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EFFECT OF SITE RESPONSE ON SPATIAL VARIABILITY OF GROUND MOTION

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ABSTRACT

The validity of a recently developed mathematical model for the site-response component of ground motion spatial variability is examined. Use is made of earthquake recordings in two downhole arrays. A procedure is described to compute an equivalent frequency response function to characterize the dynamic properties of the local soil. These functions for each pair of stations completely define the site-response component of the coherency function that characterizes the spatial variability. Predictions with the theoretical model are in close agreement with "exact" results derived from the recorded data, thus validating the model. Results also show that the site-response effect on the coherency of ground motion is most significant at resonant frequencies of each site.

KEYWORDS

Coherency function; frequency response function; ground motions; impulse response function; phase angle; power spectral density; site response; spatial variability.

INTRODUCTION

Spatial variation in earthquake ground motions arises from four sources: (1) loss of coherency of seismic waves due to scattering in the heterogeneous medium of the ground and due to differential superpositioning of the waves arriving from an extended source, collectively denoted as the "incoherence effect"; (2) difference in the arrival times of waves at different stations, commonly known as the "wave-passage effect"; (3) gradual decay of wave amplitudes with distance due to geometric spreading and energy dissipation in the ground medium, denoted the "attenuation effect"; and (4) spatially varying local soil profiles and the manner in which they influence the amplitude and frequency content of the bedrock motion underneath each station as it propagates upward, denoted herein as the "site-response effect". These effects are well characterized by the so called coherency function, which is the normalized cross-power spectral density of the motions at two stations. Use of this function in response spectrum analysis of multiply-supported structures, such as bridges, is described by Der Kiureghian and Neuenhofer (1994).

Past studies have shed considerable light on the effects of incoherence, wave passage and attenuation (e.g., Abrahamson *et al.*, 1991). However, the site response effect has only recently received attention (Somerville, *et al.*, 1988, Schneider *et al.*, 1992, Zerva and Harada, 1994, Der Kiureghian, 1994). In a recent paper(Der Kiureghian 1996), the first author has proposed a new, composite model for the coherency function that incorporates all the four effects mentioned above. This paper aims at validating the site-response component of this model. Use is made of two sets of recorded ground motions from downhole arrays and direct comparisons are made between predictions by the model and by the recorded data. The numerical comparison shows remarkable agreement between theory and observation. The paper also describes a method for characterizing the required parameters and functions for the site-response component of the coherency function. Additionally, the numerical results clearly demonstrate the importance of this component of the ground motion variability.

THE COHERENCY MODEL

The coherency model derived by Der Kiureghian (1996) is based on elementary concepts of random process theory and employs simplifying assumptions for propagation of waves in the ground medium (i.e., plane wave assumption) and through the local soil column (i.e., vertically propagating shear waves, linear stationary response). Neglecting the attenuation effect that is shown to be insignificant, the model has the form

$$\gamma(\omega) = |\gamma(\omega)| \exp\left\{ \mathrm{i}[\theta_{\mathrm{wp}}(\omega) + \theta(\omega)_{\mathrm{sr}}] \right\} \tag{1}$$

where ω denotes the circular frequency, $|\gamma(\omega)|$ denotes the modulus of the coherency function representing the incoherence effect, $\mathrm{i} = \sqrt{-1}$, and $\theta_{\mathrm{wp}}(\omega)$ and $\theta_{\mathrm{sr}}(\omega)$ are phase angles that represent the wave passage and site response effects, respectively, and are given as follows:

$$\theta_{\mathrm{wp}}(\omega) = \frac{\omega d_{kl}^{L}}{v_{\mathrm{app}}(\omega)} \tag{2}$$

$$\theta_{\mathrm{sr}}(\omega) = \tan^{-1} \frac{\mathrm{Im}[H_k(\omega) H_l(-\omega)]}{\mathrm{Re}[H_k(\omega) H_l(-\omega)]} \tag{3}$$

