

REQUIRED YIELD STRENGTH RATIO SPECTRUM FOR EVALUATION OF SEISMIC DAMAGE AND SAFETY OF SDOF STRUCTURES

K. HIRAO¹, S. SASADA² and Y. NARIYUKI¹

¹Department of Civil Engineering, The University of Tokushima,
2-Chome, Minamijosanjima-cho, Tokushima, Japan.

²Department of Construction Systems Engineering, Anan College of Technology,
265 Aoki, Minobayashi-cho, Anan, Japan.

ABSTRACT

The object of this study is to develop an evaluation method for seismic safety and damage of SDOF structures, based on the response spectra/ or its regression curves for required yield strength ratio R_r and damage parameter γ . In this study, the ratio R_r for a structure subjected to severe earthquakes is defined as the value of yield strength ratio R by which the value of a damage index D becomes a prescribed one D_r , which corresponds to a limit state of the damaged structure. Also the damage parameter γ proposed by Fajfar is introduced. Providing that the ductility capacity μ_{dr} and D_r value are known, the R_r and γ spectra and their regression curves for the SDOF structures with Q-hyst and Bilinear restoring force models are obtained by use of the Park's and Krawinkler's damage indexes D and three artificial earthquake groups, *i.e.* GROUP(H), (M) and (L), which are varying in their A/V (A = maximum acceleration, V = maximum velocity) values. In this paper, therefore, some of the R_r and γ spectra are shown discussing the effect of difference in restoring force models, damage indexes and earthquake groups on those spectra, and the method to evaluate the seismic safety and damage of the structures is also demonstrated, by the use of their regression curves.

KEYWORDS

required yield strength ratio; damage parameter; seismic safety and damage; SDOF structures.

INTRODUCTION

In order to establish a rational dual (serviceability and ultimate safety) level seismic design procedure, it is necessary to evaluate the quantification of damage and develop the procedure of seismic safety verification in a structure subjected to severe earthquake motions. Therefore, extensive research on the damage of a member and structure under cyclic loading has been carried out, and a large number of damage indexes (*e.g.* Park's model (Park *et al.*, 1985) and Krawinkler's model (Krawinkler *et al.*, 1983) *etc.*) have been proposed. However these damage indexes are too complex to incorporate directly into the design process, because the indexes are composed of inelastic seismic demand parameters, such as maximum displacement, cumulative hysteretic energy and distribution of plastic cycles *etc.*, whose values should be obtained from complicated inelastic response analyses and/or experiments. So it is suggested that these indexes should be use with statistical information on seismic demand parameters and analytical (or experimental) data on structural performance parameters to transform the structural damage into structural parameters.

Krawinkler *et al.* (1992) and Vidic *et al.* (1992) examined the reduced ductility capacity μ_{dr} weighted with respect to anticipated cumulative damage demands, and developed inelastic design spectra of the reduction factor R' for the structures with given ductility capacity μ_{dr} . Then, they proposed a design procedure by using these reduced ductility μ_{dr} and R' - μ_{dr} relationship, although the procedure is a little bit circuitous. Moreover

Reinhorn *et al.* (1992) examined the relation between strength reduction factor R' and ductility capacity μ_d , correlated to various degrees of damage, and suggested the principle to determine a suitable design procedure to control the damage, by the use of this relation. However, since the development of such a seismic design procedure is in the early stages, none of the proposed design procedures for damage control has been universally accepted.

In this study, therefore, the required yield strength ratio R_r to be used to develop a seismic safety verification (design procedure for damage control) for SDOF structures is proposed. Then, introducing the damage parameter γ defined by Fajfar, an evaluation method for seismic damage (such as displacement and energy ductility μ_d and μ_h) is also investigated.

