TORSIONAL RESPONSE OF STRUCTURES UNDER SEISMIC MOTION

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ABSTRACT

Evidence of torsional effects on buildings produced by earthquakes, has shown the need for a methodological approach to consider its effects, in order to avoid structural damages on buildings.

Several buildings at Mexico City has been instrumented to record earthquake motions at their base and at the upper floors, and now there are digital records of torsional motions generated by earthquakes.

In order to understand those motions, a computer program was developed for structural analyses of buildings under horizontal displacements at their base, and rotational motion around three orthogonal axis. Digital information recorded during seismic motions was fed into the program, to understand the torsional response of six regular tower-like structures.

Structural analyses were carried on, for different positions of a vertical axis of rotation, under the same earthquake record; they showed a gradual change on the response as the rotational axis moved from the centroid to one corner.

Modification of the dynamic properties of motion with respect to time, showed the existence of torsional resonance of the structures under analysis, when appropriate torsional stiffness was used.

Critical values of torsional stiffness at the base were computed by a dynamic method, and it was possible to find the values needed to initiate torsional instability of the buildings.

Once admissible values of stiffness at the interface were selected, to carry on stable dynamic analyses, mechanical elements at beams and columns were computed in order to evaluate the effects of torsional response, and the relative importance of the control variables.

Results obtained on regular structures, as the ones selected for this study, indicated that the torsional response is highly dependent on:

a) Position of the vertical axis of rotation at the base of a building

b) Torsional stiffness at the interface between the foundation and the subsoil

c) A critical value of stiffness at the base in order to avoid instability

KEYWORDS

Buildings; torsional response; center of gyration; rotational stiffnesses.

SOIL AND BUILDING MOTION DURING STRONG EARTHQUAKES

Theoretical solution of the equation of motion of stratified subsoil, during an earthquake, shows a superposition of wave patterns due to P, S, L, and R waves, each one of them with different type of displacements and rotation of soil particles, along three orthogonal axis. Those components of soil motion generate displacements and rotations at the base of buildings on top of the subsoil; those motions are amplified from the foundation to the roof of the building, to create a three dimensional response of the superstructure.

Few attention has been given to the effect of rotations generated around a vertical axis by torsional motion; most studies of structural analysis are devoted to horizontal displacements at different story levels of a building, and simplified general rules are defined to compute the torsional response of a building.

In this paper, attention is given to the effects produced by change of position of the center of gyration, on the structural response of beams and columns, when torsional motion is excited at the foundation level of a building.

Experimental evidence obtained from measurements carried out at an instrumented building at Mexico City, when earthquake motion moved the structure, has shown the existence of rotational motion around a vertical axis, as well as two horizontal orthogonal axis at foundation level (Rodriguez-Cuevas, 1992) when seismic waves interact with the foundation. Time series has been obtained from measurements; rotational motions around three orthogonal axis have been identified, both in magnitude and frequency content.

It may happen that the stiffnesses of the subsoil around the foundation are enough to transmit stable vibration to the superstructure; in such cases, a linear analysis may represent the response of a building; when foundation stiffnesses are below critical values as those shown in this paper, it may happen that unstable nonlinear vibrations may occur, and the building may collapse under seismic excitation.

From stability analyses using the dynamic method, expressions are proposed to compute critical values for rotational stiffnesses around three orthogonal axis at the base of the building. When suitable factors of safety are used to multiply the critical values, it is possible to avoid unstable behavior, and linear analyses may represent the stable structural response under flexural-torsional vibrations, generated by the six components of motion produced by the seismic excitation at the base of the building.

In order to carry out those linear analyses, it was necessary to develop a three dimensional program for digital computers (TESCOSE) that is able to consider three rotational acceleration time series at the base, as well as the horizontal time series for horizontal accelerations generated during an earthquake (Rodriguez-Cuevas, N and Sarcos-Portillo, A. 1994).

COMPUTER PROGRAM FOR THREE DIMENSIONAL ANALYSES

By means of a Hamiltonian approach, it was possible to develop equations of motion for a spring-mass-damper system that represent a structural system, in order to search for displacements and rotations of each story level forming the superstructure of the building, assuming the existence of soil-structure interaction, with five degrees of freedom at the base.

Hamilton´s principle can be expressed by Lagrangian equations of the following type:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_i}\right) - \frac{\partial T}{\partial \dot{q}_i} + \frac{\partial U}{\partial \dot{q}_i} = Q_i \qquad (1)$$

where T and U represent the kinetic and the strain energy of the system ; q_i and $\dot{q}_i$ represent the generalized displacement and velocity corresponding to the i-th degree of freedom; Q_i represents the generalized force associated to the same degree of freedom.