In the above expressions, d_{kl}^{L} denotes the distance between a pair of stations k and l measured in the longitudinal direction of wave propagation, $v_{\mathrm{app}}(\omega)$ is the "apparent" wave velocity (expressed as a function of ω in account of dispersion effects), and $H_k(\omega)$ and $H_l(\omega)$ denote frequency response functions of the soil columns at the two stations as defined in the following section. It is noted that the wave passage effect is completely defined by the apparent wave velocity and the geometry of the stations relative to the earthquake source, whereas the site response effect is completely defined in terms of the characteristics of the local soil at each station. The incoherence component usually must be determined by statistical fitting to observed data because of the complex and random nature of the underlying phenomena.

The remainder of this paper describes a method for evaluation of the frequency response function for a soil site with an approximate account of the nonlinearity in the soil behavior, and verifies the site-response component of the coherency model in (3) by comparison with the phase angle generated from recorded data.

DETERMINATION OF THE FREQUENCY RESPONSE FUNCTION

The site-response component of the coherency model in (3) is derived by considering the soil columns at stations k and l as two linear systems. The derivation makes use of elementary concepts of stationary random vibration theory involving the frequency response function $H_k(\omega)$ at each station. By definition, this function is the steady-state response of a linear system to a complex harmonic excitation of the form $\exp(\mathrm{i}\omega t)$. Of course soil behavior under strong earthquake motions is not linear. Hence, we need to define this function for some sort of an "equivalent" linear soil system. Three ideas for this purpose are described below.

If one is fortunate enough to have recorded earthquake ground motions at the base rock and surface levels at the site of interest, then, based on stationary random vibration theory, the frequency response function for an equivalent linear soil system can be computed from

$$H_k(\omega) = \frac{\Phi_{\text{base-surface}}(\omega)}{\Phi_{\text{base}}(\omega)} \tag{4}$$

where $\Phi_{\text{base}}(\omega)$ is the power spectral density of the base motion and $\Phi_{\text{base-surface}}(\omega)$ is the cross-power spectral density of the base and surface motions at station k. In practice it is rare that one has such recordings for a selected site. Nevertheless, this approach is valuable as a means to investigate the validity of the analytical methods that are described below.

A simple approach would be to consider the soil column at the site as a single-degree-of-freedom system with viscous damping. The frequency response function then has the well known form

$$H_k(\omega) = \frac{\omega_k^2 + 2\mathrm{i}\zeta_k\,\omega_k\omega}{\omega_k^2 - \omega^2 + 2\mathrm{i}\zeta_k\omega_k\omega} \tag{5}$$

where ω_k and ς_k respectively denote the natural frequency and viscous damping ratio of the equivalent soil system. This, however, is a very crude model for several reasons, including the fact that soil damping is not viscous and deep soil columns usually possess several significant modes of vibration. Additionally, it is not easy to determine the parameters ω_k and ς_k on a rational basis. Nevertheless, this model is a simple alternative when other options are not practical. It is noted that this model is consistent with the well known Kanai-Tajimi model for ground motions on soil sites (Clough and Penzien, 1993). Typical ranges of the parameter values for that model are $\omega_k = 2\pi$ - 5π rad/s and ς_k = 0.2 - 0.6.

For a better representation of the frequency response function, one must model the site itself, not the frequency response function directly. Having such a model, one can use existing time-domain, soil response analysis methods, e.g., the program SHAKE (Idriss *et al.*, 1991), to compute the frequency response function. The analysis would require computing the response of the soil column to a complex harmonic excitation for a long period in order to achieve the steady-state response, as required by the definition of that function. Naturally, such an approach can account for the non-viscous and nonlinear behavior of soils, albeit in an approximate "equivalent" linear sense. This analysis must be repeated for each frequency ω to obtain a complete description of $H_k(\omega)$. A more efficient approach is described below.