EARTHQUAKE MOTIONS AND STRUCTURAL PARAMETERS

Input Earthquake Motions

30 artificial earthquake motions, aiming for the mean acceleration response spectra of Japanese road bridge design specification V for seismic design, are simulated. In order to know the representative parameter for the earthquakes which affects the structural damage, the relations between the value of A/V (A = maximum acceleration, V = maximum velocity) and of both mean period t_m and total power P_t for each earthquake are shown in Fig. 1. As is generally known, here, these parameters t_m and P_t of input earthquakes are strongly related to the structural damage. It can be noticed from Fig. 1 that there are good correlations between the A/V and both the t_m and P_t , *i.e.*, the value of t_m and P_t for each earthquake decrease as the A/V value increases. In this study, therefore, the A/V value of an earthquake is considered to be a suitable parameter to represent the characteristics of the earthquake affecting the damage of structures. Then the 30 earthquakes are divided into 3 groups as GROUP(H), (M) and (L) of which A/V values are comparatively large, middle and small, respectively. In later numerical examples, the mean spectrum R_m , of each group will be compared in order to discuss the effect of difference in input earthquake motions on the R_r and γ spectrum.

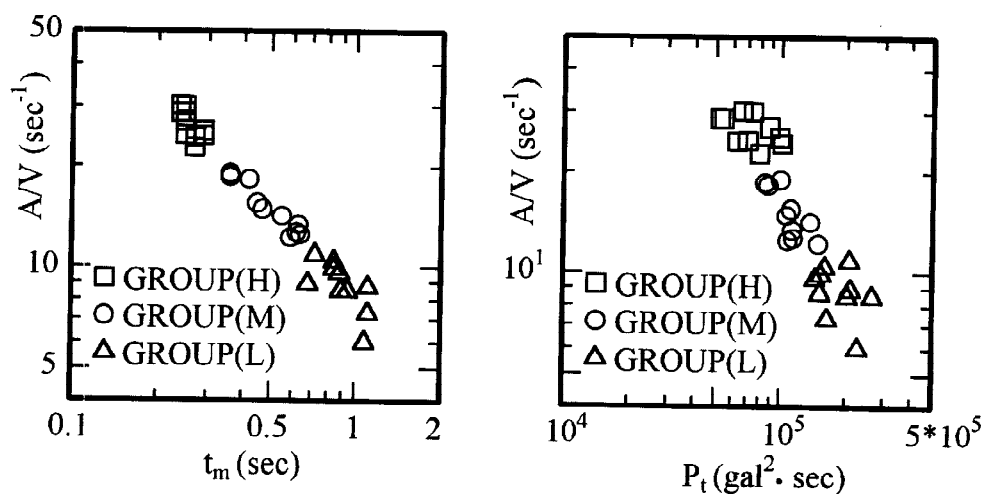


Fig. 1. Relationship between A/V and mean period t_m and total power P_t

Structural parameters

In this study, the SDOF structures with Q-hyst type and Bilinear type of restoring force model, illustrated in Fig. 2, are investigated. The 15 different values of natural period T , after dividing the range $T = 0.1$ to 5.0 sec. into equal parts on a logarithmic axis, are employed in addition to the fixed values of damping factor $h = 5\%$ and plastic stiffness ratio $p = 0.1$ (see Fig. 2) for the SDOF structures.

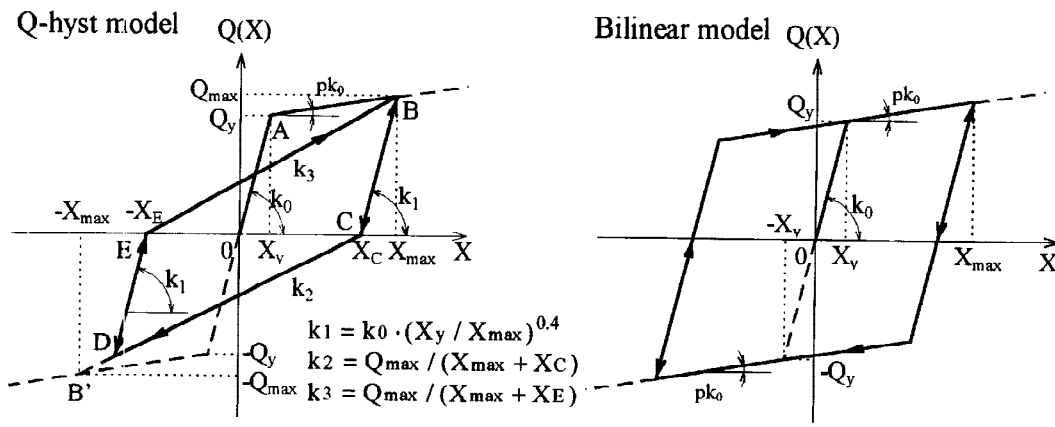


Fig. 2. Restoring force model for the SDOF structures

YIELD STRENGTH RATIO R, DAMAGE INDEX D AND DAMAGE PARAMETER γ

Yield Strength Ratio R

In this study, the yield strength ratio R, which is one of the most important structural performance parameters used to discuss the damage of a structure to severe earthquake motions, is defined as follows :

