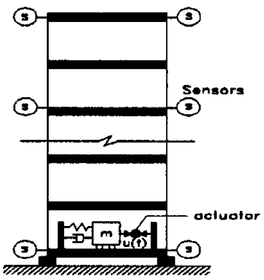
## INTRODUCTION

Mass damper systems are auxilary systems which use inertia force to reduce the vibration of a primary structure. They are passive type (Tuned Mass Dampers-TMD), active type (Active Mass Damper-AMD) or hybrid type (Hybrid Mass Damper-HMD). Generally, the seismic control capacity of a tuned mass damper on multi-degree-of-freedom systems is considered to be very low. A significant improvement of control power in building vibrations using mass damping was obtained by the introduction of active control criteria first proposed by T. Kobori in 1986 and applied for the first time on the Tokyo Kyobashi Seiwa Building in 1989 (Kobori, 1991) with an Active Mass Driver System - AMD. In 1994 the authors of this paper proposed a new system derived from the combination of base isolation strategy (BIS) and tuned mass damping (Palazzo and Petti, 1994 and 1995). The efficiency of this system (BIS+TMD) in the amplitude reduction of seismic motion has been shown to be significant when compared to the case without TMD.

As known, the reduction capacity of a frequency response produced by mass damping is limited to a narrow band of frequencies, generally chosen in proximity of the basic frequency of the main system, while a seismic event generally has significant components in a wide frequency range. The positive behaviour, of the combined system (BIS+TMD), is due to the appropriate combination of the following fundamental properties:

- the reduction of the motion transmission to the superstructure due to isolators
- the low-pass filtering capacities of the isolation
- the narrow-band attenuation produced by the damping mass tuned to the fundamental frequency.

However, the passive system presents some difficulty in tuning the TMD optimal parameters to a natural vibration frequency, variable with the excitation intensity, caused by non-linear behaviour of the isolation system. This problem can be solved using semi-active system technology, a self-tuning control system or with the introduction of an active control system (fig. 1). Moreover active control also reduces the overall response of the system.




Figg 1: Base Isolated System and HMD

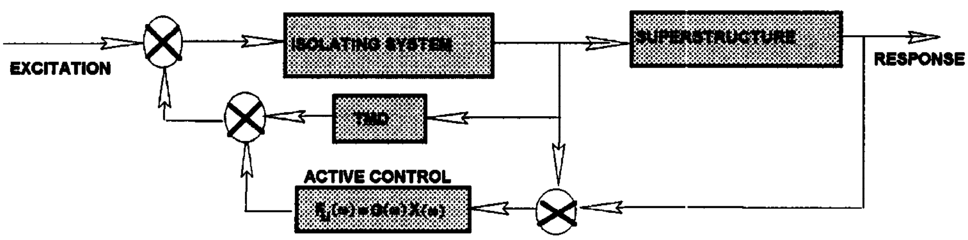



Fig. 2 : Block diagram of a BIS and HMD

The block diagram of the new hybrid control system is shown in fig. 2. The control force $F_u(\omega)$ is designed to be a linear function of the state vector $X(\omega)$ through the gain matrix $G(\omega)$ depending on the selected control algorithm. In this investigation, the response of equivalent linear hybrid systems subject to a stationary Gaussian random process is analyzed. The stationary filtered white noise modelled by Kanai (Kanai, 1957) and Tajimi (Tajimi, 1960) and modified by Clough and Penzien (Clough and Penzien, 1975) has been used as a random process. The investigation was carried out by analyzing a three-degrees-of-freedom linear model, considered as the simplest model of a general isolated building and HMD if the superstructure is represented by its $1^{st}$ modal contribution. The probabilistic characteristics of the response (mean, covariance and power spectral density) have been evaluated to explore the response reduction due to the hybrid control and sensivity to the design parameters. This analysis is carried out considering the response as a stationary process. Therefore by this method the transient response can not be taken into account.

