



## **RISK ANALYSIS FOR EARTHQUAKE - INDUCED PERMANENT DEFORMATION OF EARTH DAMS**

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### **ABSTRACT**

During an earthquake, an earth dam may experience substantial permanent deformation, causing damage or failure of the dam. Despite the significant developments in recent years, the estimation of the likelihood of such seismically-induced deformations, and of the performance of an earth dam, during the period of its functional life still remains a challenge. This paper presents a simple methodology for calculating the risk of earthquake-induced permanent deformation of earth dams. The methodology is based on the premise that, for a given dam, with known geometry and material properties, a relationship can be established between permanent deformation,  $D_r$ , earthquake magnitude,  $M$  and distance to source of energy release,  $R$ . Using this relationship and any conventional computer program for seismic hazard analysis (SHA), the annual probability of  $D_r$  exceeding a specified value  $d_r$  can be calculated. The methodology developed, can provide estimates of relative seismic risks that are useful in design and decision analysis of earth dams, and can avoid compounding conservatism. Furthermore, the procedure can be used to identify the important parameters and assumptions that influence the risk for seismic deformations of an earth dam. It can also provide rapid assessment of risk for an inventory of dams in a certain geographic region. Such estimates of risk can help prioritize the dams that would require complete risk analysis and/or program of rehabilitation. An example earth dam is analyzed to illustrate the application of the described procedure.

### **KEY WORDS:**

Seismic Risk, Earth Dams, Permanent Deformations, Probabilistic Approach, Seismic Evaluation.

### **1. METHODOLOGY**

In a conventional Seismic Hazard Analysis (SHA), the probability of exceeding a certain level of acceleration at a site is calculated using knowledge of the seismicity of the region and an acceleration attenuation relationship appropriate for that region. Such a relationship is typically of the form,

$$A = f(M, R) \quad (1)$$

or more specifically,  $\log A = c_1 + c_2 M + c_4 \log(R + c_3)$  (2)

where  $A$  is the peak ground acceleration,  $M$  is the earthquake magnitude,  $R$  is the distance of the earthquake considered from the site and  $c_1$  through  $c_4$  are regional constants. The calculation of the number of events

causing acceleration 'A' to exceed a specified value 'a',  $\lambda(A \geq a)$ , is made using any of the computer programs for SHA that are currently utilized in practice. The annual probability of acceleration 'A' exceeding 'a' is then calculated from

$$P[A \geq a] = 1 - e^{-\lambda(A \geq a)} \quad (3)$$

In an analogous manner to acceleration A, for a given dam, with known geometry and soil properties, if a relationship can be established between permanent deformation,  $D_r$ , and M and R such that

$$D_r = f(M, R) \quad (4)$$

then a conventional SHA computer program can also be used to calculate the annual number of earthquakes that would cause  $D_r$  to exceed a specified value  $d_r$ . In such an analysis, the functional relationship of  $D_r$  (Eq. 4) is read into computer program instead of an acceleration attenuation law (Eq. 2). Hence, from the SHA the number of events causing permanent deformations  $D_r$  to exceed a specified value  $d_r$ ,  $\lambda(D_r \geq d_r)$ , can be calculated from

$$P[D_r \geq d_r] = 1 - e^{-\lambda(D_r \geq d_r)} \quad (5)$$

## 2. MATHEMATICAL FORMULATION FOR $D_r$

Yegian et al. (1991b) have demonstrated that the permanent deformation,  $D_r$ , of an earth dam can be estimated from the following expression

$$D_r = D_n N_{eq} T^2 K_a \quad (6)$$

where  $D_n$  = Normalized permanent deformation =  $g(K_y/K_a)$   
 $N_{eq}$  = Number of the equivalent uniform cycles  
 $K_a$  = Average acceleration of a critical sliding mass  
 $K_y$  = Yield acceleration of the critical sliding mass  
 $T$  = Predominant period of the motion within the dam.

To develop a relationship for  $D_r$  as a function of M and R, the following mathematical formulations are made on the parameters involved in Eq. 6.