It is well known that for a linear system, the frequency response function is the Fourier transform of the unit-impulse response function, denoted $h(t)$. Thus, the frequency response function can be computed from

$$H(\omega) = \int_0^\infty h(t)\exp(-\mathrm{i}\omega t)\,\mathrm{d}t \tag{6}$$

By definition, $h(t)$ is the response of the system to an impulsive load of unit magnitude applied at time $t = 0$. For a given soil model, this can be easily computed by modeling the input excitation as a short duration pulse and scaling the response by the magnitude of the pulse. The advantage here, relative to the method described in the previous paragraph, is that $h(t)$ is computed by a single time-history analysis of the soil model. Furthermore, as $h(t)$ typically decays rapidly with time, it is not necessary to compute the soil response for too long a duration. It is important, however, to properly select the impulse magnitude so as to adequately account for the nonlinearity in the soil response.

The nonlinearity in the soil response depends primarily on the properties of the soil and the intensity of the ground motion, and secondarily on the duration and frequency content of the motion. For a given soil, the nonlinearity inherent in the computed $h(t)$ and, therefore, $H(\omega)$ depends on the magnitude of the impulse used. Naturally, it is desirable to select the impulse magnitude such that the nonlinearity inherent in the frequency response function is equivalent to the nonlinearity present in the soil response to a design earthquake.

To determine the required impulse magnitude, a parametric study with three soil models representing "stiff", "medium" and "soft" sites, shown in Fig. 1, and a large number of earthquake motions recorded on rock is performed. The selected ground motion records represent a wide variety in terms of their intensity, frequency content, and duration. For each site, the surface motion is computed with each of the rock motions as input by use of the SHAKE program and (4) is used to compute the "exact" frequency response function.

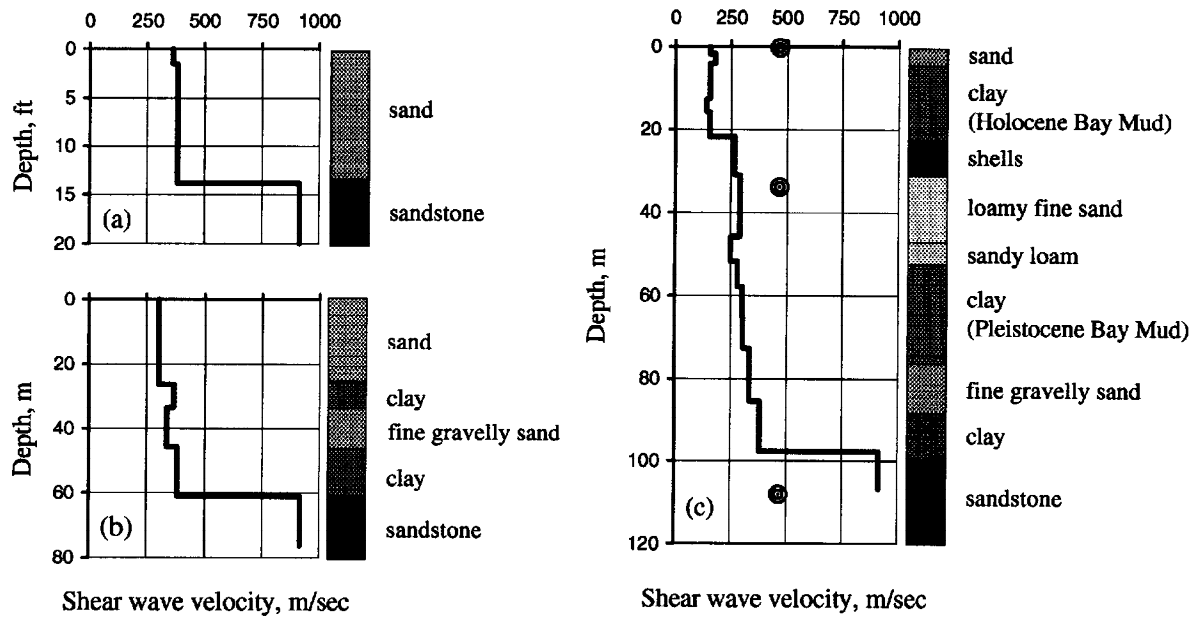

Fig. 1. Soil models for (a) stiff site, (b) medium site, and (c) soft site, Treasure Island