## STOCHASTIC MODEL

In this study, the horizontal ground acceleration $a_g(t)$ is modelled as a stationary Gaussian filtered white noise random process with a zero mean and characterized by its Power Spectral Density (PSD) $S_{ag}(\omega)$. In particular the PSD formulation proposed by Kanay (Kanay, 1957) and Tajimi (Tajimi, 1960), and later modified by Clough and Penzien (Clough and Penzien, 1975 ) by applying a low frequency filter, is used as a spectrum model:

$$S_{ag} = |H_{CP}(\omega)|^2 \cdot |H_{KT}(\omega)|^2 \cdot S_0 \tag{1}$$

where the Clough & Penzien low band frequency filter $H_{CP}(\omega)$, and the Kanai and Tajimi filter $H_{KT}(\omega)$ are:

$$H_{CP}(\omega) = \frac{\frac{\omega^2}{\omega_1^2}}{\left(1-\frac{\omega^2}{\omega_1^2}\right) + 2j\xi_1\frac{\omega}{\omega_1}} \qquad H_{KT}(\omega) = \frac{1+2j\xi_g\frac{\omega}{\omega_g}}{\left(1-\frac{\omega^2}{\omega_g^2}\right)+2j\xi_g\frac{\omega}{\omega_g}} \tag{2),(3}$$

In this model $\omega_g$ and $\xi_g$ are respectively the natural frequency and damping ratio of the soil deposit and $\omega_1$, $\xi_1$ the low band filter characteristics. $S_o$ is the white noise power density considered in the Kanai-Tajimi model. Soil is considered in the analysis by assuming the following in the spectrum model: $\omega_g$ = 31.4 rad/sec; $\xi_g$ = 0.55. For all cases the Clough-Penzien modification low-band frequency filter characteristics are assumed as: $\omega_1 = 1$; $\xi_1 = 0.7$.

## EQUIVALENT LINEAR MODEL OF BASE ISOLATED SYSTEM AND HMD

Lets consider the linear model with three-degree-of-freedom shown in Fig. 3 which represents the generic base isolated system equipped with a hybrid mass damper HMD.

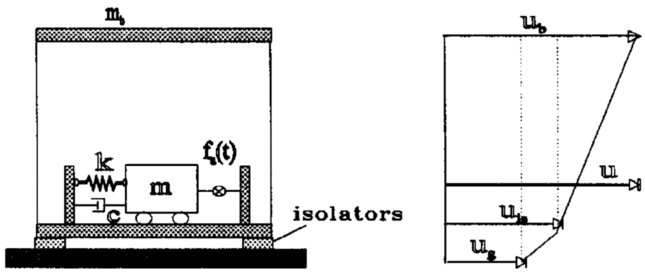



Fig. 3: equivalent linear model of BIS and HMD

Assuming an inertial reference, with the symbols indicated in Fig. 3 the motion equations of the model subject to seismic ground axceleration $\ddot{u}_g(t)$ and control force $f_u(t)$ can be written as:

$$\begin{aligned}
&m\,\ddot{u} + c\left(\dot{u} - \dot{u}_{is}\right) + k\left(u - u_{is}\right) = -f_u(t)\\
&m_b\,\ddot{u}_b + c_b\left(\dot{u}_b - \dot{u}_{is}\right) + k_b\left(u_b - u_{is}\right) = 0\\
&m_{is}\,\ddot{u}_{is} + m\,\ddot{u} + m_b\,\ddot{u}_b + c_{is}\left(\dot{u}_{is} - \dot{u}_g\right) + k_{is}\left(u_{is} - u_g\right) = 0
\end{aligned} \qquad (4)$$

By introducing the relative displacements $v = u - u_{is}$, $v_b = u_b - u_{is}$, $v_{is} = u_{is} - u_g$ and the status vector $\mathbf{x} = \begin{vmatrix}\mathbf{v} & \dot{\mathbf{u}}\end{vmatrix}$, equation (4) can be rewritten in the classic state space form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_g\,\ddot{u}_g + \mathbf{B}_u\,f_u \qquad (5)$$

whereas:

$$\mathbf{A} = \begin{vmatrix} \mathbf{0}_{33} & \mathbf{I}_{\mathbf{xx}} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{vmatrix};\mathbf{B}_g = \begin{vmatrix} \mathbf{I}_{\mathbf{x}} \\ \mathbf{M}^{-1}\mathbf{C}\mathbf{b}_{\mathbf{g}} \end{vmatrix};\mathbf{B}_u = \begin{vmatrix} \mathbf{0}_{31} \\ \mathbf{M}^{-1}\mathbf{b}_{\mathbf{u}} \end{vmatrix};\mathbf{0}_{33} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix};\mathbf{I}_{\mathbf{xx}} = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix};\mathbf{I}_{\mathbf{x}} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix};$$

