



## CHARACTERIZATION OF STRUCTURAL DAMAGE UNDER HIGH-INTENSITY SEISMIC LOADING

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### ABSTRACT

A strain-based low-cycle fatigue model commonly used for the prediction of fatigue life in metals is adapted for cumulative damage assessment of structures under seismic conditions. By defining the number of load cycles in terms of the plastic strain energy dissipated by the structure, the model is presented in a form suitable for seismic damage assessment. The plastic strain energy is expected to decrease rapidly with increased displacement in the small displacement range and to decrease gradually in a near linear manner with increased displacement in the large displacement range. At the ultimate limit state, the modified Park and Ang damage model may be considered as a linear approximation to the low-cycle fatigue model in the large displacement range. In the small displacement range, however, the total plastic strain energy predicted by the low-cycle fatigue model deviated rapidly from the modified Park and Ang model.

### KEYWORDS

Cumulative damage; low-cycle fatigue; hysteretic energy; earthquake duration.

### INTRODUCTION

One of the concerns during a large magnitude earthquake is the potential damage that may be inflicted on structures by the possible long duration of the strong ground motion. While the peak ground acceleration may be limited by the non-linear behavior of the soil as a result of large strains associated with the motion, the duration of intense ground shaking may be significantly lengthened due to a larger fault rupture occurring in a large earthquake. With an extended duration of ground shaking, yielding structures are expected to undergo a larger number of cycles into the inelastic range. The increased number of inelastic cycles is important for the survival of structures during large earthquakes, particularly for relatively stiff systems. For overall damage assessment of structures, it is important to include both the damage due to peak response and the damage accumulated through all non-peak inelastic cycles.

Although the damage potential of long duration ground motions on structures is generally recognized, most design codes do not explicitly provide guidelines to account for the damaging effect of the long duration. In the United States, the Commentary for the NEHRP's provisions (NEHRP,

1988) suggests an increase in the design lateral force for earthquakes with anticipated long duration. A larger lateral strength for the structure will presumably result in a smaller number of yield excursions which would then compensate for the cumulative damage effect associated with a long duration event. The Commentary (NEHRP, 1988), however, does not provide a procedure for increasing the design force for the structures.

The damaging effect of long duration ground motions on structures may be assessed using a cumulative damage model which takes into account the damage due to all inelastic excursions including the peak cycle which occur during the response of the structure. In this paper, a strain-based low-cycle fatigue model for predicting the fatigue life of metals is adapted for assessment of cumulative damage under seismic conditions. By defining the number of cycles in terms of the plastic strain energy dissipated by the structure, the model can be adapted to relate the total plastic strain energy capacity of the structure to the maximum displacement imposed on the structure. In the large displacement range, the model compares well with an energy-based linear damage model which has been extensively used for damage assessment of structures under seismic conditions.

## STRAIN-BASED FATIGUE-LIFE PREDICTION

A strain-based model commonly used for the prediction of fatigue life of metals assumes that the damage process in metals can be adequately characterized by the total strain imposed by the cyclic loading process. The total strain amplitude is assumed to be consisted of an elastic and a plastic strain component (Collins, 1992):

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c \quad (1)$$

where  $\Delta\epsilon/2$  = total strain amplitude,  $2N_f$  = number of load reversals,  $E$  = Young's modulus of the material,  $\sigma'_f$  = fatigue strength coefficient,  $b$  = fatigue strength exponent,  $\epsilon'_f$  = fatigue ductility coefficient, and  $c$  = fatigue ductility exponent. The first term on the right-hand-side of Eqn. 1 corresponds to the elastic strain component and represents damage in the high-cycle fatigue range while the second term on the right-hand-side of Eqn. 1 corresponds to the plastic strain component and represents damage in the low-cycle fatigue range. Eqn. 1 has been extensively calibrated, and for most metals, the exponent  $b$  varies between -0.05 and -0.15, and the exponent  $c$  varies between -0.5 and -0.8 (Bäumel and Seeger, 1990). An average value of  $c = -0.6$  is commonly assumed for steel. The plastic component, i.e. the second term on the right-hand-side of Eqn. 1, is sometimes referred to as the Coffin-Manson equation.