### 2.1 Normalized Permanent Deformation, $D_n$

Using data from actual earthquake records, Yegian et al. (1991b) have established a mathematical function  $g(K_y/K_a)$  that defines  $D_n$ . Expressing this function in terms of the natural log of  $D_n$  yields,

$$\ln D_n = 0.5135 - 23.3 \left( \frac{K_y}{K_a} \right) + 37.72 \left( \frac{K_y}{K_a} \right)^2 - 26.43 \left( \frac{K_y}{K_a} \right)^3 + S \zeta_{D_n} \quad (7)$$

in which S is the standard normal variate and  $\zeta_{D_n}$  is the standard deviation of  $\ln D_n$  and is equal to 0.45.

### 2.2 Uniform Number of Equivalent Cycles, $N_{eq}$

Based on data from Seed et al. (1983), Number of uniform cycles,  $N_{eq}$ , can be related to Richter magnitude, M, by the following equation

$$N_{eq} = 0.16 e^{(0.6 M)} \quad (8)$$

### 2.3 Predominant Period, T

The parameter T in Eq. 6 represents the predominant period of the motion within a dam. Previous investigators including Gazetas (1987), Takahashi et al. (1977) and Okamoto (1984) have shown that this period over the height of an earth dam is almost constant; and, for a wedge shape geometry of dam, is close to the first mode fundamental period of the dam (Ambraseys & Sarma 1967). Hence, in Eq. 6, T is substituted by the fundamental period of the dam. For a homogeneous dam, considering a linear-elastic response, this period can be approximated by the following expression (Ambraseys & Sarma 1967)

$$T = 2.61 (H/V_s) \quad (9)$$

where H is the height of the dam and  $V_s$  is the shear wave velocity of the dam material. Alternatively, T can be calculated using dynamic response analysis of the dam. In such an analysis, nonlinear material behavior under large strains can be incorporated using normalized modulus reduction curves. From dynamic response analysis of typical earth dams, Ghahraman (1993) has demonstrated that, the calculated period, T, of a dam can be expressed in general terms as

$$T = T_0 + \beta \text{ pga}/g \quad (10)$$

where  $T_0$  is the period of the dam at small strains (Eq. 9) where material nonlinearity is of little importance, and  $\beta$  is a parameter that introduces the effect of large acceleration, hence, material nonlinearity upon T, pga is the peak ground acceleration at the base of the dam, and g is the acceleration of gravity.

### 2.4 Average Acceleration within a Dam, $K_a$

According to Makdisi & Seed (1978), Ambraseys (1960) and Ambraseys & Sarma (1967) the acceleration of any sliding mass within a dam is a function of the maximum acceleration at the crest of the dam,  $\ddot{U}_{\max}$ . For a case where the failure surface of a sliding mass passes through the base of the dam, Makdisi & Seed (1978) suggest that

$$K_a = 0.35 \ddot{U}_{\max} \quad (11)$$

Also, Makdisi & Seed (1979) have shown that

$$\ddot{U}_{\max} = [(1.6S_{a1})^2 + (1.06S_{a2})^2 + (0.86S_{a3})^2]^{0.5} \quad (12)$$

where  $S_{a1}$ ,  $S_{a2}$  and  $S_{a3}$  are the spectral accelerations corresponding to the fundamental period of the first three modes of the motion of the dam. Considering the contribution of only the first mode to  $\ddot{U}_{\max}$ ,  $K_a$  can be approximated by

$$K_a = 0.56 S_{a1} \quad (13)$$

$S_{a1}$  can be related to pga and T through the average normalized spectral acceleration curves proposed by Seed et al. (1974). For stiff soil conditions, and for a period range of most dams (i.e.  $0.3 \text{ s} < T_p < 1.5 \text{ s}$ ), the curve is expressed by

$$S_a = 0.893 \text{ pga}/T \quad (14)$$

Substituting Eq. 14 into Eq. 13 results in

$$K_a = 0.50 \text{ pga}/T \quad (15)$$

The results from the dynamic response analysis of a typical dam (Ghahraman 1993) also confirms the

general form of the Eq. 15. Thus, in general  $K_a$  can be expressed as

$$K_a = \alpha (pga/T) \quad (16)$$

where  $\alpha$  can either be estimated from dynamic response analysis or be approximated as  $\alpha = 0.5$ . It is noted that the effect of material nonlinearity on  $K_a$  (Eq. 16) can be conveniently introduced through  $T$  (Eq. 10).