$$\mathbf{0}_{31} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\mathbf{M} = \begin{vmatrix} m & 0 & 0 \\ 0 & m_b & 0 \\ m & m_b & m_{is} \end{vmatrix};\mathbf{C} = \begin{vmatrix} c & 0 & -c \\ 0 & c_b & -c_b \\ 0 & 0 & c_{is} \end{vmatrix};\ \mathbf{K} = \begin{vmatrix} k & 0 & 0 \\ 0 & k_b & 0 \\ 0 & 0 & k_{is} \end{vmatrix};\mathbf{u} = \begin{vmatrix} u \\ u_b \\ u_{is} \end{vmatrix};\mathbf{v} = \begin{vmatrix} v \\ v_b \\ v_{is} \end{vmatrix};\mathbf{b}_{\mathbf{g}} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix};\mathbf{b}_{\mathbf{u}} = \begin{vmatrix} -1 \\ 0 \\ 0 \end{vmatrix};$$

Matrix $\mathbf{A}$ is of order 6x6, while vectors $\mathbf{x}$, $\mathbf{B}_g$, $\mathbf{B}_u$ have dimensions 6x1. In (5) the acceleration and velocities appear in absolute terms while the displacements appear in relative terms. For the Isolation System equipped with HMD the following main parameters are introduced:

$\omega_b = \sqrt{k_b / m_b}$; $T_b = 2\pi / \omega_b$; $\xi_b$ — superstructure natural frequency, fundamental period and damping factor;

$\omega_{is} = \sqrt{k_{is} / m_{is} + m_b}$; $\xi_{is}$ — isolating system natural frequency and damping factor;

$\chi = m_b / m_{is} + m_b$; $I_d = \omega_b / \omega_{is}$ — mass ratio and Isolation degree;

$\omega^2 = k/m$ ; $\xi_{TMD}$ natural TMD frequency and damping;

$\alpha_{TMD} = \omega/\omega_{is}$ ; $\mu = m/m_{is} + m_b$ TMD frequency and mass ratios;

The optimum TMD parameters have been assigned according to Ioi and Ikeda, 1978 formulation. In the equation (5) the control force $f_u(t)$ is an unknown quantity to be determined from the optimization of the dynamic behavior of the system according to a given criteria. Optimization is intended as a modification of status variables with the aim of keeping the system as close to a reference configuration as possible. The optimization approach is hereby formulated with the introduction of a performance index $J$ to be minimized, in the linear quadratic problem (LQ), function of the system status and of the regulation force $f_u(t)$:

$$J(t) = \frac{1}{2}\int_{t_0}^{t}\left\{\mathbf{x}^T(t)\,\mathbf{Q}\,\mathbf{x}(t) + f_u(t)^2\,R\right\}dt \tag{6}$$

This index penalizes the system state through the use of square matrix $\mathbf{Q}$ which penalizes the distance from the motionless configuration for each time fraction. $R$ directly penalizes the control force; its value, with respect to the value of $\mathbf{Q}$, determines the influence of active control on the system performance. The problem is finding the control force $f_u(t)$ that minimizes the performance index $J$. Solving the well known Riccati problem, we obtain:

$$f_u(t) = -\mathbf{G}(t)\ \mathbf{x}(t) \tag{7}$$

In equation (7) matrix $\mathbf{G}(t)$ is called gain matrix.
The transfer functions of the linear differential equation (5), considering the (7), are obtained applying the Fourier transform assuming the initial conditions to be zero:

$$\mathbf{X}(\omega) = \left(i\omega\mathbf{I} - \mathbf{A} + \mathbf{B}_u\mathbf{G}\right)^{-1}\mathbf{B}_g i\omega U_g(\omega) = \mathbf{H}_g(\omega)\,U_g(\omega) \tag{8}$$

where $\mathbf{X}(\omega)$ and $U_g(\omega)$ respectively represent the Fourier transform of $\mathbf{x}(t)$ and $u_g(t)$.
The complex frequency response $\mathbf{H}(\omega)$ (6x1) contains all information necessary to obtain the system response to an arbitrary excitation. As known, the spectral densities of the response $\mathbf{S}_{out}(\omega)$ are related to the acceleration spectral density $S_{ug}(\omega)$ by:

$$\mathbf{S}_{out}(\omega) = \mathbf{H}^*(\omega)\mathbf{S}_{ug}(\omega)\mathbf{H}^T(\omega) \tag{9}$$

The mean square of the response is given by:

$$\sigma^2_{out} = \int_{-\infty}^{+\infty}\mathbf{S}_{out}(\omega)d\omega \tag{10}$$

RESULTS

Using the modified Kanai-Tajimi excitation input model applied to the equivalent three-degree-of freedom structural model, the root mean square values (rms) of the response have been evaluated for several values of main parameters.