$$R = Q_y / Q_{ye} = Q_y / (m \cdot S_{ao} \cdot \ddot{X}_{o\max}) \quad (1)$$

where Q_y is yield strength, $Q_{ye} (= (m \cdot S_{ao} \cdot \ddot{X}_{o\max}))$ is elastic maximum restoring force which defines the yield strength required of the SDOF structure in order to respond elastically to a ground motion, S_{ao} is elastic acceleration response factor, and $\ddot{X}_{o\max}$ is maximum acceleration of an input earthquake motion.

Damage Index D

As this is a fundamental study, the following two typical damage index models are employed.

Park's model. The damage index proposed by Park and Ang (1985) can be written in the form:

$$D \cdot \mu_u = (\mu_d + \beta \cdot \mu_h) \quad (2)$$

where D is damage index, β is parameter which depends on the structural characteristics, μ_d , μ_h and μ_u are displacement ductility, cumulative energy ductility and ductility capacity defined as follows:

$$\mu_d = X_{\max} / X_y, \mu_h = E_h / (Q_y \cdot X_y), \mu_u = X_u / X_y \quad (3)$$

where Q_y is yield strength, X_{\max} is maximum displacement, X_y is yield displacement, X_u is ultimate displacement and E_h is total hysteretic energy.

Moreover, in this Park's model, since the exact values of damage index D corresponding to the limit states of structure, such as usable state, repairable state, collapse state and so on, have not been clearly defined (Reinhorn *et al.*, 1992), and for the sake of decreasing analytical parameters, the product of prescribed value D_r and ductility capacity μ_u , $D_u = D_r \cdot \mu_u$, is adopted as the parameter which represents the allowable degree of damage of the structures. Then the 6 values of $D_u = 2.0, 2.5, 3.5, 5.0, 7.5$ and 10.0 employed, together with the 3 values of parameter β in Eq.(2), i.e., $\beta = 0.05$ (suggested value for concrete structures (Reinhorn *et al.*, 1992)), 0.15 (realistic mean value (Vidic *et al.*, 1992)) and 0.25 (rather large value examined in this study).

Krawinkler's model. The damage index proposed by Krawinkler and Zohrei (1983) may be written as follows (Conseza *et al.*, 1992):

$$D = \sum_{i=1}^N \{(\mu_i - 1) / (\mu_u - 1)\}^c \quad (4)$$

where μ_i is the ductility relative to the plastic displacement range of excursion i , c is are structural parameter.

Moreover, in this model, the prescribed value of damage index $D = D_r = 1$ and the 5 values of ductility capacity $\mu_u = 2.0, 3.5, 5.0, 7.5$ and 10.0 are employed, together with 2 values of parameter c in Eq.(4), $c=1.6$ and 1.8 .

Damage Parameter γ

The damage parameter γ proposed by Fajfar (1992) can be written in the form:

$$\gamma = \sqrt{\mu_h} / \mu_d \quad (5)$$

From Eq.(5) and (2) the following relations between μ_d and μ_u can be obtained:

$$\mu_d = (\sqrt{1 + 4 \cdot D \cdot \mu_u \cdot \beta \cdot \gamma^2} - 1) / (2 \cdot \beta \cdot \gamma^2) \quad (6)$$

REQUIRED YIELD STRENGTH RATIO AND SAFETY VERIFICATION

Required Yield Strength Ratio R_r

The required yield strength ratio R_r in this study, is defined as the required value of yield strength ratio R , by which the value of damage index D employed for a structure excited by an intense ground motion will result in a prescribed value D_r , which corresponds to a limit state, *e.g.*, repairable state or collapse state, of the structure. From the above definition it is easy to see that the ratio R_r will be a function of the prescribed damage D_r , ductility capacity μ_u , natural period T , damping factor h and restoring force model for the structure and the characteristics of earthquake motion. In this study the R_r and its mean spectra for a given SDOF structure is obtained by repetition of ordinary inelastic response analysis.