## 2.5 Yield Acceleration, $K_y$

In the analysis of permanent deformations, the yield acceleration,  $K_y$ , plays a very important role. For a selected sliding mass, the value of  $K_y$  is very much dependent on the shear strength of the dam material in the region of the slide. If the dam material experiences loss of shear strength, then  $K_y$  will consequently decrease. Reduction in shear strength (i.e. reduction in  $K_y$ ) can be attributed to larger values of  $K_a$  and/or larger number of cycles of motion,  $N_{eq}$ , associated with larger earthquake magnitude. Again, the results from the analysis of a typical dam (Ghahraman 1993) shows that  $K_y$  can be expressed as follows,

$$K_y = K_{y0} - a_1 K_a^{a_2} e^{a_3 M} \quad (17)$$

where  $K_{y0}$  is the yield acceleration considering no loss in shear strength, and  $a_1$ ,  $a_2$  and  $a_3$  are parameters that define the relative influence of larger accelerations and magnitudes (or  $N_{eq}$ ).

## 2.6 Peak Ground Acceleration, $pga$

The peak ground acceleration is commonly related to earthquake magnitude  $M$  and distance  $R$  through an attenuation relationship having typically the form,

$$\ln pga = c_1 + c_2 M + c_4 \ln(R + c_3) \quad (18)$$

The uncertainty associated with this relationship is given by  $\sigma_{\ln pga}$ , the standard deviation of  $\ln pga$ .

## 3. PERMANENT DEFORMATION, $D_r$

Rewriting Eq. 6 in a natural logarithmic form results in

$$\ln D_r = \ln D_n + \ln N_{eq} + 2 \ln T + \ln K_a \quad (19)$$

Substituting expression for  $\ln D_n$  and  $K_a$  (from Eqs. 7 and 16, respectively) into Eq. 19 results

$$\begin{aligned} \mu_{\ln D_r} = & 0.5135 - 23.3 \left( \frac{\mu_{K_y} \mu_T}{\alpha \mu_{pga}} \right) + 37.72 \left( \frac{\mu_{K_y} \mu_T}{\alpha \mu_{pga}} \right)^2 - 26.43 \left( \frac{\mu_{K_y} \mu_T}{\alpha \mu_{pga}} \right)^3 \\ & + \ln(\mu_{N_{eq}}) + \ln(\mu_{pga}) + \ln(\mu_T) + \ln \alpha + \mu_S \zeta_{D_n} \end{aligned} \quad (20)$$

where  $\mu_{K_y}$  = mean value of  $K_y$  given by Eq. 17  
 $\mu_T$  = mean value of  $T$  given by Eq. 10

- $\mu_{N_{eq}}$  = mean value of  $N_{eq}$  given by Eq. 8  
 $\mu_{pga}$  = mean value of  $pga$  given by Eq. 18  
 $\mu_S$  = mean value of  $S = 0$ .

Using Taylor's series expansion for the variances

$$\begin{aligned}
 \text{Var}[\ln D_r] = & \left( \frac{\partial \ln D_r}{\partial K_y} \right)^2 \text{Var}[K_y] + \left( \frac{\partial \ln D_r}{\partial T} \right)^2 \text{Var}[T] + \left( \frac{\partial \ln D_r}{\partial pga} \right)^2 \text{Var}[pga] \\
 & + \left( \frac{\partial \ln D_r}{\partial N_{eq}} \right)^2 \text{Var}[N_{eq}] + \left( \frac{\partial \ln D_r}{\partial S} \right)^2 \text{Var}[S]
 \end{aligned} \quad (21)$$

where

$$\frac{\partial \ln D_r}{\partial K_y} = -23.3 \frac{T}{\alpha pga} + 75.44 \frac{K_y T^2}{(\alpha pga)^2} - 79.29 \frac{K_y^2 T^3}{(\alpha pga)^3} \quad (22)$$

$$\frac{\partial \ln D_r}{\partial T} = -23.3 \frac{K_y}{\alpha pga} + 75.44 \frac{K_y^2 T}{(\alpha pga)^2} - 79.29 \frac{K_y^3 T^2}{(\alpha pga)^3} + \frac{1}{T} \quad (23)$$

$$\frac{\partial \ln D_r}{\partial pga} = 23.3 \frac{K_y T}{\alpha pga^2} - 75.44 \frac{K_y^2 T^2}{\alpha^2 pga^3} + 79.29 \frac{K_y^3 T^3}{\alpha^3 pga^4} + \frac{1}{pga} \quad (24)$$

$$\frac{\partial \ln D_r}{\partial N_{eq}} = \frac{1}{N_{eq}} \quad (25)$$

$$\frac{\partial \ln D_r}{\partial S} = \zeta_{D_n} \quad (26)$$

These expressions are evaluated at mean values of  $K_y$ ,  $T$ ,  $pga$ ,  $N_{eq}$ , and  $S$ .