Then (6) is used to compute the theoretical frequency response function with varying impulse magnitude, and the "required" magnitude is determined by minimizing the difference between the two frequency response functions. Figure 2 summarizes the results of the analysis for the three sites where the required impulse magnitude is plotted versus the intensity of the input motion as described by the peak ground acceleration computed by SHAKE on the surface. No appreciable difference between the required impulse magnitudes for the three sites is observed. A least-square fit to the combined data gives the following relation between the required impulse magnitude, I, and the peak ground acceleration, A:

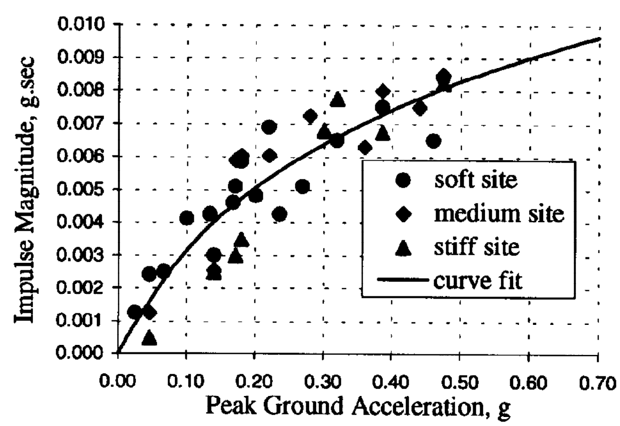

Fig. 2. Required impulse magnitude for equivalent linear system

$$I = -0.00213 + 0.01835A^{0.5} - 0.00512A \quad (7)$$

The procedure described above is clearly an approximate one. Hence, there is need to provide a verification of its accuracy. Given the complexity of the problem and lack of exact analytical solutions, comparison with recorded data is the obvious choice. However, it is necessary to select records that isolate the site-response effect from the effects of ground motion incoherence, wave passage, and attenuation. Downhole array recordings achieve this objective. The following section describes the selected records used in the subsequent analysis.

SELECTED DOWNHOLE ARRAY RECORDINGS

For experimental verification of the models in (3) and (6), two sets of downhole array recordings are used. The first set was recorded at the Treasure Island Geotechnical Downhole Array Station during the Gilroy, California, Earthquake of January 16, 1993 (Darragh *et al.*, 1993). The magnitude of the earthquake was 5.3 and the peak acceleration recorded on the ground surface and at 104m below were 0.0142g and 0.0032g, respectively. This motion obviously is not sufficiently strong to cause a significant amount of nonlinearity in the soil response. On the other hand, a good description of the site geology is available, thus allowing a refined modeling of the site. Figures 1c, the "soft" site model, describes the shear wave velocity and soil types with depth for this site. Figure 3 shows the first 20 seconds of the recorded acceleration time histories at three elevations. Strong amplification of the motion as it propagates upward is observed. For the purpose of this analysis, it is assumed that the bottom of the downhole represents the base rock. The second set of recordings was taken at Chiba Experimental Station during Tokyo area earthquake of November 6, 1985 (Katayama *et al.*, 1990). The magnitude of the earthquake was 5.0 and peak ground accelerations recorded on the surface and at 40m below the surface were 0.077g and 0.0135g, respectively, which again shows a very strong amplification. The recorded acceleration time histories at three elevations are shown in Fig. 4.