Safety Verification

The required yield strength ratio R_r described above means that the damage of a structure with yield strength ratio $R = R_r$ reaches the targeted level regulated by the value of a damage index $D = D_r$, and it is well known that the damage of a structure having a large R value is lower than that of one with a small R . Therefore, once the R_r spectra for the required values of D_r , μ_u and h in addition to the necessary restoring force models and input earthquake motions are prepared, it is easy to verify the seismic safety of a structure for the limit state prescribed by the value of damage index $D = D_r$. This is done by comparing the value of yield strength ratio R for the structure and its required value R_r obtained from the corresponding R_r spectrum, as shown in Fig. 3.

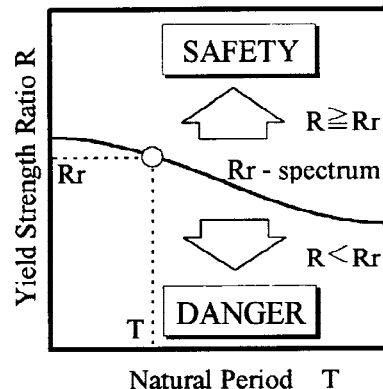


Fig. 3. Safety verification by the use of R_r -spectrum

NUMERICAL EXAMPLES

R_{rm} Spectrum and its regression equation

Influence of earthquake group on R_{rm} Spectrum. Fig. 4 shows the mean spectra, R_{rm}, of the required yield strength ratio R_r for 10 earthquakes in each GROUP(H), GROUP(M) and GROUP(L), comparing the effect of difference in earthquake groups on the R_{rm} spectra for each restoring force model, Q-hyst type and Bilinear type. From this figure, the R_{rm} spectra show a tendency to decrease their values with increasing value of the natural period T in the range of about T ≤ 1.0 sec. And, it can be seen that decreasing value of the A/V makes this decremental tendency larger in the both restoring force models.

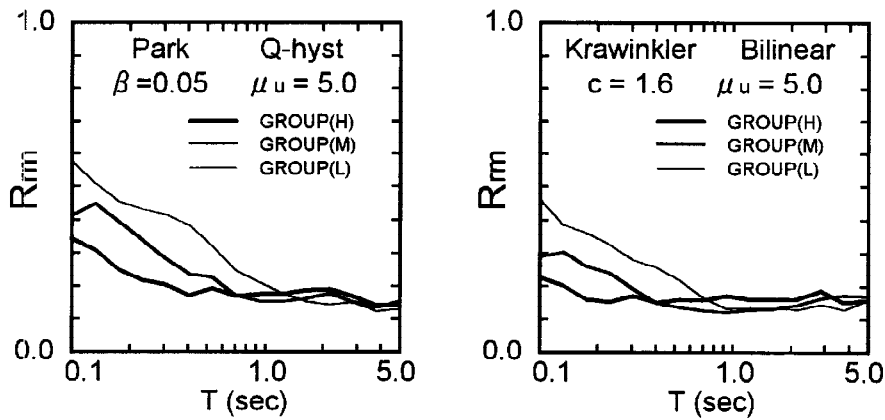


Fig. 4. Influence of earthquake group on R_{rm} spectrum.

Influence of ductility capacity μ_u on R_{rm} Spectrum. Fig. 5 shows the effect of ductility capacity μ_u on R_{rm} spectra. From this figure it can be seen that the R_{rm} value becomes smaller as the ductility capacity μ_u gets larger for whole period range (T = 0.1 to 5.0 sec.), irrespective of the restoring force model and damage index model.