When the material that forms the superstructure is assumed to be linearly elastic, it is possible to compute the lateral stiffness matrix of the building, by means of matrix analysis, in order to obtain the reduced lateral stiffness matrix that take into consideration the existence of axial force, shear and bending in all members of the building.

Soil-structure interaction was considered by the use of an infinitely rigid foundation, supported by five viscoelastic springs. The rigid foundation was connected to the columns of the building; the superstructure was assumed to be formed by plane frames with their corresponding direction, interconnected by infinitely rigid slabs to torsion , supported by elastic beams to reproduce the floor system, with continuous joints to the columns.

Figure 1 shows the idealized model of the building, formed by plane frames and rigid slabs; it also shows the foundation model, supported by five viscoelastic springs. Viscous dashpots were selected at each level, to reproduce the damping associated to each degree of freedom.

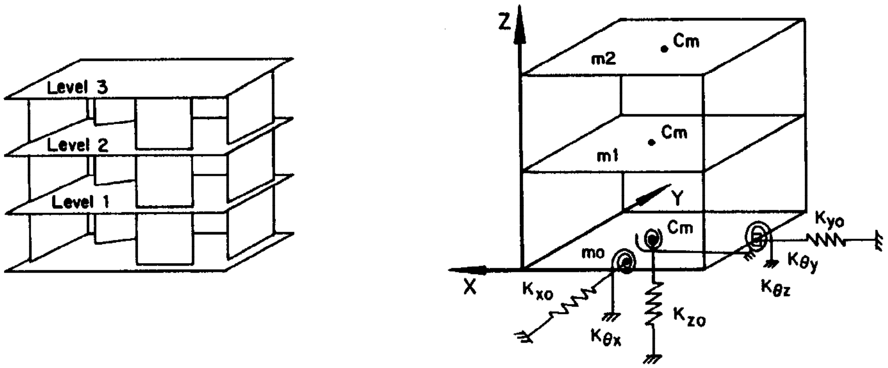

Fig. 1. Idealized model of a building used to develop the equations of motion under seismic excitation.

CRITICAL ROTATIONAL STIFFNESSES AT THE BASE OF A BUILDING

To study the possible instability of the spring-mass system of n masses located along the height of the building, each mass, with weight W_i, was located at height h_i over foundation level. Mathematical analyses were carried out to find the critical value of the rotational stiffnesses at the base of the building, denoted by $(K_{\theta X}$ or $K_{\theta Y})_{cr}$, and $(K_{\theta Z})_{cr}$. The dynamic method gave the following expressions to evaluate them (Rodriguez-Cuevas, 1984):

$$(K_{\theta X})_{cr} = \sum_{i=1}^{n} W_i \, h_i \tag{2}$$

$$(K_{\theta Y})_{cr} = \sum_{i=1}^{n} W_i \, h_i \tag{3}$$

and for the torsional response motion around a vertical axis:

$$(K_{\theta Z})_{cr} = w^2 \sum_{i=1}^{n} a_i \, J_{Zi} \, h_i \tag{4}$$

where: a_i = amplification value for the rotation at the base of the building, to obtain the rotation of the i-th mass, with respect to the base, when the first torsional mode of vibration is developed at the structure

J_{iZ} = mass moment of inertia of the i-th floor around the center of gyration

w = frequency of the first torsional mode of vibration around a vertical axis z

When critical values of the angular stiffnesses at the base were fed in the program, the Jacobi routine used in the program became ill conditioned and unstable behavior was detected in the model of the building. When rotational stiffnesses above the critical values were fed into the program, with time series representing seismic accelerations at the base, the program gave time series for displacements and velocities at each level of the superstructure.

TORSIONAL RESONANCE OF A BUILDING

Time series for accelerations recorded at the base of an instrumented building at Mexico City were used to compute time series representing the torsional motion at the base of the building. Rotational accelerograms are digital records of rotational motion formed by magnitudes of acceleration, recorded at instants separated Δ_t seconds.

By an adequate selection of the time interval Δ_t, it is possible to change the frequency content implicit in the time record, and therefore it is possible to feed them to the model of a building, to observe the change in response of the model.

In order to illustrate this point, Table 1 shows the maximum angular rotation at the top of a twelve story high structure, when Δ_t was varied between 0.001 and 0.10 s. Near 0.030 s the rotation increased, indicative of the existence of torsional resonance of one of the torsional modes of vibration of the structure.