One of the approaches to cumulative damage assessment of structures under intense ground motion is to assume that the damage sustained by the structure during a strong earthquake is similar to the damage experienced in metal fatigue under large strain reversals (Kasiraj and Yao, 1969, Krawinkler and Zohrei, 1983) Since the number of load cycles experienced during earthquakes is significantly smaller than the number of cycles needed for damage in high-cycle fatigue, seismic damage needs only be assessed in terms of the plastic strain component. In structural application, however, characterization of damage in terms of the plastic strain amplitude is less convenient since the response displacement is the more commonly computed response parameter instead of strains. In this paper, a correspondence between material and structural damage under large inelastic strain reversals is assumed (McCabe and Hall, 1989):

$$\Delta_m - \Delta_y = (\Delta_{um} - \Delta_y)(2N_f)^c \quad (2)$$

where  $\Delta_m$  = peak response displacement under cyclic loading;  $\Delta_{um}$  = ultimate displacement under

monotonic loading; and  $\Delta_y$  = first-yield displacement. By adopting a definition for cyclic displacement ductility factor:

$$\mu_c \equiv \frac{\Delta_m}{\Delta_y} \quad (3)$$

and monotonic displacement ductility factor:

$$\mu_m \equiv \frac{\Delta_{um}}{\Delta_y} \quad (4)$$

Equation 2 can be written as:

$$\left[ \frac{\mu_c - 1}{\mu_m - 1} \right]^{1/c} = 2N_f \quad (5)$$

Under a constant amplitude cyclic displacement condition, the number of load reversals  $2N_f$  that may be imposed on the structure decreases exponentially with the magnitude of the imposed displacement as characterized by the displacement ductility factor  $\mu_c$ .

### EQUIVALENT HYSTERETIC ENERGY CYCLES

The use of Eqn. 5 for cumulative damage assessment of structures under seismic conditions requires a suitable combination of damages from all unequal displacement cycles which would occur during the response of the structure. A common approach uses the Miner's rule which assumes that the total damage inflicted on the structure, as signified by the index  $D_t$ , can be computed as a linear summation of all damage fractions  $D_k$  (Krawinkler, 1987, Mander *et al.*, 1995):

$$D_t = \sum_{k=1}^{n_t} D_k \quad (6)$$

where  $n_t$  = number of cycles experienced by the structure during the response, and  $D_k$  = damage fraction representing the 'consumption' of the fatigue life by the  $k^{th}$  cycle, and is defined as:

$$D_k = \frac{1}{(N_f)_k} \quad (7)$$

where  $(N_f)_k$  = number of cycles to cause failure upon cycling to a given displacement. Failure of the structure is assumed if the total damage index  $D_t$  exceeds unity.

In the case of an uniaxially loaded member where the strain field is uniform, the number of cycles to cause failure of the member can be easily identified, even in cases where strain-softening of the material may occur. Upon cycling to a constant displacement, failure of these members would invariably involve a rather distinctive drop in strength as fracture propagates rapidly across the member. The number of load cycles  $(N_f)_k$  in this case would be inclusive of all the cycles up to the point of fracture. On the other hand, the number of cycles  $(N_f)_k$  to cause failure in a flexural member where the strain field is non-uniform may not be as easily identified. Depending on the displacement amplitude, failure of these members may not involve a distinct drop in strength but rather a slow degradation of strength as cracks develop more gradually across the member upon cycling to the same displacement. In this case, inclusion of all cycles as complete load cycles may not be justified especially when the degraded strength is small compared to the peak strength.

In order to overcome the difficulty of identifying the number of cycles to cause failure in members where a gradual degradation of strength may occur, the approach based on the plastic strain energy

