



SCATTERING ATTENUATION OF SEISMIC WAVES IN INHOMOGENEOUS MEDIA

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ABSTRACT

Earthquake records shows that the attenuation of seismic waves propagated in near-surface ground depends on their frequency. This can be attributed to the scattering effect of inhomogeneous soil on seismic waves. This research investigates : (1) the relationship between the properties of inhomogeneity and scattering attenuation by using two-dimensional FEM simulation, a method that allows the consideration of multiple statistical properties of spatial variation, and (2) the statistical properties of the velocity structure of the Tokyo Bay area by comparing the frequency-dependent attenuations identified from the actual earthquake records and the single scattering theory.

KEYWORDS

Scattering Attenuation; Inhomogeneous Medium; Anisotropy; Correlation Length

INTRODUCTION

The attenuation characteristics of soil deposits are very significant, together with the velocity structure, they characterize the amplification factor of near-surface ground area. The frequency-dependence of the Q-value in near-surface ground was pointed out from the vertical array observation records (Takemura *et al.*, 1992). It is thought that the frequency-dependence of the Q-value is caused by inhomogeneities in the ground (Sato, 1990; 1991). Sato and Kawase (1992) also investigated the scattering attenuation characteristics of inhomogeneous media based on a numerical simulation using the two-dimensional finite element method, with the inhomogeneities expressed by fluctuations of the velocity structure. The statistical properties of the velocity fluctuation of an inhomogeneous medium include variance, function type of auto-correlation and correlation length. In general, the horizontal correlation length is rather longer than the vertical correlation length because the sedimentary layers are assumed to be horizontally layered soil deposits. In this study, the anisotropic properties of the correlation of velocity fluctuation in near-surface ground were investigated by numerical simulation. Comparison between the frequency-dependent attenuations identified from actual earthquake records on the coast of Tokyo Bay and the single scattering theory was also performed. Finally, the statistical properties of velocity

structure were estimated.

SCATTERING ATTENUATION OF SEISMIC WAVES IN RANDOM MEDIA WITH ANISOTROPIC VELOCITY FLUCTUATION

Methodology

The calculation of apparent attenuation in this study's numerical simulation followed the method of Sato and Kawase(1992). A 2-D FEM model shown in Figure 1 was also used. Each element had a fluctuating S-wave and P-wave velocity around the mean value. The Ricker wavelet was used as the incident SV plane wave. An inhomogeneous field was constructed using the so-called spectral representation method(Yamazaki and Shinozuka,1988). A Gaussian type function was chosen to describe the auto-correlation function of the inhomogeneous field.

$$R(\xi_x, \xi_y) = \sigma^2 \exp\left\{-\left(\frac{\xi_x}{a_x}\right)^2 - \left(\frac{\xi_y}{a_y}\right)^2\right\} \quad (1)$$

where ξ_x and ξ_y = separation distance in the x and y direction, respectively, σ^2 = variance of wave velocity, and a_x and a_y = correlation length in the x and y direction, respectively. Parameters of random media are listed in Table 1. Here, the unit weight and Poisson's ratio are constant, and the absolute displacement time history at the 144 node points (9 columns \times 16 rows) around the central axis can be observed.