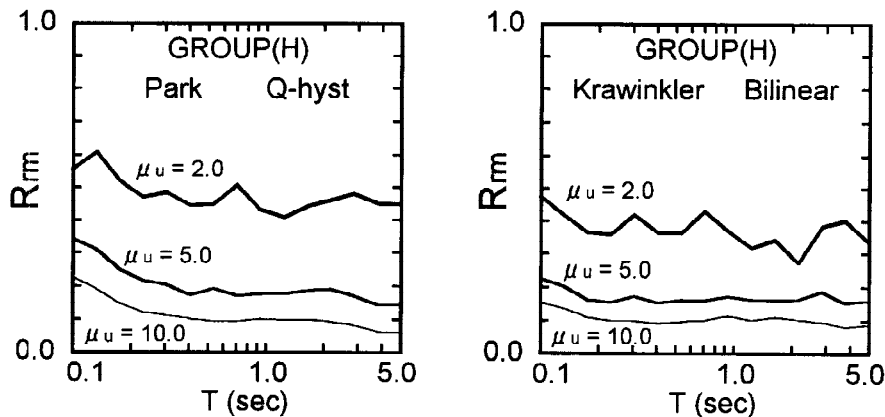


Fig. 5. Influence of ductility capacity μ_u on the R_{rm} spectrum.

Influence of damage index model on R_{rm} Spectrum. Fig. 6 shows the effect of difference in damage index model and restoring force model on the R_{rm} spectra. From the figure, it can be seen that the R_{rm} spectrum depends on the damage index model, i.e., the value of R_{rm} by Park's model is larger than the one by Krawinkler's model in the range of about T < 1.0 sec.. It is also noticed that the value of R_{rm} spectrum is

influenced by the difference in restoring force model, especially for Park's model.

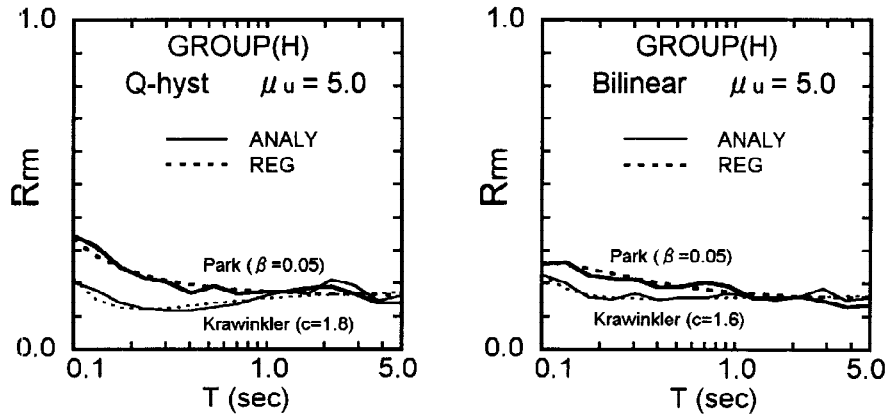


Fig. 6. Influence of damage index model on the R_m spectrum.

R_m Regression Equation. Above results suggested that R_m depend on the natural period T , ductility capacity μ_u , damage index model, restoring force model and A/V value. Thus, the authors (Hirao *et al.* 1995) have tried to derive regression equations of the R_m spectra for three earthquake groups, GROUP(H), GROUP(M) and GROUP(L), for the case of Q-hyst & Park's model and Bilinear & Krawinkler's model. The results are as follows :

$$R_m = C_0 + C_1 \cdot \alpha + (C_2 + C_3 \cdot \alpha) / (D \cdot \mu_u) + \{C_4 + C_5 \cdot \alpha + (C_6 + C_7 \cdot \alpha)\} \cdot (D \cdot \mu_u) / T + \{C_8 + C_9 \cdot \alpha + (C_{10} + C_{11} \cdot \alpha)\} \cdot (D \cdot \mu_u) / T^2 \quad (7)$$

where α is β in Park's model or α is c in Krawinkler's model, T is natural period, and C_1, \dots, C_{11} are regression coefficients shown in Table 1.

In Fig. 6, the regression results of R_m obtained from Eq. (7) shown by broken line together with the analytical ones (solid lines), so as to examine the accuracy of Eq. (7). As it is clear from the figure, the regression curves mach the analytical ones well, although the form of Eq. (7) is a little complicated to be employed for practical use.

Table 1 Coefficients in the regression Eq. (7).