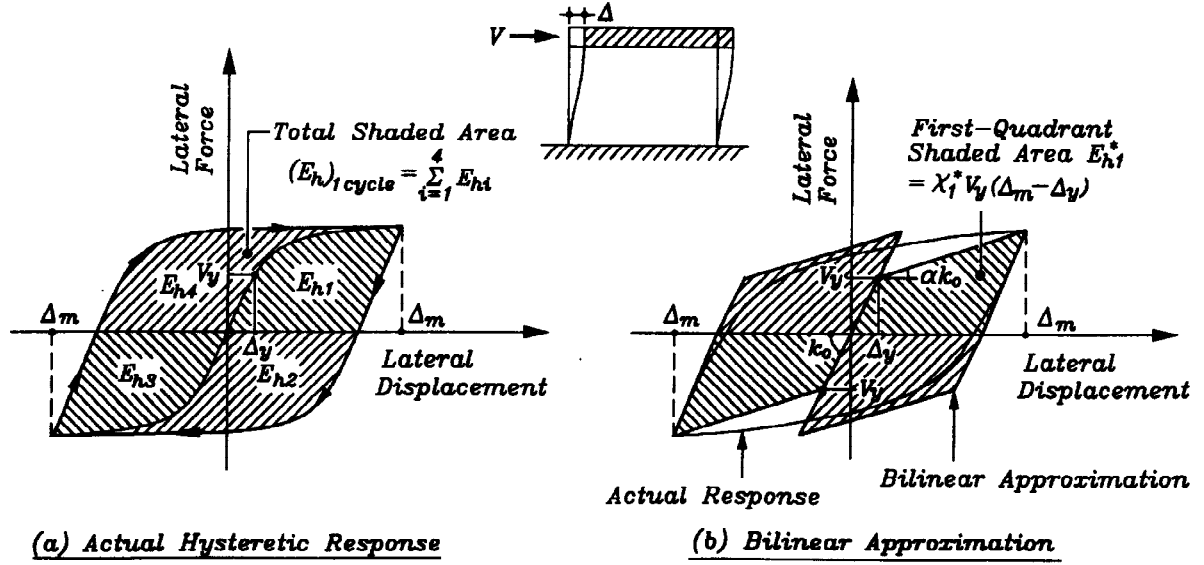


Figure 1: Representation of Plastic Strain Energy in One Displacement Cycle

dissipated by the structure is followed (McCabe and Hall, 1989). Figure 1(a) shows a lateral force-displacement hysteresis loop for a simple structure. The plastic strain energy dissipated by the loop can be written as a summation of the areas enclosed by the four quadrants of the loop:

$$(E_h)_{cycle} = \sum_{i=1}^4 E_{hi} \quad (8)$$

The plastic strain energy dissipated by the  $i^{th}$  quadrant can be written as:

$$E_{hi} = \chi_i V_y (\Delta_m - \Delta_y) \quad i = 1 \text{ to } 4 \quad (9)$$

where  $V_y$  = first-yield lateral force,  $\Delta_m$  = post-yield peak displacement,  $\Delta_y$  = first-yield displacement, and  $\chi_i$  = correction factor for the plastic strain energy dissipated above the first-yield response for the  $i^{th}$  quadrant. In the case of an elasto-plastic response, the correction factor  $\chi_i$  assumes a value of unity for all quadrants, and the plastic strain energy dissipated by one cycle  $(E_h)_{cycle}$  is equal to four times the energy dissipated by the first quadrant,  $E_{h1}$ . By extending from the considerations of an elasto-plastic response, an equivalent number of 'energy' cycles  $N_f^e$  can be defined as the ratio of the total plastic strain energy dissipated by the structure  $E_{ht}$  to the plastic strain energy dissipated by one cycle  $(E_h)_{cycle}$ :

$$N_f^e \equiv \frac{E_{ht}}{(E_h)_{cycle}} \quad (10)$$

In this paper, the actual nonlinear hysteresis loop is approximated by four identical bilinear quadrants with an unloading stiffness assumed to be the same as the initial stiffness, as shown in Figure 1(b). The plastic strain energy dissipated by one cycle can thus be written as:

$$(E_h)_{cycle} \approx 4E_{h1}^* = 4\chi_1^* V_y (\Delta_m - \Delta_y) \quad (11)$$

where  $E_{h1}^*$  = plastic strain energy dissipated by one bilinear quadrant, and the correction factor  $\chi_1^*$  for a bilinear response can be written as:

$$\chi_1^* = \frac{(-\alpha(\frac{\Delta_m}{\Delta_{ym}})^2 \mu_m^2 + \alpha^2(\frac{\Delta_m}{\Delta_{ym}})^2 \mu_m^2 - 2(\frac{\Delta_m}{\Delta_{ym}})\mu_m + 4\alpha(\frac{\Delta_m}{\Delta_{ym}})\mu_m - 2\alpha^2(\frac{\Delta_m}{\Delta_{ym}})\mu_m + 2 - 3\alpha + \alpha^2)}{2(1 - (\frac{\Delta_m}{\Delta_{ym}})\mu_m)} \quad (12)$$