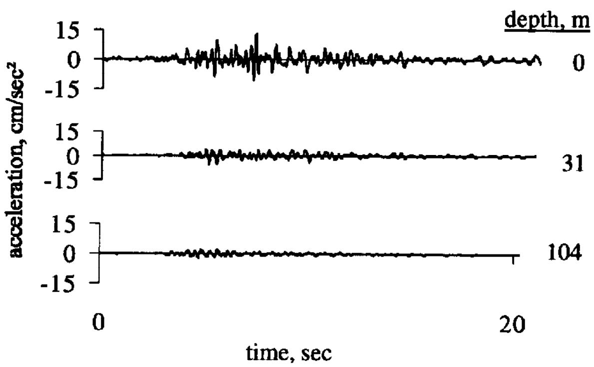

Fig. 3. Recorded accelerations at Treasure Island downhole array for Gilroy earthquake of January 16, 1993

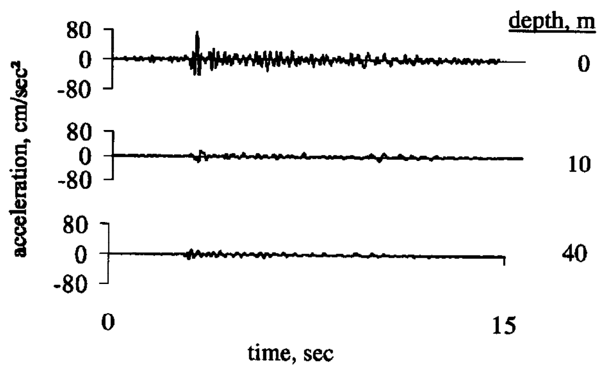

Fig. 4. Recorded accelerations at Chiba downhole array for Tokyo area earthquake of November 6, 1985

Unfortunately geologic information available for this site only allowed a crude modeling of the soil, which is shown in Fig. 5.

It is appropriate at this point to stress the need for downhole recordings at soil sites during strong earthquakes, especially near the source. The significant amplifications observed in the above two sets of recordings highlight the importance of the soil response effect and its influence on the spatial variability of ground motions in regions with rapidly varying local geology.

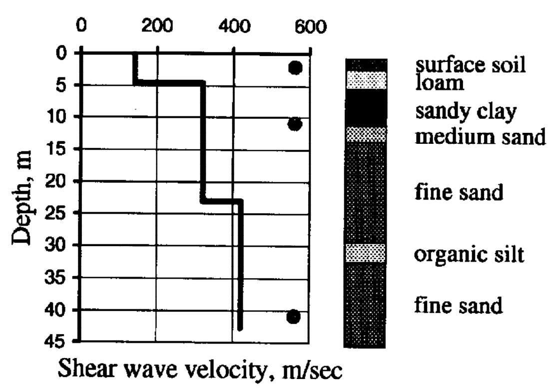

Fig. 5. Soil model for Chiba site

MODEL VERIFICATION

To verify the procedure for determining the site frequency response function, each of the soil models in Figs. 1c and 4 are subjected to a short-duration triangular acceleration impulse at the base level. The required impulse magnitudes (area of the triangle) is obtained from (7) resulting in $I = 0.049\,\text{cm/s}$ for the Treasure Island records and $I = 0.0686\,\text{cm/s}$ for the Chiba records. The program SHAKE (Idriss *et al.*, 1991) is used to compute the soil unit impulse response function $h(t)$ at each site. The Fourier transforms of these functions represent the theoretical predictions of the frequency response function for each site. The moduli of these functions are shown as dashed lines in Figs. 6ab and 7a. The "exact" moduli of the frequency response functions, shown as solid lines in Figs. 6ab and 7a, are computed by use of (4) and the recordings at each site.