It is noted that in the expression for  $D_r$  (Eq. 20) the parameters on the right hand side are functions of  $M$  or  $M$  and  $R$ . However, the expression is complicated and does not have the normal form of an attenuation relationship needed for use in a conventional SHA computer program. Using regression analysis and the expression for  $D_r$  (Eq. 20), a more convenient relationship for  $D_r$  as a function of  $M$  and  $R$  was established, as is described in the following section.

#### 4. ATTENUATION RELATIONSHIP FOR $D_r$

For a given earth dam,  $K_{y0}$  and  $T_0$  are known and depend on the geotechnical material properties. The rest of the parameters defining  $D_r$ , depend on  $M$  and  $R$ . The concept followed in here was to develop a procedure that provides the constants of the equation for  $D_r$  that has the attenuation relationship form shown in Eq. 27.

$$\ln D_r = B_0 + B_1 M + B_2 \ln(R + 25) \quad (27)$$

For a given dam ( $K_{y0}$  and  $T_0$  known) and for a selected values of  $M$  and  $R$ ,  $\ln D_r$  and  $\sigma_{\ln D_r}$  can be calculated using Eq. 27. Thus, the constants in Eq. 27 can be obtained from a multi-variate regression analysis if an array of  $D_r$  versus  $M$  and  $R$  is generated. To accomplish this, a computer program "ARPED" was developed in which  $M$  and  $R$  are varied from minimum to maximum values of interest based on the knowledge of the local seismicity. Thus, values of  $D_r$  and  $\sigma_{\ln D_r}$  are calculated for every pair of  $M$  and  $R$ .

using Eqs. 20 and 21. Then, a subroutine in the program performs a regression analysis and computes the values of the B parameters of the Eq. 27. The error of residuals from the regression analysis,  $\sigma_R$  is combined with the  $\sigma_{\ln D_r}$  by

$$\sigma_{\text{tot}}^2 = \sigma_R^2 + \sigma_{\ln D_r}^2 \quad (28)$$

## 5. RISK ANALYSIS FOR PERMANENT DEFORMATIONS

As described in Section 1 the number of earthquakes causing  $D_r$  to exceed specified value  $d_r$  can be obtained using a conventional SHA program. In this approach the value of the B parameters of Eq. 27 are read in for constants of attenuation relationship. The standard deviation,  $\sigma_{\text{tot}}$ , is also read into the computer program replacing standard deviation of pga (or  $\ln$  pga).

It is noted that  $\sigma_{\text{tot}}$  calculated from the regression analysis is not constant but varies slightly with the value of  $D_r$ . Thus,  $\sigma_{\text{tot}}$  used in the SHA program should correspond to the typical range of  $D_r$  determined by the maximum and minimum values of M and R for the region. In the next section the sensitivity of the calculated probabilities of exceeding  $D_r$  to  $\sigma_{\text{tot}}$  is discussed.

## 6. ANALYSIS OF AN EXAMPLE DAM

To illustrate the details of the risk analysis procedures described in the preceding sections, an example dam shown in Fig. 1 was analysed. The values of the parameters that define the relationship for the period T, average acceleration  $K_a$  and yield acceleration  $K_y$  were obtained based on the results of the dynamic response analysis of the dam, for various levels of pga, reported by Ghahraman (1993). The results are as follows,

- |   |   |
|---|---|
| 1. $T = T_0 + \beta \text{ pga}/g$ ;          | $T_0 = 0.28 \text{ s}$ , $\beta = 1.3$                                    |
| 2. $K_y = K_{y0} - a_1 K_a^{a_2} e^{a_3 M}$ ; | $K_{y0} = 209 \text{ gals}$ , $a_1 = 0.01$ , $a_2 = 1.62$ , $a_3 = 0.247$ |
| 3. $K_a = \alpha (\text{pga}/T)$ ;            | $\alpha = 0.6$  |