Table 1. Maximum rotation at the top of a 12 story-high building, under torsional excitation at the base

Δ_t, s	0.001	0.010	0.020	0.025	0.030	0.035	0.040	0.050	0.100
Maximum rotation, 10^{-4} rad	0.146	0.699	1.024	1.205	1.269	1.021	0.793	0.677	0.710

ANALYSIS OF REGULAR BUILDINGS UNDER TORSIONAL EXCITATION

Six tower-like structures, formed by four symmetrical column and beams, were designed following Mexico City Code recommendations, and were selected to compute their torsional response under rotational time series for accelerations, obtained during an earthquake. Geometrical plain view, and their elevations are shown on figure 2. Stiffnesses at their base were selected to induce stable vibrations; their values were selected following Mexico City Code recommendations. It was considered that the structures were supported by a clay stratum, with 10 meter depth, and the natural period of the subsoil was equal to one second.

The horizontal dimensions of all slabs were kept equal in shape; their sides were equal in length, measuring 12.70 m. No eccentricity in mass distribution was assumed during the computation carried out to find their torsional response. All story height was equal to 3 m, in all structures under analysis. The column and beam dimensions were obtained though a structural analysis; the structure was designed in reinforced concrete. Every four stories, the columns changed their sides. Beams depths varied between 0.80 m and 0.90 m in all the structures.

In order to evaluate the effect of the position of the center of gyration, on the torsional response of the structures, different analysis were carried out under the same torsional response at their base, but with a different position of the z axis of rotation. Five positions were selected along the diagonal connecting two opposite columns. The first position coincided with the centroid of the base; other four position fall on the diagonal, equally separated among them; the last position coincided with the center of the column at the corner of the structure.

Structures with two, four, six eight, ten and twelve stories were considered for analysis, loaded with live loads and its own dead weight, without any eccentricity. Torsional time series were applied at the foundation level of the structures, assuming one of the positions of the center of gyration above mentioned.

In order to illustrate the type of results obtained from the analyses, results of the maximum rotations achieved at the rigid slabs of the structure, around the selected axis of rotation, are shown in tables 2 and 3 for the same torsional excitation at the base of the structure.

Table 2. Maximum rotation (10^{-4} rad) obtained at the slabs of a six story high structure under torsion

Building	Center of gyration coordinates		Level					
	x	y	1	2	3	4	5	6
3A	0.00	0.00	0.58	1.18	1.49	1.72	2.60	2.73
3B	1.59	1.59	0.99	2.34	3.17	4.19	4.73	5.76
3C	3.17	3.17	1.16	3.31	6.92	9.76	11.69	14.79
3D	4.76	4.76	1.46	4.42	7.68	10.6	13.03	17.74
3E	6.35	6.35	3.46	7.39	11.6	16.4	20.64	24.23

Table 3. Maximum rotation (10^{-4} rad) obtained at the slabs of a twelve story high structure under torsion

Building	Center of gyration coordinates		Level											
	x	y	1	2	3	4	5	6	7	8	9	10	11	12
6A	0.00	0.00	0.31	0.89	1.44	1.20	2.35	2.56	2.79	3.09	3.67	3.82	4.14	4.62
6B	1.59	1.59	0.77	2.21	3.52	4.31	6.07	7.82	9.37	10.45	10.87	11.63	12.09	16.64
6C	3.17	3.17	0.86	2.29	4.50	7.00	9.70	12.42	14.99	17.35	19.41	21.10	22.49	24.24
6D	4.76	4.76	1.18	2.75	5.43	8.49	11.08	15.23	18.53	21.61	24.35	26.65	28.55	30.17
6E	6.35	6.35	1.97	7.00	13.96	22.10	31.17	40.75	50.31	59.50	68.00	75.42	81.76	87.20

Similar results were obtained for all other four structures under analysis and all of them indicated the same tendency of the torsional response: as the center of gyration moved toward the column at the corner, the rigid rotations of the slab increased; although not shown, the results showed an almost linear increase with height for the rotations induced at intermediate levels.

These motions also increased the mechanical elements at beams and columns of the structure under torsional excitation at their base.

MECHANICAL ELEMENTS FOUND WITH DIFFERENT AXIS OF ROTATION

Bending moments, axial forces, torsional moments and shear forces at all the structural beams and columns were computed from the displacement field, and maximum values were looked for, in order to understand the effect of the change in center of gyration, at all the structures under analysis.