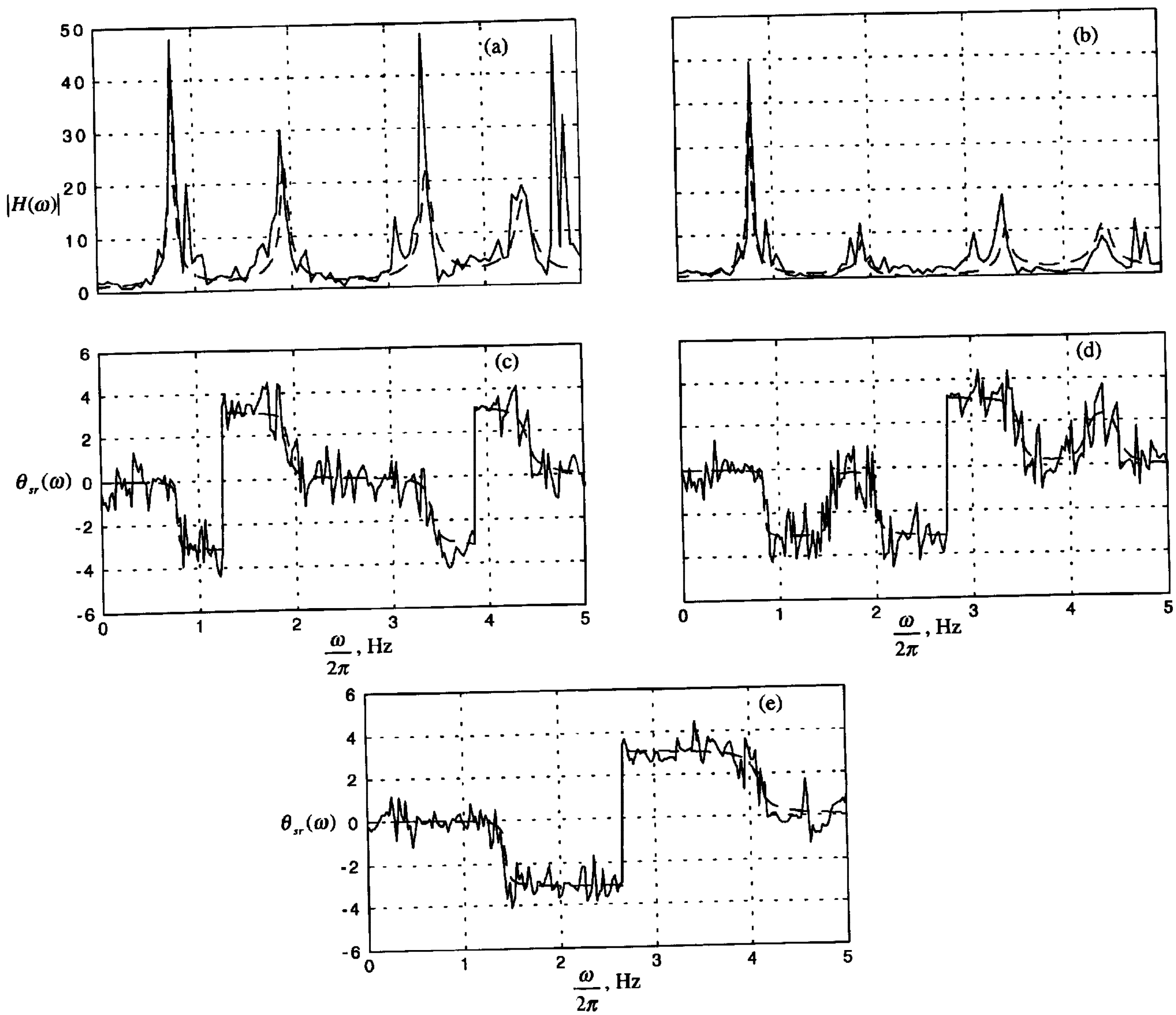

Fig. 6. Comparison of model predictions (dashed lines) with "exact" results (solid lines) for Treasure Island site: (a) frequency response function at surface, (b) frequency response function at depth 31m, (c) phase angle between motions at 0m and depth 104m, (d) phase angle between motions at depths 31m and 104m, (e) phase angle between motions at 0m and depth 31m.

