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A SECOND ORDER ANALYSIS METHOD, SUITABLE FOR MODELLING INELASTIC BEHAVIOR OF STRUCTURES SUBJECTED TO EARTHQUAKES

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ABSTRACT

A new second order analysis method called *Geometric Load* Method is introduced, for the first time, in a context of dynamic inelastic analysis method and applied to plane frames. The objective is to enhance the modelling of non-linear inelastic behavior of steel structures subjected to earthquakes. Since existing methods employ geometric stiffness matrices, the analysis has to follow the load-deflection path, during the snap-through (post-buckling) region. For a path-dependent analysis, such as cyclic inelastic, the load reduction during snap-through region(s), unrealistically changes the loading history, and hence the prediction of stress/strain behavior. In the proposed method, second order effects are not considered in the calculation of stiffness matrices. In each static analysis, a series of iterative steps follows each load step to account for unbalanced forces produced by change of geometry. Consequently, the proposed method can continue with the loading, and produces more realistic results. Members are modelled as beam columns, with simple but comprehensive set of differential equations. For inelastic analysis plastic-zone method is used, while for dynamic loading, time history analysis is employed. The method is evaluated by calibrating its results with those from a static elastic second order analysis of the shallow circular arch problem, treated elsewhere.

KEYWORDS

Non-linear Analysis; Inelastic Analysis; Second Order Analysis; Geometric Non-linear Analysis; Cyclic Loading; Dynamic Analysis; Post-Buckling Analysis; Plastic-Zone Analysis; Steel Structures; P-Δ Effect.

INTRODUCTION

Cyclic nature of earthquake loading demands different type of resistance in structures. Researchers have been working to find and compare different types of earthquake resistant structures. Most of the work in this area has been on actual tests rather than analytical modelling. UBC (1988) recognises *Special Moment-Resisting Space Frames* (*SMRSF*) as reliable ductile systems. The most difficult task faced by such researchers is to find ways to balance the strength of columns, beams and panel zones of buildings, in such a way to prevent their premature collapses. Topic of *column hinge formation vs. beam hinge formation* has been the source of controversial discussion among engineers and researches which indicates the need for further work in this area. As search continues for finding better alternatives, attention was paid to bracing systems. While concentric bracing systems are not ductile, eccentric bracing systems (shear links) are promising alternatives as they may utilise material more economically, from energy absorbent point of view. Since experimental testing is expensive, only limited number of tests may be conducted, which is insufficient to provide conclusive information. On the other hand, since inelastic design of structures has not reached its maturity phase, building codes avoid direct inelastic analysis. Based on the above discussion, it is most desirable to develop an analytical model that can predict the behavior of structures subjected to earthquakes, as closely as possible. Such a system simulates the behavior of a structure during earthquakes up to its collapse, and evaluates its overall performance as well as the performances of its members.

With above objective in mind, the first author started a research program, on modelling non-linear inelastic behavior of plane steel frames subjected to earthquakes. Such modelling could be used as a substitute for costly tests. Similar attempts have been made by Challa and Hall (1994), Prakash (1992) and Allahabadi (1987). The reason for choosing steel is the fact that it is extremely ductile material compared to concrete (or even reinforced concrete). Such a research also establishes basis for the three dimensional modelling. While the existing second order methods are not suitable for inelastic analysis, as will be explained, the method proposed in this paper is suitable and crucially important for such analysis. Being original, the method has been independently developed and will be explained in more detail in (Nassiri, to be published).