All mechanical elements were highly dependent on the center of gyration position, when torsional excitation was introduced at the base of the structures under consideration. In all cases, all the mechanical elements increased as the center of gyration moved from the centroid to the corner column.

In order to illustrate this tendency, mechanical elements computed at the top and bottom end of column B-2, are shown at tables 4 and 5, for two of the buildings under analysis, when the same acceleration time series were applied at their base.

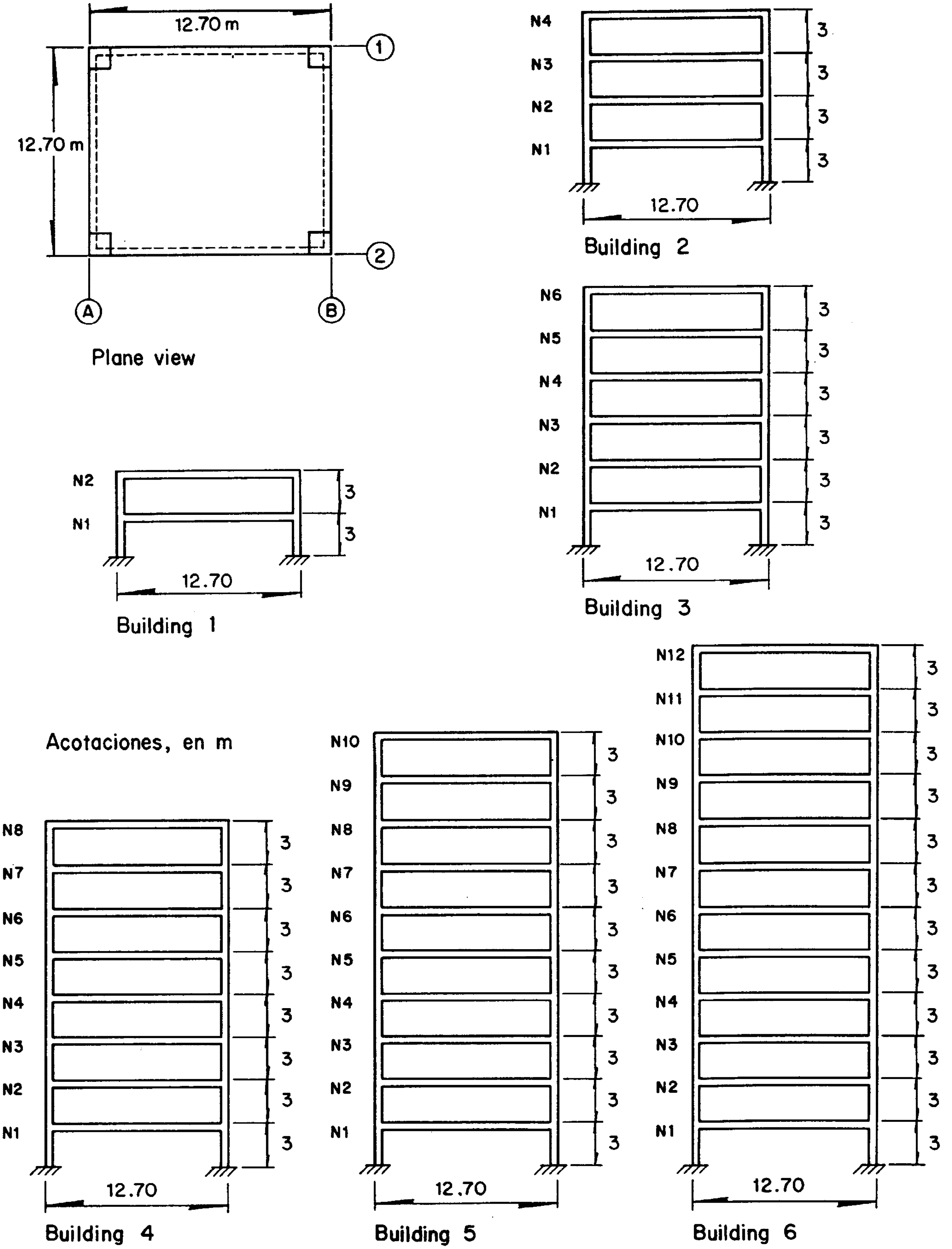

Fig. 2 . Tower-like structures selected to compute their torsional response, under seismic rotational excitation at their base

Table 4. Mechanical elements produced at both ends of columns B-2, at the lower level of a two story high building