To establish the attenuation relationship for  $D_r$ , the program ARPED was utilized with the above input parameters. The value of standard deviation of  $\ln D_r$ ,  $\sigma_{\ln D_r}$  was taken as 4.5. The attenuation relationship of Donovan (1973) worldwide, with standard deviation of  $\ln$  pga,  $\sigma_{\ln \text{ pga}}$  equal to 0.84 was used,

$$\ln \text{ pga} = 7.185 + 0.58 M - 1.52 \ln (R + 25)$$

Using the program ARPED, the values of the coefficients in Eq. 27 were calculated as following

$$\ln D_r = 19.904 + 8.911 M - 17.415 \ln (R + 25)$$

The standard deviation,  $\sigma_{\ln D_r}$ , of the predicted  $D_r$  from Eq. 21 was also calculated and ranged between 4.3 and 5.7 for  $D_r$  ranging between 2 and 300 cm, respectively.

Using the attenuation relationship for  $D_r$  (Eq. 27) and the seismicity parameters for the example site, the annual number of earthquakes causing  $D_r$  to exceed  $d_r = 0$  to 300 cm (0 to 10 ft) were calculated using the computer program MITRISK (Schumacker & Whitman 1978). Figure 2 shows the probability of exceedance in 50 years as a function of  $d_r$ . The two curves in the figure correspond to the lower and upper bound values

of  $\sigma_{\ln D_r}$ . The results show that variations in  $\sigma_{\ln D_r}$  has only a small effect on the calculated probabilities.

To further illustrate the benefits of using the procedure in sensitivity analysis, the acceleration attenuation relationship of Donovan (1973-worldwide) was replaced with that of Idriss (1991). The results shown in Figure 3 indicate that the computed probability values are very sensitive to the attenuation law selected in the analysis.

Another observation that can be made from this example is that the probability of exceedance does not change significantly for  $d_r > 30$  cm. Hence, the definition of catastrophic damage is not very sensitive to the value of  $d_r$ , provided that  $d_r > 30$  cm.

## CONCLUSION

In summary, the risk analysis procedure for earthquake-induced permanent deformation of earth dams presented in this paper can easily provide preliminary estimates of probability of such deformations exceeding selected value  $d_r$ . Such analysis can be used to identify the important parameters and assumptions that influence seismic risk of an earth dam. Furthermore, the procedure can provide rapid assessment of permanent deformations risk for an inventory of dams in a certain geographic region. Such estimates of risk can help prioritize the dams that would require complete risk analysis and/or program of rehabilitation.

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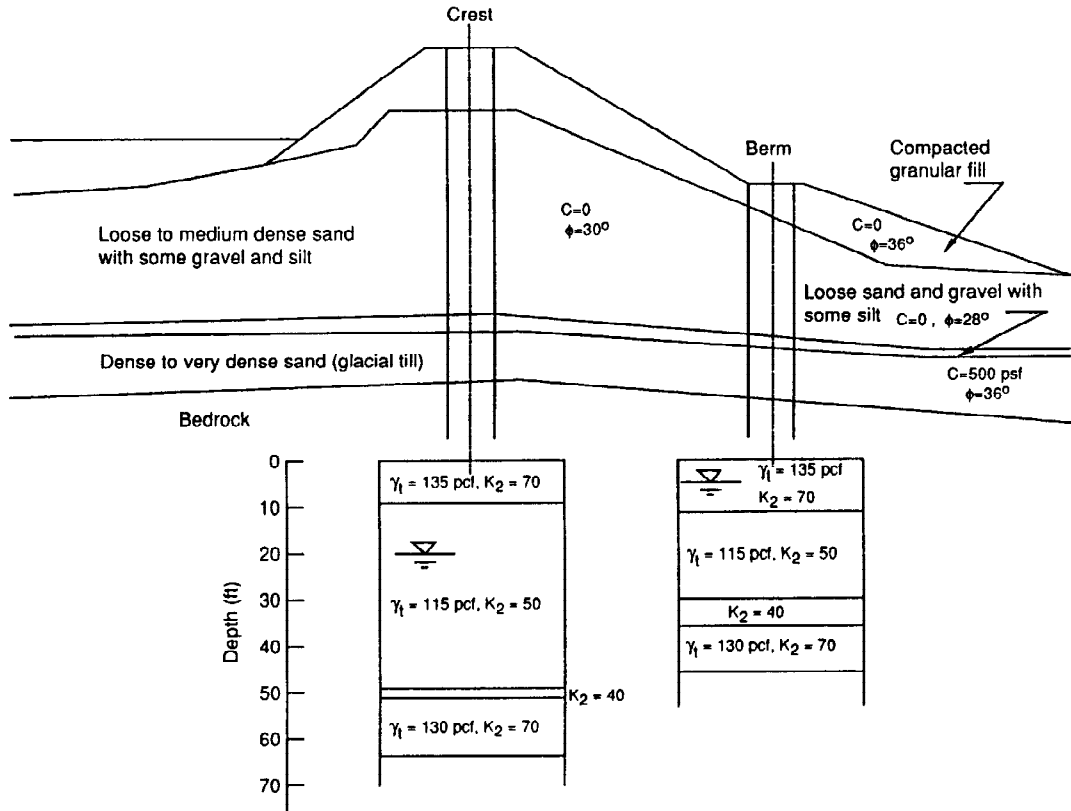