Results, represented in figures 4-6, show the rms response ratios of hybrid system respect to the corrispondent passive ones. Figures 4 and 6 show the influence of the isolation degree $I_d$ and mass ratio $\mu$ in comparison with the cases with and without TMD. Figure 5 shows the influence of the fixed-base structure main period $T_b$ and the isolation damping $\xi_{is}$ compared to the case with and without TMD. It is noted that in all examined cases the hybrid system response is significantly reduced respect to the passive system. In most cases the rms response gain is about 50% in comparison with BIS+TMD and about 70% in comparison with the correspondent BIS. Maximum gains occur for a low value of damping factor $\xi_{is}$ and mass ratio $\mu = 0.05 \div 0.10$. As shown in the figures, the main reduction effect is produced on the isolators relative displacements for high $I_d$ values.

Tab. 1 shows the results of the response varying active control parameters. Moreover, tab. 1 shows the minimum rms ratios of the hybrid system in respect to the passive ones.

| Case | q1 | q2 | q3 | q4 | q5 | q6 | r | Response | BIS+HMD /BIS | BIS+HMD /BIS+TMD |
|---|---|---|---|---|---|---|---|---|---|---|
| a | $10^3$ | $10^5$ | $10^5$ | $10^3$ | $10^5$ | $10^5$ | $10^{-2}$ | $v_b$ | 0.2307 | 0.3878 |
| | | | | | | | | $v_{is}$ | 0.1640 | 0.4043 |
| | | | | | | | | $\dot{u}_b$ | 0.1926 | 0.4810 |
| | | | | | | | | $\ddot{u}_b$ | 0.2308 | 0.3878 |
| b | $10^3$ | $10^5$ | $10^5$ | $10^3$ | $10^5$ | $10^5$ | $10^{-4}$ | $v_b$ | 0.2290 | 0.3876 |
| | | | | | | | | $v_{is}$ | 0.1631 | 0.4042 |
| | | | | | | | | $\dot{u}_b$ | 0.1914 | 0.4810 |
| | | | | | | | | $\ddot{u}_b$ | 0.2290 | 0.3877 |
| c | $10^8$ | $10^8$ | $10^8$ | $10^8$ | $10^8$ | $10^8$ | $10^{-5}$ | $v_b$ | 0.2447 | 0.5560 |
| | | | | | | | | $v_{is}$ | 0.3691 | 0.6709 |
| | | | | | | | | $\dot{u}_b$ | 0.2101 | 0.4624 |
| | | | | | | | | $\ddot{u}_b$ | 0.1404 | 0.3189 |
| d | 0 | $10^3$ | $10^5$ | 0 | $10^3$ | $10^5$ | $10^{-5}$ | $v_b$ | 0.0766 | 0.0933 |
| | | | | | | | | $v_{is}$ | 0.1231 | 0.1762 |
| | | | | | | | | $\dot{u}_b$ | 0.1326 | 0.1912 |
| | | | | | | | | $\ddot{u}_b$ | 0.0766 | 0.0933 |
| e | 0 | $10^5$ | $10^7$ | 0 | $10^5$ | $10^7$ | $10^{-6}$ | $v_b$ | 0.0414 | 0.0479 |
| | | | | | | | | $v_{is}$ | 0.1230 | 0.1761 |
| | | | | | | | | $\dot{u}_b$ | 0.1132 | 0.1296 |
| | | | | | | | | $\ddot{u}_b$ | 0.0414 | 0.0479 |

## CONCLUSION

The capacity of new hybrid control strategy based on the combination of base isolation and hybrid mass damping was investigated. The response reduction of the hybrid systems subject to random excitations was evaluated and compared to the correspondent passive systems. From the obtained results it is evident that the new BIS+HMD hybrid system shows significant gain on the seismic vibration reduction compared to the corresponding BIS+TMD system which, in turn, shows sensible gains compared to the simple isolated system (BIS). Moreover by adding an active control system to a tuned mass damper the necessity of having a perfect tuning of the additional system over the primary systems can be avoided. The new strategy can be considered as a useful combination of the low-pass BIS filter with the HMD vibration suppressing capacity.