Since existing methods employ geometric stiffness matrices, explained in the next section, analysis has to follow the load-deflection path, which means, the load needs to be reduced during snap through region(s). Since in cyclic inelastic loading, stress/strain behavior is dependent on the history of the loading (path dependant), such a load reduction unrealistically changes loading history, which consequently affects prediction of stress/strain behavior. One such example is frequent inelastic buckling of bracing system without collapse of the whole building. In the proposed method, second order effects are not included in the calculation of stiffness matrices. Unbalanced forces are calculated and applied separately to the structure. Therefore, the proposed method is superior to the previous methods, since loads do not need to be reduced. This new method may be called *Geometric Load* method, in future references as opposed to all methods based on the *Geometric Stiffness Matrix* approach. The proposed new method also is easier to implement and more accurate, since it captures all the second order effects (including P-Δ and P-δ). Other aspects of the method including the set of employed differential equations add to its accuracy. The method is initially applied to the analysis of plane frames, but it may be extended to three dimensional finite element structural systems.

In this paper, the main focus is on elastic second order analysis which may be used as the basic and crucial tool for dynamic and/or inelastic analysis. Other issues regarding realistic modelling of steel structures subjected to earthquakes, including stress/strain modelling, treatment of flexible joints (panel zones), and effect of shear strains in member deflections are under investigation.

REVIEW OF STATIC ANALYSIS METHODS

Studies on limit state design of structures have shown that first order analyses can not closely predict building behaviors. On the other hand studies on earthquakes show that the so called P-Δ effects are greatly responsible for building failures. The second order inelastic-zone method based on the finite element approach is known to be the best static analysis method that closely models inelastic behavior of structures (Ziemian, 1990). In inelastic (plastic) zone method, line members of structures are divided into segments, while their cross sections are divided into smaller areas, resulting in discretisation of structures into fibres as shown in Figs. 1 and 2 (White, 1985; Morales, 1994). Each fibre in this type of analysis has its own stress/strain history. Tangent modulus (E_t) of such fibres are obtained, based on the constitutive stress/strain models (Chen and Han, 1988).

To account for second order effects, geometric stiffness matrices are calculated, using energy (work) based methods. These methods are extensively covered in the literature (Bathe, 1982; Bergan *et al.*, 1978; Chen 1991). Two widely used approaches are *Total Lagrangian* and *Updated Lagrangian* methods. Because of the nonlinearity of the problem, incremental methods are mostly used for static analysis. To solve the incremental equations of equilibrium, Newton-Raphson method is employed using load controlled steps. Since this method fails near the limit points, several other methods have been developed to follow load-deflection path in the pre- and post-critical point regions (such as snap-through and snap-back). These methods include displacement control method, arc-length method, work control method (Bathe, 1982; Bergan *et al.*, 1978; Chen, 1991; Clarke and Hancock, 1990).

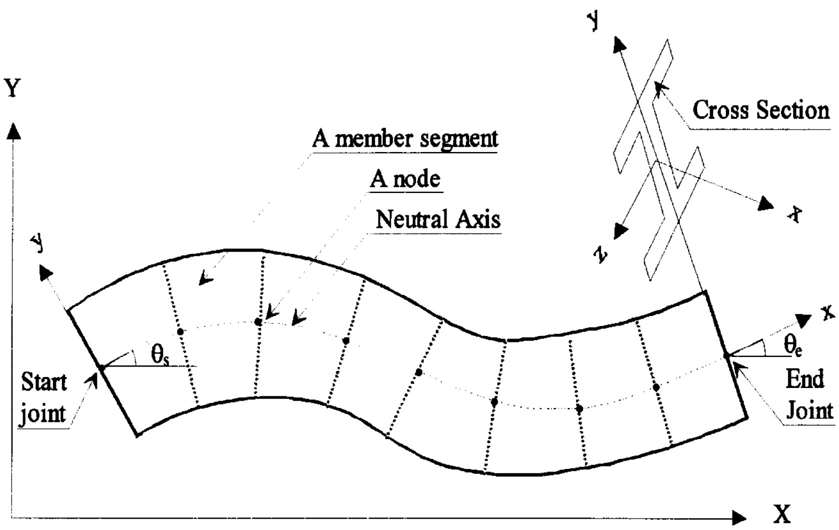

Fig. 1. Global (X, Y, Z) and Local (x, y, z) Cartesian coordinates.