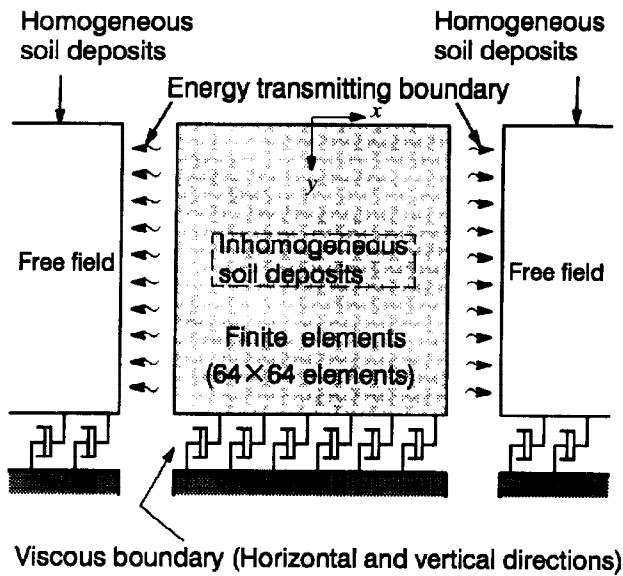


Table 1. Parameters of random media

Unit weight	(t/m ³)	1.8
Average Vp	(m/s)	1000.0
Average Vs	(m/s)	200.0
Vp/Vs		Constant
Auto-correlation function		Gaussian type
Correlation length (m)		2.0~4.0
Coefficient of variation		0.2
Intrinsic attenuation		0

Figure 1. FEM model

Scattering attenuation of direct S-wave was determined by the least square method with the following relationship.

$$\frac{A(y)}{A_0(y)} = \exp\left[-\frac{2\pi f_c y}{2V_{sm}Q_s(f_c)}\right] \quad (2)$$

where,

- $A(y)$: Peak amplitude of direct wave (Random velocity medium)
- $A_0(y)$: Peak amplitude of direct wave (Constant velocity medium)
- V_{sm} : Average S-wave velocity of random medium
- $Q_s(f_c)$: Quality factor of S-wave
- f_c : Peak frequency of incident wave

Equation (2) represents the amplitude ratio of a direct wave against propagation distance. Attenuation as a function of frequency can be obtained from the calculation by varying the peak frequency of the incident wave .

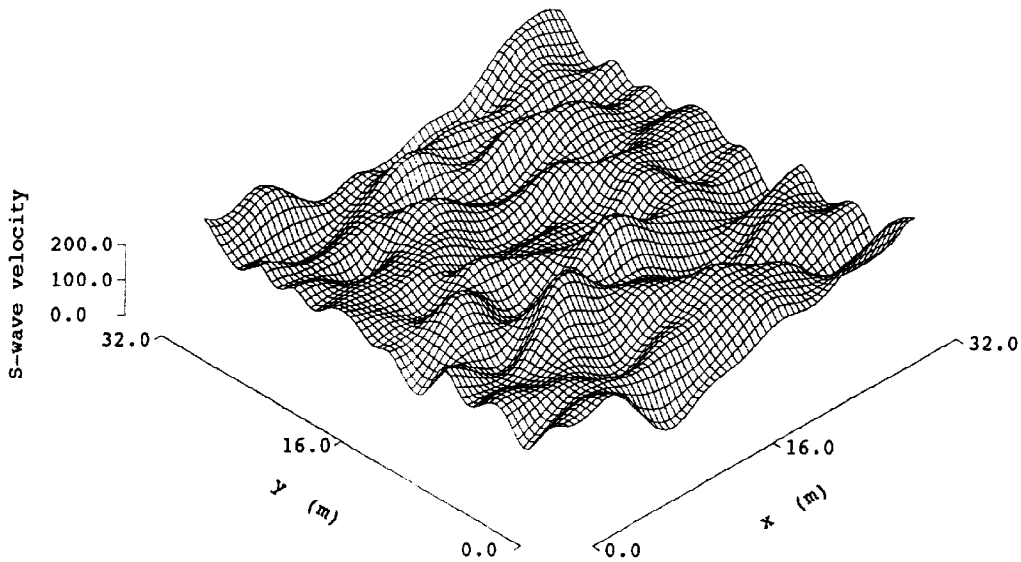
Influence of the Anisotropy of Fluctuation Velocity on Scattering Attenuation

Two types of combinations of anisotropic correlation length are considered. One is $a_x=4m$ and $a_y=2m$, and the other is $a_x=3m$ and $a_y=2m$. Also, a two-case simulation of random media with isotropic correlation lengths, was carried out as part of the comparative study. Figure 2 shows a sample anisotropic velocity distribution and its auto-correlation coefficient with correlation length $a_x=4m$ and $a_y=2m$. The peak frequency of the incident wave varied from 3Hz to 32Hz (when the peak frequency of scattering attenuation was included). Figure 3 shows the decay of the amplitude ratio of direct S-wave against the propagation distance for the random velocity model of Figure 2 with a 12Hz incident wave. In this figure, solid circles denote the mean value of the peak amplitude ratio and the line indicates the least square fit of the amplitude decay. The line can be used to determine Q_s . Error bars represent one standard deviation of the peak amplitude ratio.