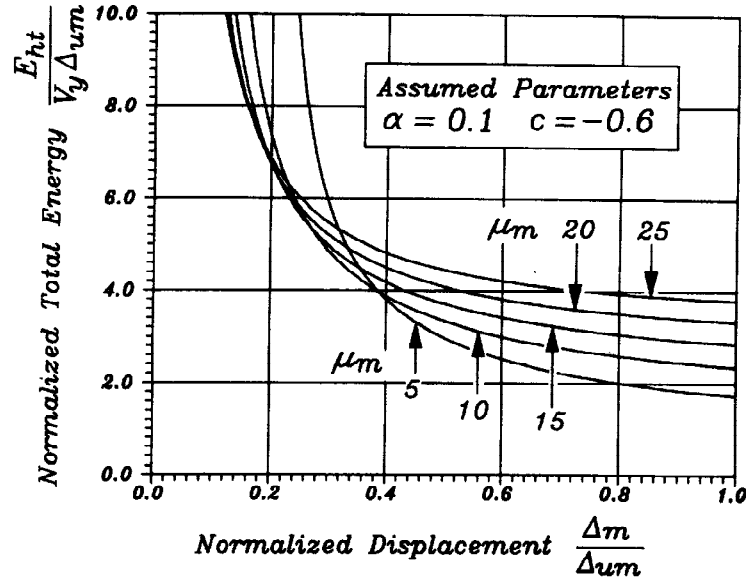


Figure 2: Coffin-Manson Model - Total Plastic Strain Energy versus Displacement

where  $\alpha$  = post-yield-to-elastic stiffness ratio (see Figure 1(b)). The combination of Eqn. 10 and 11 gives the number of 'energy' reversals as:

$$2N_f^e = \frac{E_{ht}}{2\chi_1^*(\mu_c - 1)V_y\Delta_{um}} \quad (13)$$

Assuming that the number of 'energy' reversals  $2N_f^e$  is the same as the number of load reversals  $2N_f$ , Eqn. 13 and Eqn. 5 can be combined, which after some re-arranging, gives:

$$\frac{E_{ht}}{V_y\Delta_{um}} = 2\chi_1^* \left[ \frac{\mu_m}{\mu_m - 1} \right]^{1/c} \left[ \frac{\Delta_m}{\Delta_{um}} - \frac{1}{\mu_m} \right]^{1+1/c} \quad (14)$$

The predictive form of the low-cycle fatigue model in Eqn. 14 is suitable for seismic damage assessment of structures as the total plastic strain energy is now related to the peak response displacement. Figure 2 shows a plot of the total normalized plastic strain energy  $E_{ht}/V_y\Delta_{um}$  versus the normalized peak response displacement  $\Delta_m/\Delta_{um}$  for members with monotonic displacement ductility factor  $\mu_m = 5$  to 25. A fatigue ductility exponent of  $c = -0.6$  and a stiffness ratio of  $\alpha = 0.1$  are assumed in Figure 2. It can be seen from Figure 2 that the total normalized plastic strain energy  $E_{ht}/V_y\Delta_{um}$  decreases rapidly with increased normalized displacement  $\Delta_m/\Delta_{um}$  in the small displacement range, whereas the total normalized plastic strain energy decreases more gradually in a near linear manner with increased displacement in the large displacement range. It can also be seen from Figure 2 that the total plastic strain energy capacity of ductile systems, as characterized by the large monotonic displacement ductility factor ( $\mu_m > 10$ ), is not very sensitive to the ductility capacity of the system in the small displacement range.