	Q-hyst & Park's model $2.5 \leq D \cdot \mu_u \leq 10$			Bilinear & Krawinkler's model $D=1, 3.5 \leq \mu_u \leq 10$		
	GROUP(H) $A/V \geq 20$	GROUP(M) $A/V=11 \sim 20$	GROUP(L) $A/V \leq 11$	GROUP(H) $A/V \geq 20$	GROUP(M) $A/V=11 \sim 20$	GROUP(L) $A/V \leq 11$
C_0	-0.0206	-0.0591	-0.0630	0.1065	0.0548	0.0563
C_1	0.0965	0.1100	0.1450	-0.0438	-0.0375	-0.0357
C_2	0.8664	0.8727	0.8737	0.6995	0.6749	0.7400
C_3	0.6821	0.8330	0.9342	-0.0328	-0.0407	-0.0389
C_4	0.0076	0.0787	0.1189	-0.0231	-0.0236	-0.0009
C_5	0.1670	0.0820	-0.0150	0.0070	-0.0041	-0.0234
C_6	-0.0005	-0.0040	0.0120	0.0040	0.0043	-0.0021
C_7	-0.0070	0.0055	-0.0043	0.0008	0.0004	0.0024
C_8	0.0008	-0.0040	0.0070	0.0014	0.0027	0.0004
C_9	-0.0130	-0.0050	-0.0020	0.0003	0.0004	0.0025
C_{10}	0.0001	0.0004	0.0004	-0.0003	-0.0005	0.0001
C_{11}	0.0001	-0.0010	-0.0010	-0.0000	-0.0001	-0.0003

γ_m Spectrum and its Regression Equation

Influence of earthquake group, μ_u and β on γ_m Spectrum. Fig. 7 shows the effect of difference in ductility capacity μ_u and parameter β on the mean spectra, γ_m , of damage parameter γ , in the case of Q-hyst model and prescribed value of $D = D_r = 1$ for Park's model. It can be seen that the γ_m spectrum depends on the natural period T , i.e., the value of γ_m decreases with increasing period T . Also it is noticed that the γ_m spectrum depends somewhat on the A/V value, i.e., earthquake GROUP(H), (M) and (L). There is, however, no clear difference among the γ_m spectra for each value of the μ_u and β .

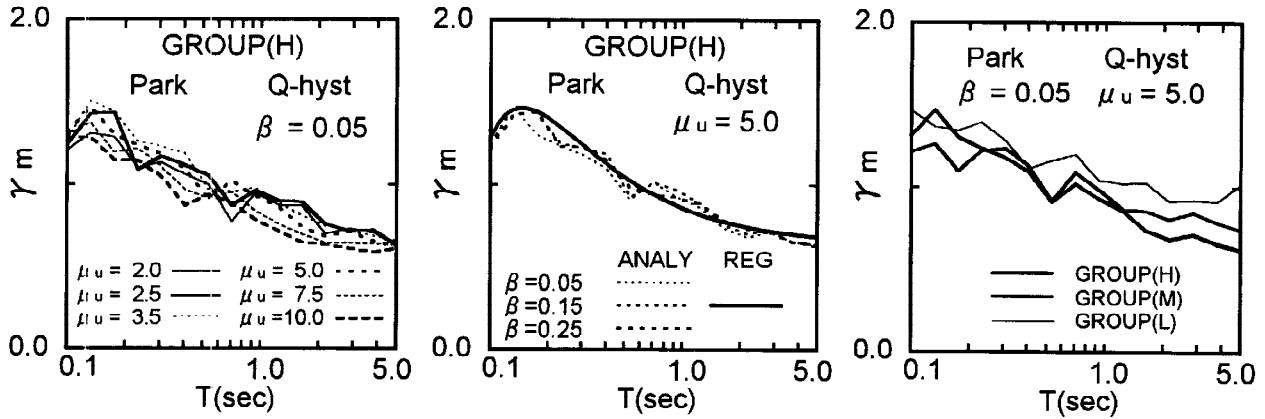


Fig. 7. Influence of earthquake group, μ_u and β on γ_m spectrum.