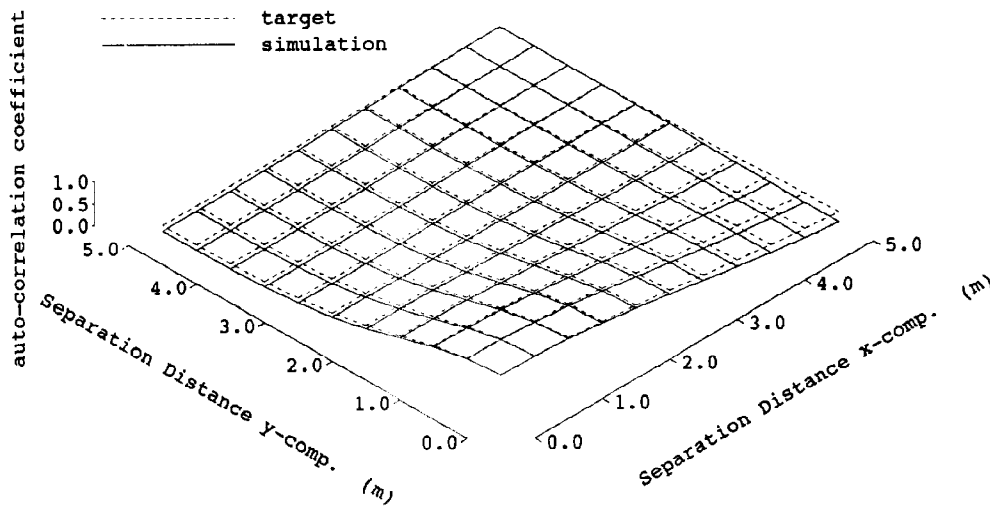
Figure 4 compares the scattering attenuation of anisotropic and isotropic velocity fluctuation media. In this figure, solid and open symbols denote the scattering attenuation for anisotropic and isotropic velocity fluctuation media, respectively. A comparison between the open circle, which shows an isotropic medium ($a_x=2m$ and $a_y=2m$), and the solid triangle, which denotes an anisotropic one ($a_x=3m$ and $a_y=2m$), shows that the peak frequency of the anisotropic medium is lower than the isotropic medium. The peak frequency of the anisotropic medium ($a_x=4m$ and $a_y=2m$), denoted by a solid square, shifts towards a lower frequency. The linear relation between the horizontal correlation length and the peak frequency of the attenuation curve can also be seen. Viewed in this way, the horizontal correlation length of a random medium is very sensitive to scattering attenuation characteristics. Next, the significance of the vertical correlation length was investigated. A comparison between the open triangle denoting an isotropic medium ($a_x=4m$ and $a_y=4m$) and the solid square indicating anisotropic medium ($a_x=4m$ and $a_y=2m$), shows that the peak frequencies of the attenuation curves agree with each other. However, the magnitude of the attenuation of anisotropic medium at peak frequency is about 20 percent lower than that of the isotropic medium. Thus, the vertical correlation length of the inhomogeneous field is not as sensitive to the scattering attenuation as the horizontal correlation length. Furthermore, the attenuation of an anisotropic medium with $a_x=3m$ and $a_y=2m$ at a peak denoted by a solid triangle agrees with the attenuation of an anisotropic medium with $a_x=4m$ and $a_y=2m$, represented by the solid square.