## ENERGY-BASED LINEAR DAMAGE MODEL

The use of plastic strain energy as a characterizing parameter for cumulative damage assessment of structures under intense ground motions has been extensively studied (Williams and Sexsmith, 1995). The original work by Park and Ang (Park and Ang, 1985) assumes that structural damage

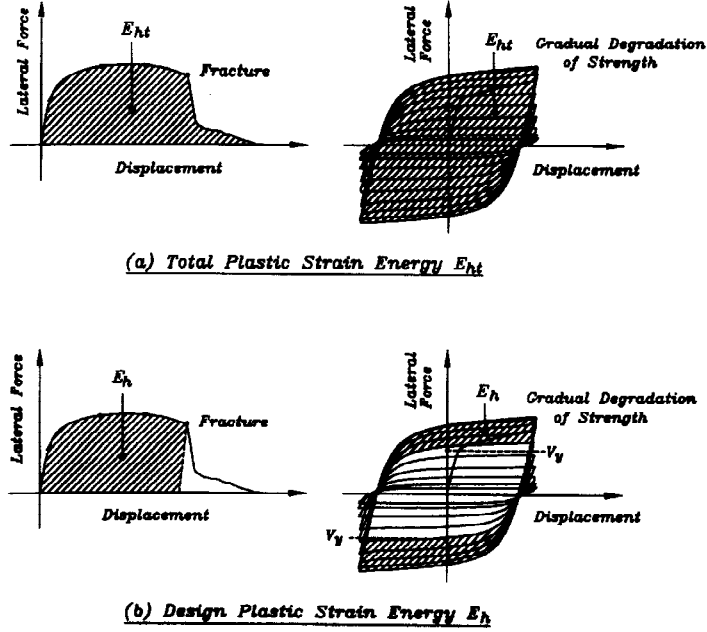


Figure 3: Definition of Total and 'Design' Plastic Strain Energy

can be assessed by a linear combination of damage due to peak response displacement and damage due to plastic strain energy dissipated by the structure. The damage model was recently modified to account for the plastic strain energy dissipated under monotonic loading, and the modified model can be written as (Chai *et al.*, 1995):

$$D_i = \frac{\Delta_m}{\Delta_{um}} + \frac{\beta^*(E_h - E_{hm})}{V_y \Delta_{um}} \quad (15)$$

where  $D_i$  = damage index,  $\beta^*$  = strength deterioration parameter,  $E_h$  = plastic strain energy dissipated during the response of the structure,  $E_{hm}$  = plastic strain energy dissipated under a monotonic loading. In Eqn. 15, only the portion of the plastic strain energy dissipated above that of the monotonic response is considered as contributing to damage. At the ultimate limit state, the damage index assumes a value of unity i.e.  $D_i = 1$ , and Eqn. 15 can be re-arranged into:

$$\frac{E_h}{V_y \Delta_{um}} = \left( \frac{1}{\beta^*} + \frac{E_{hm}}{V_y \Delta_{um}} \right) - \frac{1}{\beta^*} \frac{\Delta_m}{\Delta_{um}} \quad (16)$$

Eqn. 16 indicates that the ultimate limit state can be represented by a straight line with a slope of  $-1/\beta^*$  in the normalized plastic strain energy versus normalized displacement space (Chai *et al.*, 1995).

For comparison between the Coffin-Manson model and the modified Park and Ang model, it is important to note the difference in the definitions of the total plastic strain energy  $E_{ht}$  as used in Eqn. 14, and the plastic strain energy  $E_h$  used in Eqn. 15. Although the Coffin-Manson model is adapted for flexural members where the strain field is non-uniform, the plastic strain energy must be identified with the total plastic strain energy  $E_{ht}$  to cause complete fracture, i.e. physical separation, of the member. This energy is indicated as shaded areas in Figure 3(a) for both fracture and gradual degradation type of failures. In Eqn. 15, however, the plastic strain energy  $E_h$  is defined in terms of the performance of the structure. For a satisfactory response of the structure under high intensity seismic loading, the ultimate limit state must be associated with a stable hysteretic response, that