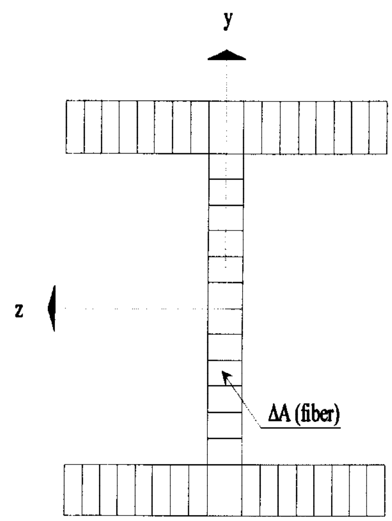

Fig. 2. Discretisation of member segments.

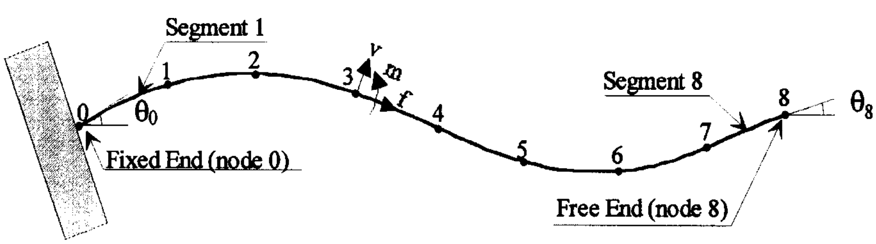

Fig. 3. Cantilever beam-column discretised into 8 segments.

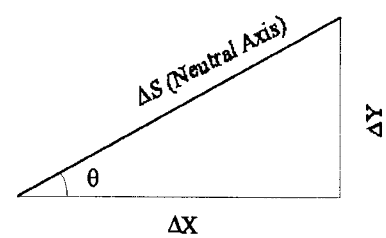

Fig. 4. Beam-Column increment.

DESCRIPTION OF THE PROPOSED METHOD

The proposed method is applied to plane frame analysis (which may be extended to any finite element structural system) based on the following assumptions and approaches.

- Each line member is treated as a beam-column. The principle of beam theory applies which implies after bending cross sections of members remain plane. Effect of shear strains in member deflections are ignored in this study.

- No out of plane behavior is considered in this study. For this reason, the cross section is assumed to be symmetrical in regards to local y axis (Fig. 2), which is significant in inelastic analysis.

- The right hand Cartesian global (X, Y, Z) and local (x, y, z) coordinate systems are chosen, in which the local x axis is tangent to the neutral axis, the local y axis is the axis of symmetry of the cross section, while the local z is parallel to global Z (Figs. 1 and 2). Member stiffness are calculated based on the global coordinate system.

- All external loads are defined in the global system providing that loads retain their original direction at all times, even when the structure is dramatically deformed (Fig. 1). However, internal forces (axial and shear forces as well as bending moments) on member sections (at the intersection with neutral axis) are calculated in the local system (Fig. 3). Such a system helps to accumulate unbalanced forces, and apply them slowly using small steps (without losing their validity).

- For any differentiation or integration along a member, parameter s, length of the curved member, is used instead of local x. This increases accuracy in the case of extreme member deflections.

- Each member is divided into certain number of segments. Such a member, from global point of view, is assumed to be one member having one start joint and one end joint. This means the segments of each member could be used to calculate the member stiffness matrix and member end forces, but will not directly participate in the assemblage of global stiffness matrix. The intermediate points along each member representing ends of segments are called nodes in this study (Fig. 3).