The above investigation indicates that the peak frequency of scattering attenuation is determined by the horizon-

tal correlation length of velocity fluctuation. The attenuation of an anisotropic medium at the peak frequency is smaller than that of an isotropic medium. However, the vertical correlation length of an inhomogeneous medium is not very sensitive to the magnitude of scattering attenuation. It can be concluded, therefore, that one-dimensional analysis, which considers fluctuation in the vertical direction only, may not be sufficient to evaluate the frequency-dependency of scattering attenuation.



(1) Velocity distribution



(2) Auto-correlation coefficient

Figure 2. Sample anisotropic random medium (correlation length $a_x=4m$ and $a_y=2m$)

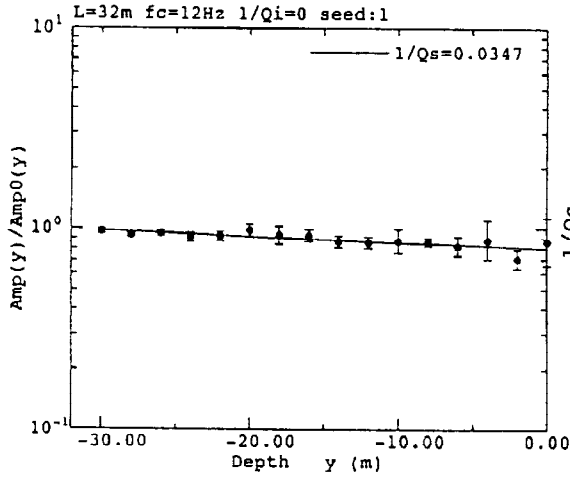


Figure 3. Amplitude ratio decay against propagation distance

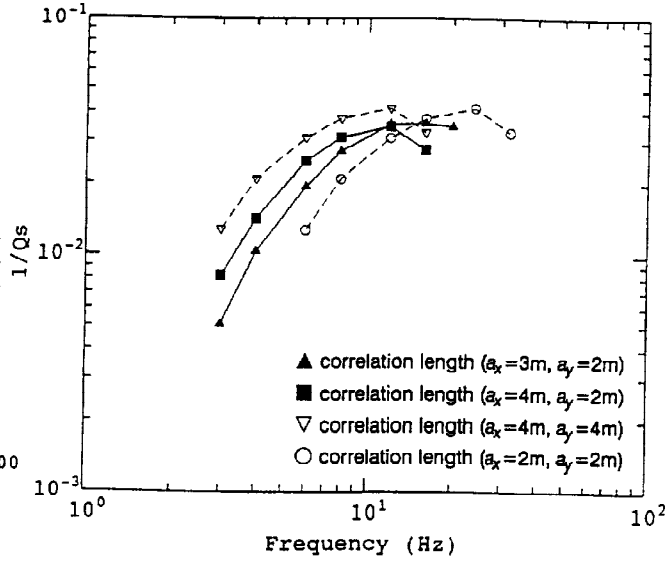


Figure 4. Scattering attenuation of an anisotropic medium

ATTENUATION CHARACTERISTICS OF ACTUAL SOIL DEPOSITS

Relationship between the Fluctuation of S-wave Velocity and Scattering Attenuation

Scattering attenuation caused by S-wave velocity fluctuation is expressed by the single scattering theory (Aki and Richards, 1980). Frankel and Clayton (1986) derived the following equation, which expresses the scattering attenuation of a two-dimensional inhomogeneous field, from the single scattering theory.

$$Q^{-1}(k) = 2k^2 \left(\frac{\delta c}{c} \right)^2 \int_{\theta_{min}}^{\pi} P[2k \sin(\theta/2)] d\theta \quad (3)$$

where, k = wave number, $\delta c/c$ = coefficient of variation of fluctuation velocity, $P[\cdot]$ = two-dimensional Fourier transform of auto-correlation coefficient, θ_{min} and π represent the range of scattering angles.