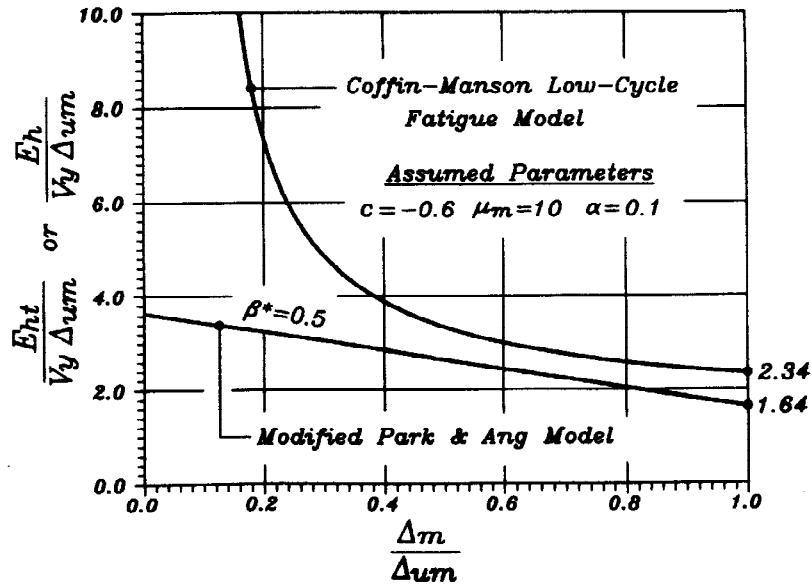


Figure 4: A Comparison Between Coffin-Manson and Modified Park and Ang Models

is, the structure must be able to maintain sufficient lateral strength without excessive degradation of strength upon cycling to the same displacement. In this case, the plastic strain energy  $E_h$  is defined as the plastic strain energy dissipated up to initial fracture for fracture-type failures, or the energy dissipated up to the point where the lateral strength degrades to the first-yield strength  $V_y$  for gradual degradation type failures. The plastic strain energy  $E_h$  is shown as shaded areas in Figure 3(b), and will be referred to as the 'design' plastic strain energy in this paper. The ratio of the 'design' plastic strain energy  $E_h$  to the total plastic strain energy  $E_{ht}$  is denoted by:

$$\eta \equiv \frac{E_h}{E_{ht}} \quad (17)$$

The plastic strain energy capacity predicted by the Coffin-Manson model (Eqn. 14) and the modified Park and Ang model (Eqn. 16) are compared in Figure 4 using some representative values for the models. The total normalized plastic strain energy for the Coffin-Manson model is calculated assuming a fatigue ductility exponent of  $c = -0.6$ , a monotonic displacement ductility factor of  $\mu_m = 10$ , and a post-yield-to-elastic stiffness ratio of  $\alpha = 0.1$ . Under monotonic loading where  $\Delta_m/\Delta_{um} = 1$ , the total plastic strain energy is computed as  $E_{ht}/V_y\Delta_{um} = 2.34$ . For the modified Park and Ang model, a strength deterioration parameter of  $\beta^* = 0.5$  is assumed. For comparison between the two models, a constant plastic strain energy ratio of  $\eta = 0.7$  is used, and the 'design' plastic strain energy under monotonic loading is  $E_h/V_y\Delta_{um} = 1.64$ . It can be seen from Figure 4 that the 'design' plastic strain energy line lies below the total plastic energy line, and is almost parallel to the total plastic strain energy line in the large displacement range of  $\Delta_m/\Delta_{um} > 0.5$ . Although a relatively large value of  $\beta^* = 0.5$  is required for the slope of the 'design' plastic strain energy line, the modified Park and Ang model may be considered as a linear approximation to the Coffin-Manson low-cycle fatigue model in the large displacement range. In the small displacement range, however, a rapid deviation of the total plastic strain energy line from the 'design' plastic strain energy line occurs.

## CONCLUSIONS

A strain-based low-cycle fatigue model commonly used for the prediction of fatigue life in metals is adapted for cumulative damage assessment of structures under seismic conditions. By defining the number of load cycles in terms of the total plastic strain energy dissipated by the structure, the model is presented in a form suitable for seismic damage assessment. The total plastic strain energy is expected to decrease rapidly with increased displacement in the small displacement range and to decrease gradually in a near linear manner with increased displacement in the large displacement range. When compared to the modified Park and Ang damage model, the modified Park and Ang model may be considered as a linear approximation to the low-cycle fatigue model in the large displacement range. For practical damage assessment of structures, however, only a fraction of the total plastic strain energy should be considered, and this plastic strain energy should be based on a performance criterion to ensure a stable hysteretic response of the structure.

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