Differential Equations of Beam-Column *Deflections*

An increment of beam column is shown in Fig. 4. Before the load step, $\Delta X = \Delta s.\cos\theta$ and $\Delta Y = \Delta s.\sin\theta$, while after the load step, $\Delta X+\delta X = (\Delta s+\delta s)\cos(\theta+d\theta)$ and $\Delta Y+\delta Y = (\Delta s+\delta s).\sin(\theta+d\theta)$. In these equations, δ is a shorthand symbol showing combination of d and Δ [eg. δX means $d(\Delta X)$], while ε_n is total axial (normal) strain at a point along the neutral axis. Combining the above equations and using $\delta s = \Delta s.d\varepsilon_n$.

$$\delta X/\Delta s = \cos(\theta+d\theta) - \cos\theta + d\varepsilon_n.\cos(\theta+d\theta) \qquad (1)$$

$$\delta Y/\Delta s = \sin(\theta+d\theta) - \sin\theta + d\varepsilon_n.\sin(\theta+d\theta) \qquad (2)$$

Equations 1 and 2 give change of ΔX and ΔY relative to Δs for large load steps. For small steps it may be assumed that $\sin d\theta = d\theta$, $\cos d\theta = 1$, and $d\varepsilon_n.\sin d\theta = 0$. Replacing these terms with their corresponding values in 1 and 2.

$$\delta X/\Delta S = -\sin\theta.d\theta + \cos\theta.\,d\varepsilon_n \qquad (3)$$

$$\delta Y/\Delta S = \cos\theta.d\theta + \sin\theta.\,d\varepsilon_n \qquad (4)$$

In this case, displacements remain linear during each load step. Equations 3 and 4 are simple and comprehensive, comparing to conventional beam-column differential equations given in the literature. These equations are used in the method, since it is based on small steps.

Cross Sectional Properties

Effective properties for cross sections are calculated as follows.

$$\bar{A}=\frac{1}{E}\sum_{i=1}^{i=N}(E_t\Delta A)_i \quad ; \quad \bar{Q}=\frac{1}{E}\sum_{i=1}^{i=N}(E_t y\Delta A)_i \quad ; \quad \bar{I}=\frac{1}{E}\sum_{i=1}^{i=N}(E_t y^2\Delta A)_i$$

In the above equations, E and E_t are modulus of elasticity and tangent modulus of elasticity of increment i, respectively, while N is the number of increments of area on the cross section. Similar equations are given in (White, 1985; Clarke and Hancock, 1989).

Beam-Column Cantilever Model

The model of beam-column cantilever is used as the basic model to calculate member stiffness matrices and member fixed end forces. Member deflections are calculated using numerical integration technique. Values at the nodes along each member are maintained and used for integration. Three point Gausian quadrature method is used for numerical integration, with possibility of unequal segment lengths (originally and/or after the loading). The following equations are employed in the procedure.

$$\begin{Bmatrix} df \\ dm \end{Bmatrix} = E \begin{bmatrix} \bar{A} & -\bar{Q} \\ -\bar{Q} & \bar{I} \end{bmatrix} \begin{Bmatrix} d\varepsilon_n \\ d\phi \end{Bmatrix} \quad ; \quad \begin{Bmatrix} d\varepsilon_n \\ d\phi \end{Bmatrix} = \frac{1}{E(\bar{A}\bar{I}-\bar{Q}^2)} \begin{bmatrix} \bar{I} & \bar{Q} \\ \bar{Q} & \bar{A} \end{bmatrix} \begin{Bmatrix} df \\ dm \end{Bmatrix}$$

$$d\theta_j = d\theta_{j-1} + \int_{segment_j} [(d\phi)\, dS] \quad ; \quad dX_j = dX_{j-1} + \int_{segment_j} \left[\left(\frac{\delta X}{\Delta S} \right) dS \right]$$

$$dS_j = dS_{j-1} + \int_{segment_j} [(d\varepsilon_n) dS] \quad ; \quad dY_j = dY_{j-1} + \int_{segment_j} \left[\left(\frac{\delta Y}{\Delta S} \right) dS \right]$$

In the above equations, f and m are total axial force and bending moment, while θ shows total angle that local x axis makes with global X axis, and ϕ represents member curvature, at a node along the member.