In the identification of the frequency-dependent attenuation of soil deposits from the vertical array observation data, the following type function is considered.

$$Q^{-1}(f) = \alpha \cdot f^{-\beta} \quad (4)$$

Here, f = frequency, and α and β are identified parameters. From the comparison between Eq.(3) and Eq.(4), statistical properties of velocity structure at each objective point can be estimated. Eq.(3) denotes the theoretical equation for an isotropic inhomogeneous field. However, the peak frequency of scattering attenuation is determined from the correlation length in the horizontal direction (based on the aforementioned argument). Consequently, the correlation length in the horizontal direction is estimated from a comparison of peak frequencies

only.

Comparison between the Identified Attenuation of Actual Soil Deposits and Theoretical Attenuation

In this study, the objective points are located on the coast of Tokyo Bay. At all of the objective points, vertical array systems are being installed in the ground and many earthquake records are observed. In the identification(Annaka *et al.*,1994) of attenuation model, the average Fourier spectral ratio of weak motion from 10 gal to 30 gal was used, as their strain can be assumed to be in the elastic range. Figure 5 compares the identified results and the theoretical attenuation curve. In this figure, each theoretical curve is calculated using the optimum auto-correlation function that is shown out of each frame. Each auto-correlation function is determined based on level of the agreement with the identified attenuation. From this fitting, the auto-correlation function type of velocity fluctuation in the horizontal direction can be estimated.