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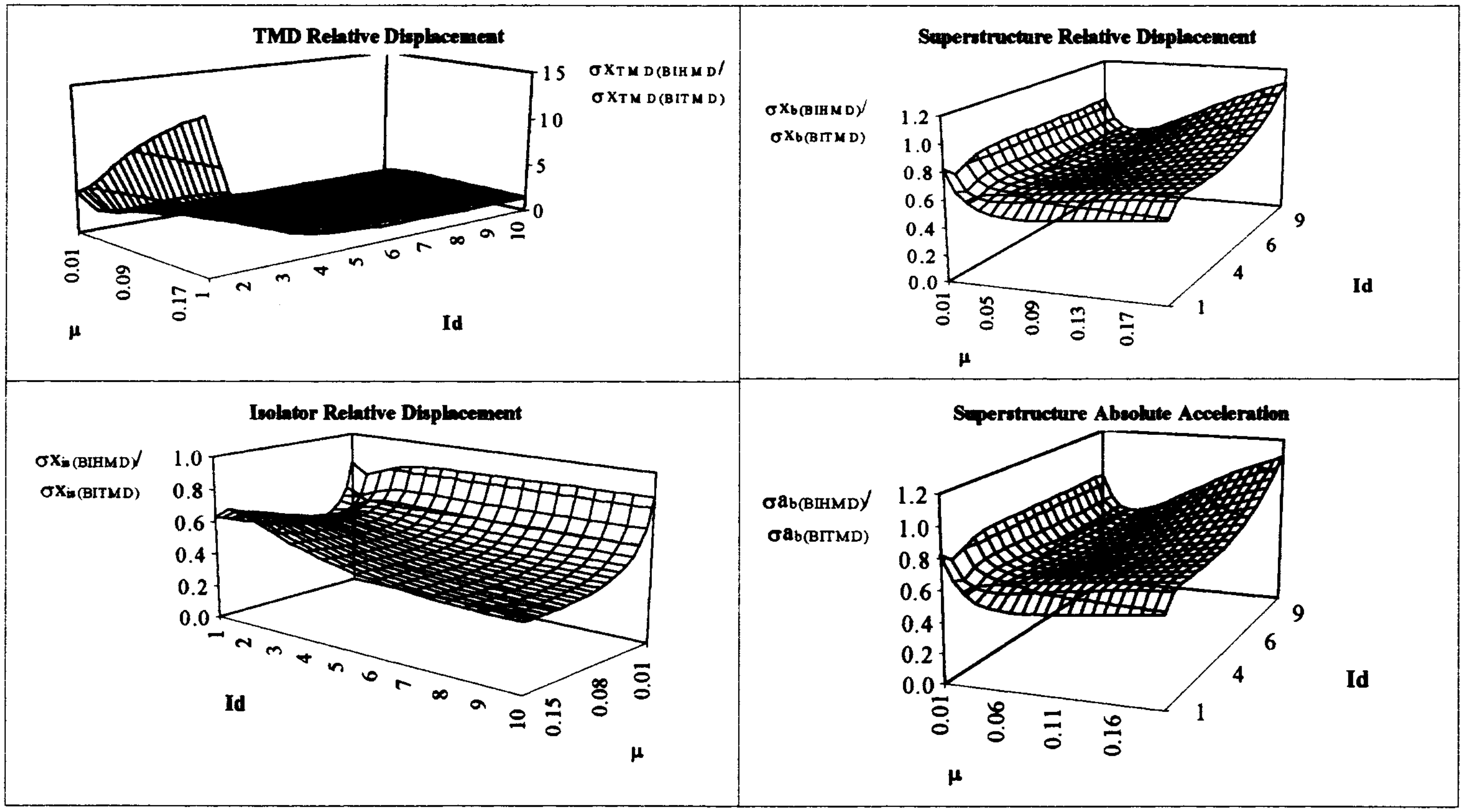
**HYBRID SYSTEM TO PASSIVE SYSTEM WITH TMD**
**RMS RESPONSE RATIOS**
Influence of Isolation Degree Id and TMD mass ratio μ