Fig. 1. Cross Section and Geotechnical Properties of the Example Dam Analyzed

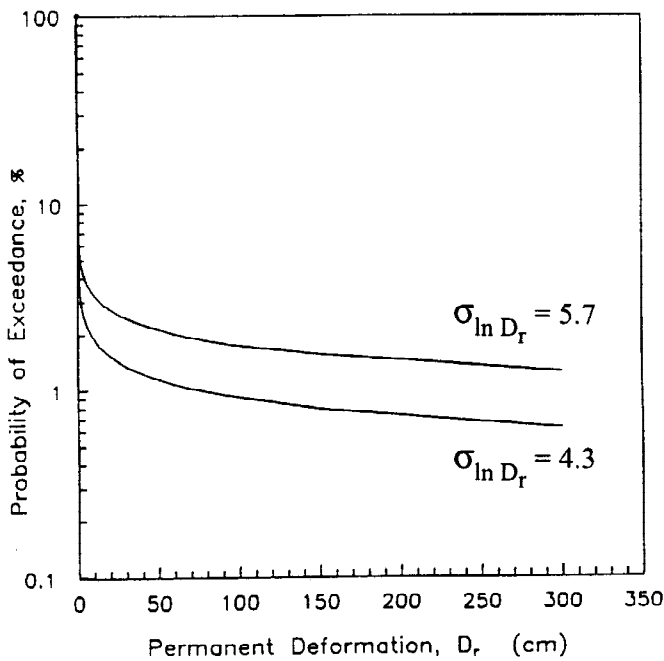


Fig. 2. Probability of Exceeding Permanent Deformations in 50 Years, (for the Range of Uncertainties Considered)

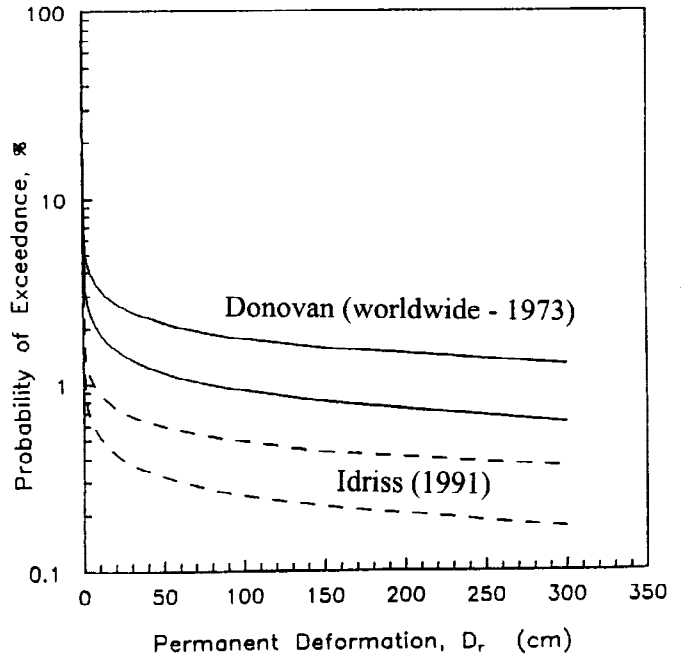


Fig. 3. Probability of Exceeding Permanent Deformations in 50 years, Using the Attenuation Relationships of Donovan (1973) and Idriss (1991) and for the Range of Uncertainties Considered