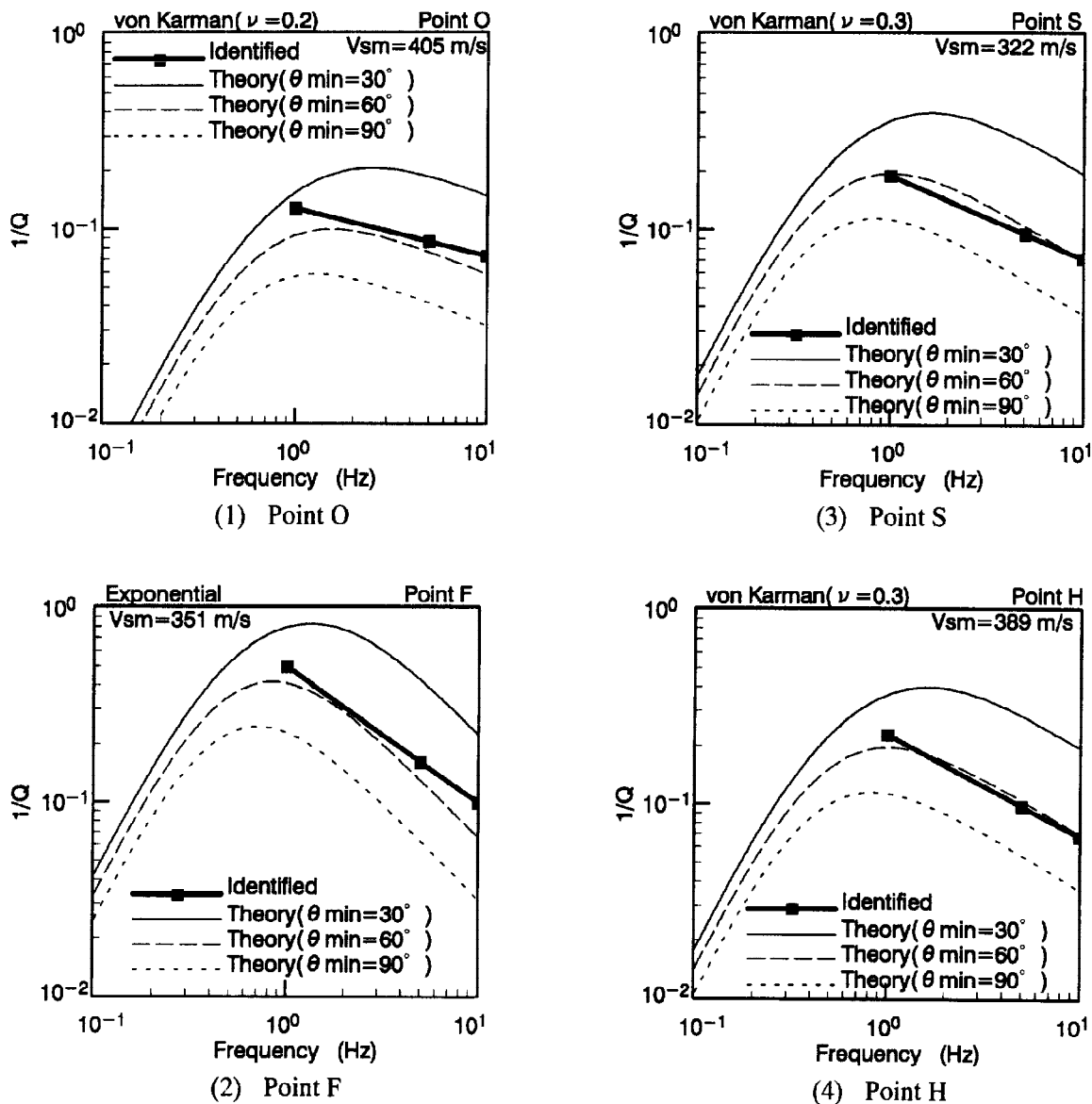


Figure 5. Comparison between identified attenuation and single scattering theory

Estimation of the Statistical Properties of Velocity Structure

Figure 6 shows the attenuation curve based on the single scattering theory as a function of normalized frequency ka (wave number \times correlation length) in a von Karman type ($\nu = 0.2$) random field. The scattering attenuation reaches a peak between $ka = 1.2$ and $ka = 2.5$. In general, the frequency-dependent attenuation of seismic wave obtained from observation data peaks at about 1 Hz. From this disposition and analyzed results, the correlation length of point O in the horizontal direction can be estimated between 77 m and 161 m. The statistical properties of velocity structure for all points are listed in Table 2, which shows that each correlation property differs clearly. The correlation length in the horizontal direction varies between about 40 m and 160 m.

Table 2. Statistical properties of velocity structure in the horizontal direction

Point	Function Type of Auto-correlation func.	Correlation Length in Horizontal Direc. (m)
O	von Karman ($\nu = 0.2$)	77~161
F	Exponential	39~84
S	von Karman ($\nu = 0.3$)	51~102
H	von Karman ($\nu = 0.3$)	62~124

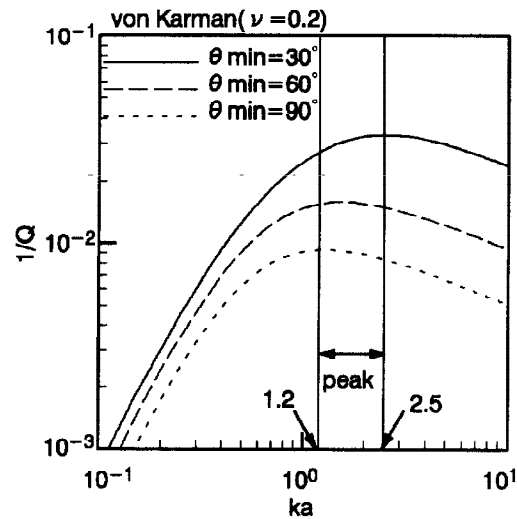


Figure 6. Peak frequency of $1/Q_s$ in von Karman type ($\nu = 0.2$) field

CONCLUSIONS

This study has investigated the relationship between the scattering attenuation of seismic wave and the properties of inhomogeneous media. The conclusions obtained in this study are as follows:

- (1) The horizontal correlation length of a random medium is highly sensitive to scattering attenuation characteristics. The peak frequency of scattering attenuation can be determined by the horizontal correlation length of velocity fluctuation.
- (2) The statistical properties of the fluctuation of velocity structure can be estimated by comparing the identified frequency-dependent attenuation with the results determined by the single scattering theory for four points located on the coast of Tokyo Bay.
- (3) The estimated correlation length in the horizontal direction varies between about 40 m and 160 m, which seems to be valid based on previous study investigated on sedimentary ground.

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