γ_m Regression Equation. From the above results, it is suggested that γ_m depends mainly on the natural period T and somewhat on the A/V value. Thus the regression equations of γ_m spectrum for each earthquake group (A/V value) are derived as follows:

$$\gamma_m = B_0 + B_1 / T + B_2 / T^2 \quad (8)$$

where, T is natural period, and B_0 , B_1 and B_2 are regression coefficients shown in Table 2.

Moreover the γ_m spectrum obtained from Eq. (8) is illustrated by solid line in the middle figure of Fig. 7.

Table 2. Coefficients in the regression Eq. (8).

	GROUP(H) $A/V \geq 20$	GROUP(M) $A/V = 11 \sim 20$	GROUP(L) $A/V \leq 11$
B_0	0.6377	0.7759	0.9114
B_1	0.2422	0.1648	0.1574
B_2	-0.0177	-0.0124	-0.0102

Damage index = Park's model ($D=1$)
Restoring force model = Q-hyst type

Damage Evaluation

Fajfar (1992) suggested that seismic damage, i.e., displacement ductility μ_d and energy ductility μ_h for given D , μ_u and β values can be obtained from Eqs. (6) and (5), by using the damage parameter γ . So, as for Park's model, the μ_d and μ_h for a structure with yield strength ratio R , μ_u and T are evaluated as follows: (1) Substituting the R , μ_u and T in Eq.(7), then estimate the value of $D = D'$; (2) Substituting the D' , μ_u and β values together with the value of $\gamma = \gamma'$ obtained from Eq. (8), then calculate the value of $\mu_d = \mu_d'$; (3) Then obtain the value of $\mu_h = \mu_h'$ from Eq. (5), by using the μ_d' and γ' . The μ_d and μ_h spectra evaluated by the procedure above are illustrated by solid lines in Fig. 8, comparing the analytical ones by broken lines. From the figure, it can be seen that the evaluated values match the analytical ones well.

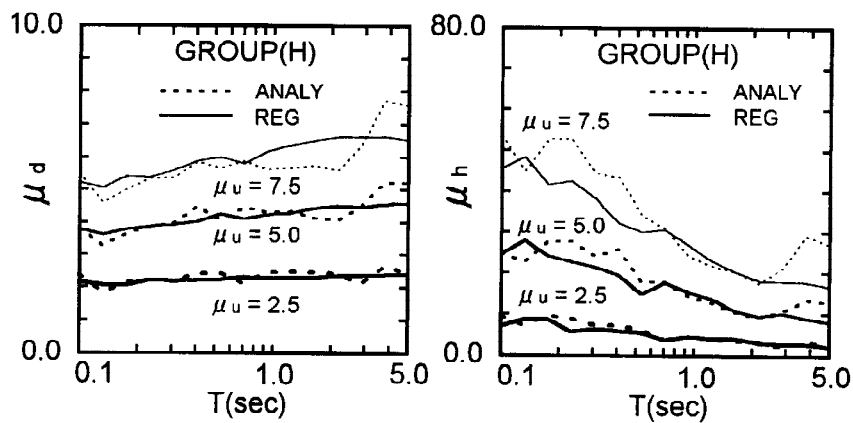


Fig. 8. Comparing the target values and regression values each μ_d and μ_h , where damage index D is Park's model ($D=1$, $\beta=0.05$) and restoring force model is Q-hyst type.

CONCLUSIONS

The main results obtained in this study are summarized as follows:

- (1) The method proposed herein to evaluate the seismic safety and damage for a prescribed limit state of a structure may be valid for future development of a seismic design procedure for damage control, if the required specific R_r and γ spectra or their regression equations are prepared.
- (2) The R_r spectrum depends on the values of prescribed damage $D = D_r$, ductility capacity μ_u , natural period T , restoring force model, damage index model and its parameter (β in Park's model and c in Krawinkler's model) as well as the A/V value for input earthquake motions. So it is necessary that the regression equation of R_r spectrum is derived as a function of the D_r , μ_u , T and damage index parameter (β or c), considering the difference of R_r spectrum in the restoring force model, damage index model and A/V value.
- (3) The γ spectrum depends mainly on the values of natural period T and restoring force model, and somewhat on the A/V value.

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