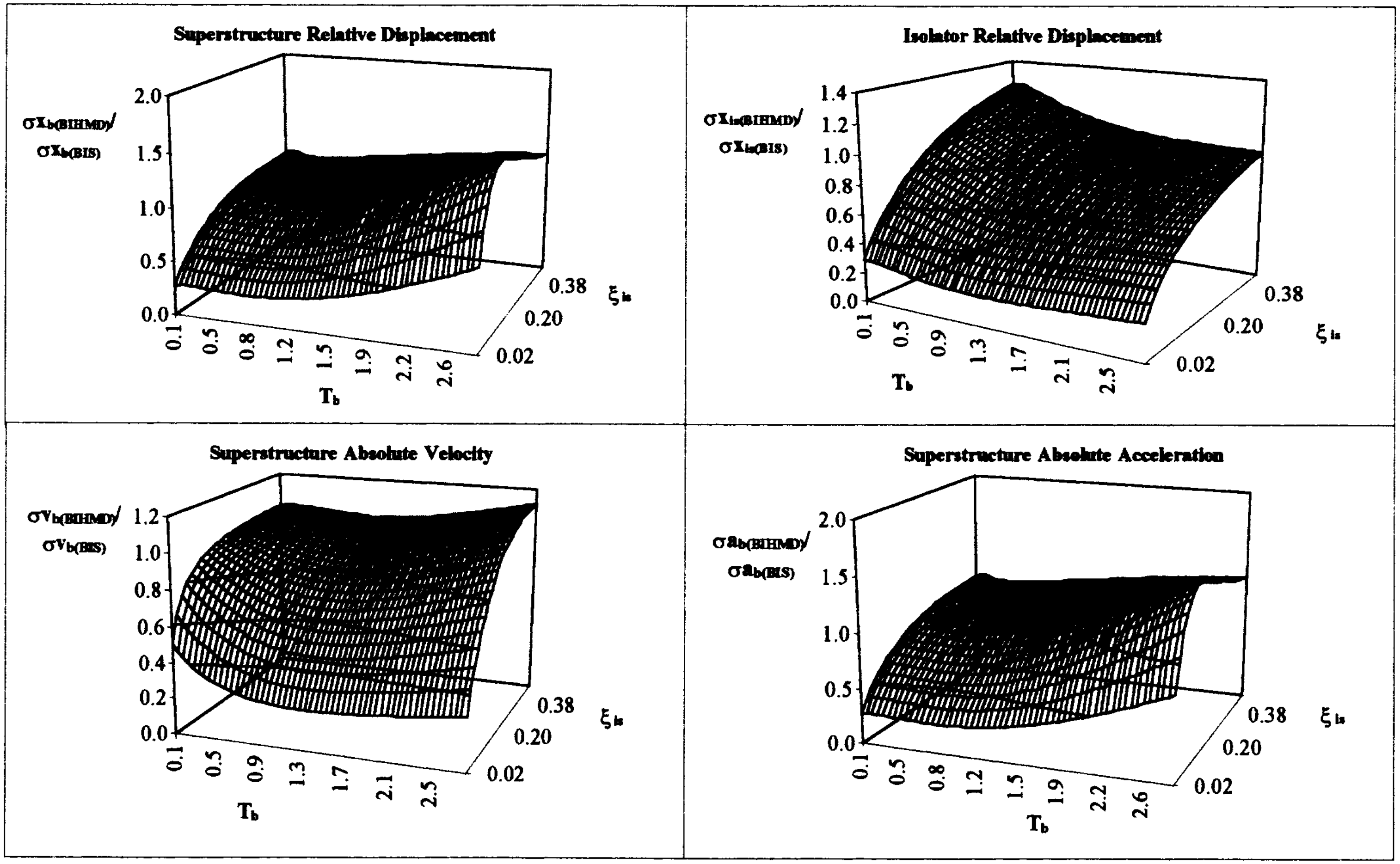



$\xi_b$=0.02; $\xi_{is}$=0.02; χ=0. 7; $T_b$=0.8 sec
diag**Q**=[10 1000 1000 10 100 1000]; R=0.01

Fig. 4

## HYBRID SYSTEM TO PASSIVE SYSTEM WITHOUT TMD
## RMS RESPONSE RATIOS

Influence of isolator damping factor $\xi_{is}$ and superstructure main period $T_b$