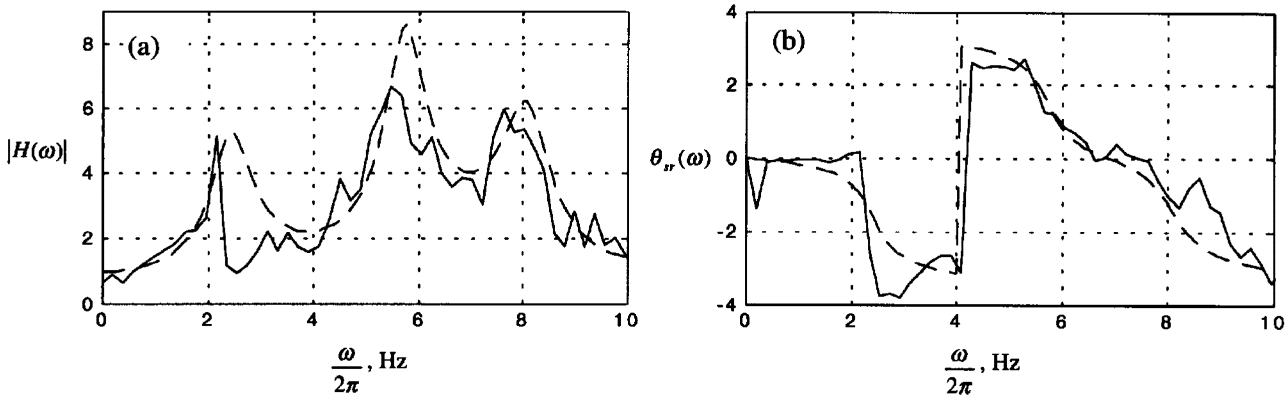

Fig. 7. Comparison of model predictions (dashed lines) with "exact" results (solid lines) for Chiba site: (a) frequency response function at surface, (b) phase angle between motions at 0m and 40m.

For both sites the agreement between the theoretical predictions and "exact" estimates of the frequency response function are remarkable. This is partly due to the low intensity of the motion and hence linearity of the soil response. It is interesting to observe that the two sites have multiple resonant frequencies. Obviously, if a single-degree-of-freedom soil model as in (5) is used, at best one can model one resonant frequency. However, one should not discard this simple model too soon, since there are many structures, e.g., long-span bridges, for which the high-frequency components of ground motion are not significant and even this simple model may be adequate if it captures the first resonant frequency well.

We now turn to the verification of the model in (3) for the coherency phase angle due to the site-response effect. An "exact" estimate of the phase angle is computed for each site by using the formula

$$\theta_{\text{exact}}(\omega) = \tan^{-1}\left(\frac{\operatorname{Im}\Phi_{\text{base-surface}}(\omega)}{\operatorname{Re}\Phi_{\text{base-surface}}(\omega)}\right) \tag{8}$$

where $\Phi_{\text{base-surface}}(\omega)$ is the cross-power spectral density of the recorded motions at the base and surface levels. The predictions of the phase angle based on the proposed model are computed by use of (3) and the frequency response function obtained by the procedure described earlier (i.e., the dashed curves in Figs. 6ab and 7a. The results are shown in Figs. 6c-e and 7b. The agreement between the "exact" and theoretical values is remarkable for both sites. It is worth noting that sharp changes of the phase angle occur at points of resonant frequency for each site, the sharpness being dependent on the bandwidth of the frequency response function. It follows that in predicting the site-response effect, it is essential to accurately predict the resonant frequencies for the site of interest. It is also evident that the site response effect will be most significant for multiply supported structures that have supports situated on sites with different resonant frequencies.

SUMMARY AND CONCLUSIONS

Downhole array recordings are used to verify a theoretical model for the site-response component of the ground motion spatial variability. A procedure is developed and verified for computing the frequency response function of a soil site that is required for the model. Predictions by the theoretical model are in close agreement with "exact" results obtained from the recorded motions at two downhole arrays.

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