Axis of rotation		Bending moments Dir. x		Bending moments Dir. y		Axial force	Torsional moments	Shear force Dir. x		Shear force Dir. y	
x	y	top	bot	top	bot			top	bot	top	bot
0.00	0.00	6.10	25.80	6.10	30.10	74	2.50	27.30	27.30	12.50	12.50
1.59	1.59	8.50	27.60	8.50	31.00	95	17.20	27.60	27.60	12.20	12.20
3.17	3.17	8.50	63.90	8.50	55.70	95	52.10	35.10	35.10	2.60	2.60
4.76	4.76	56.60	159.0	17.50	80.30	85	78.60	42.60	42.60	23.10	23.10
6.35	6.35	34.00	86.60	56.90	134.2	77	325.60	41.00	41.00	12.60	12.60
Moments are given in ton-m; forces in tons											

Table 5. Mechanical elements produced at both ends of columns B-2, at the lower level of a twelve story high building

Axis of rotation		Bending moments Dir. x		Bending moments Dir. y		Axial force	Torsional moments	Shear force Dir. x		Shear force Dir. y	
x	y	top	bot	top	bot			top	bot	top	bot
0.00	0.00	201	344	184	357	409	2.5	67.6	67.6	56	56
1.59	1.59	294	612	246	535	375	17.2	113.2	113.2	125	125
3.17	3.17	633	1481	459	759	217	52.1	107.0	107.0	302	302
4.76	4.76	1565	4961	1136	3445	943	78.6	905.2	905.2	1332	1332
6.35	6.35	266	702	2470	7507	2677	325.6	1975.0	1975.0	171	171
Moments are given in ton-m; forces in tons											

All the mechanical elements of the structures studied changed when the center of gyration moved from the centroid towards the corner column, along the diagonal. It was observed that the values were kept at low values, when the center of gyration was displaced from the centroid to the second position selected along the diagonal. This interval was named as central core of the foundation. When the center of gyration left the central core, all the mechanical elements increased their values.

These results suggest the existence of the central core in other directions; as the vertical axis of rotation leaves the central core, moments and forces at the structural elements increased; therefore, in order to understand the behavior of a regular structure, as the ones selected for this investigation, it is necessary to define the position of the vertical axis of rotation, if the torsional response of a structure is computed for design purposes.

IMPORTANCE OF THE FOUNDATION SCHEME ON THE STRUCTURAL RESPONSE

All the analyses carried out and their results, indicated the paramount importance of the foundation characteristics on the structural response. The selection of the type of foundation and the construction details used for the foundation , play an important role on the rotational and linear stiffnesses at the base of a building; those details also fix the position of the center of gyration; this position defines the general response of the structure, when torsional response is developed when seismic waves moves the building, during an earthquake.

The results obtained from the structural analyses, indicated the need for a rather stiff foundation to resist angular rotation in three orthogonal directions, in order to obtain a linear response of the superstructure. Also, it became evident the need of define the center of gyration position at the base, to get small displacements and rotations at the differents levels of the structure, as well as to control the mechanical elements generated during the motion generated by an earthquake.

Based on the evidence obtained, it became clear the existence of a central core at the foundation level, that controls the generation of small rotations and displacements at the upper floors of the structure; once the vertical axis of rotation abandons the central core, it was observed a rather steep increase of rotation and displacements, producing a remarkable increase of the mechanical elements computed at beams and columns, on regular structures without load eccentricities. Further studies on the amplitude of the central core should be carried on, on future research work.

It became evident from the work done, that the height of a building over the foundation level, plays an important role on the stable response of a structure with soil-structure interaction, as well as the weight of each floor, its mass polar of inertia around the center of gyration, and its height over the foundation.

Additional research efforts should be perform, for a better understanding on the angular stiffness of the soil surrounding different types of foundation, in function of soil properties obtained from common geotechnical studies. A review of values obtained from several authors' approach, showed high variation of numerical values obtained from their computation, when the same soil properties and foundation scheme were used; none of the published expressions take into consideration the relative position of the center of gyration with respect to the centroid, as control variable for computations.

FINAL COMMENTS

Stable flexo-torsional vibration of building with soil-structure interaction, are dependent on mass distribution along their height, as well as on the rotational moment of inertia around the vertical axis of rotation. Soil properties and foundation scheme selected at their base also play an important role on the resulting vibrations generated by earthquakes.

Regular buildings without any load eccentricity, may have linear vibrations under torsional excitation, depending on the rotational stiffnesses at their base. Inadequate selection of foundation scheme, may produce rotational instability or collapse of the building. Damage on structural elements of the superstructure is more liable, when the center of gyration, at the base, falls outside the central core.

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