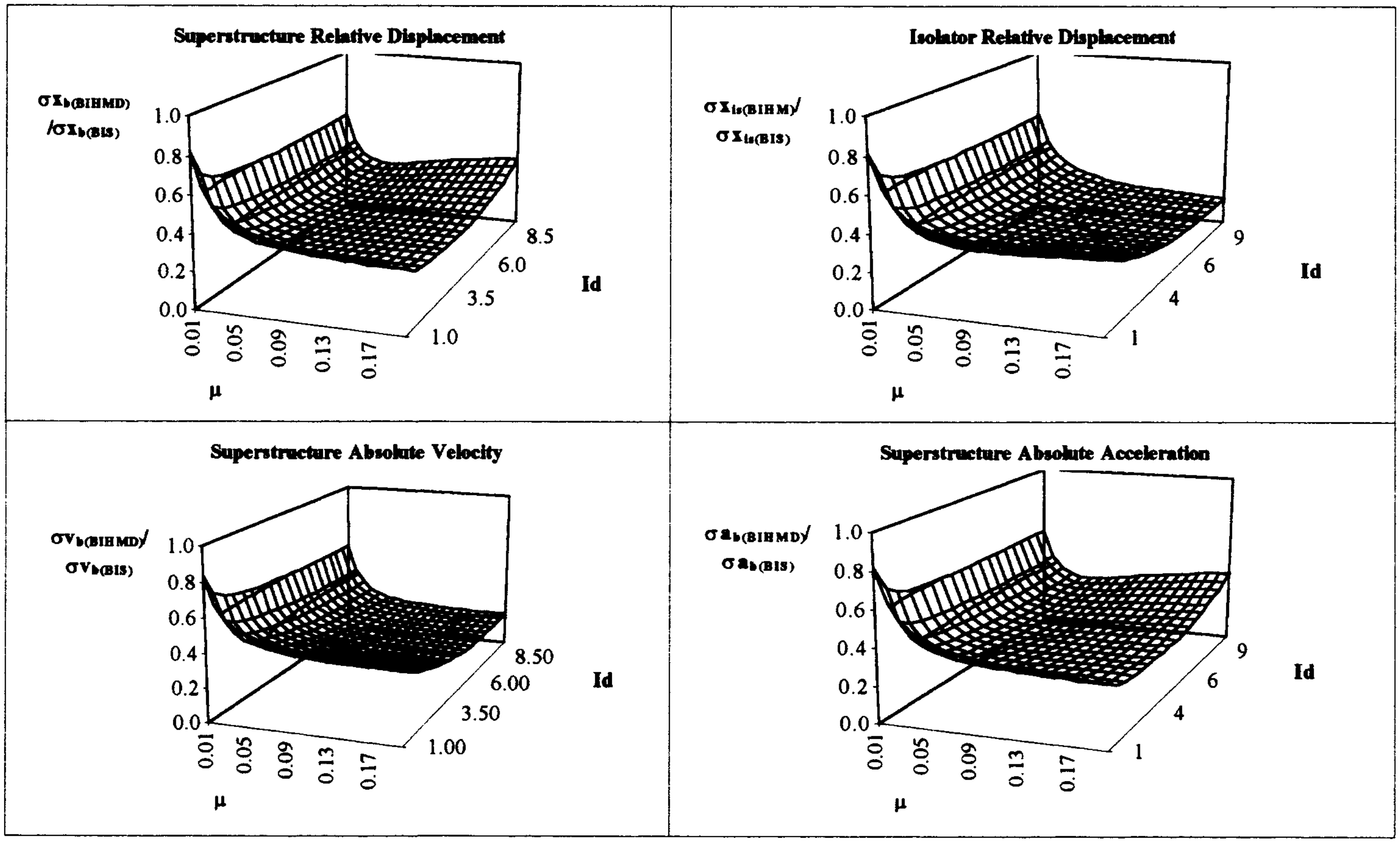



$\xi_b$=0.02; $\mu$=0.1; $\chi$=0.7; Id=5

diag**Q**=[10 $10^3$ $10^3$ 10 $10^2$ $10^3$]; R=0.01

Fig. 5

**HYBRID SYSTEM TO PASSIVE SYSTEM WITHOUT TMD**
**RMS RESPONSE RATIOS**
Influence of Isolation Degree Id and TMD mass ratio μ




$\xi_b$=0.02; $\xi_{is}$=0.02; $\chi$=0.7; $T_b$=0.8 sec
diagQ=[10 $10^3$ $10^3$ 10 $10^2$ $10^3$]; R=0.01

Fig. 6