Treatment of Different Loading Types

Loading are treated at different levels, where dynamic analysis is at the highest level. Dynamic analysis is performed as a time history analysis, where the acceleration is assumed to change linearly during each time step. The method is unconditionally stable for elastic analysis, but for inelastic analysis, further investigations are needed. No out of balance energy in this process is measured or controlled as explained in (Allahabadi, 1987).

Each static loading whether being a direct static loading or the loads produced by one time step of dynamic loading, is analysed based on the incremental analysis. In this method each incremental loading consist of one external loading step plus zero or more correctional loading steps.

Since in this method second order effects are not accounted for, in the calculation of tangent stiffness matrices, correctional (geometric) load steps are applied to the frame, in which, each load step is a fraction of existing unbalanced forces. Such forces are accumulated after application of each load step as functions of change in the geometry of the nodes. Within each incremental loading, correctional loading is continued in small steps until total unbalanced forces are negligible. Since unbalanced forces are in terms of internal forces defined in local systems, any deflections during the delay in their applications have no effect on their validity. As long as external loads are not extremely large, such simple iterative process always converges. During buckling, unbalanced forces are increased after each correctional step, causing divergence from equilibrium, but either such forces subside after a while, or the building collapses. However, in the unlikely case that loads (hence internal forces) become extremely large, before the building collapses, different methods may be used to prevent divergence. As far as dynamic analysis is concerned, it is assumed that all the sub steps of each time step are performed in no time and finished within such a step (even during buckling).

Each loading step, whether being external or correctional is analysed using conventional stiffness method, with some exceptions as will be explained. For each loading step, a fraction of set of existing loads are applied (load control method). Different means may be used to control the load factor, including increments of fibre axial strains ($d\varepsilon$) and/or increments of node curvatures ($d\phi$). For inelastic analysis, fibres' tangent modulus's of elasticity (E_t) and members' cross sectional properties are updated, at the beginning or end of each step. Emphasise is on simplicity, using small load steps. Therefore material properties (and hence cross sectional properties) are assumed to remain constant during each step. It is further assumed that no unloading (stress/strain reversal) happens during each step. In addition, no event-to-event approach is used to control steps as explained in (Allahabadi, 1987), and no correction is made after each step for any discrepancy which may exit, since such differences are extremely small. For greater accuracy, no closed form expressions are assumed for stiffness matrixes, since using such expressions may compromise accuracy. These matrices are obtained during each step, using the beam-column cantilever model as was mentioned in the relevant section.

The provisions are made to have more than one loading present (eg. on a building, dead loads are applied as the first loading, then earthquake loads are applied as the second loading, which may be used to test effect of P-Δ on the building).

EVALUATION OF THE PROPOSED METHOD

To evaluate the method, an example is taken from (Clarke and Hancock, 1990) as shown in Fig. 5a. This problem is widely used in the literature for the complex behavior it shows during snap through region (Fig. 5b). The results produced by the proposed method are depicted in Fig. 6, which closely match the results given in Fig. 5b. To verify the accuracy, starting from no load (point *A*), the arch was loaded up to 2000 force unit downward (point *D* on the curve), then unloaded and reloaded (loading reversed) up to 1200 force unit upward (point *G*), then unloaded again to no load (point *H*). Points *A* and *H* are theoretically supposed to match (since the problem is elastic), but there is about 5 length unit (1/2000 of total span) gap, which is relatively a small error. Lines *BC* and *EF* correspond to the buckling regions. Following the loading and unloading paths, and connecting significant points produces the sequence *ABCDCEFGFH*. As it can be seen paths *CD* matches with *DC* and *FG* with *GF*. The error produced in the (loading-unloading) process, is neither the real error for real case loading, nor may reflect errors associated with linear analysis. However, it may reflect errors directly associated with non-linear analysis, and may cautiously be used to compare performances of different second order methods of analysis. Other details of the example are as follows. The whole arch was discretised into 50 segments. The limits to control load factor were chosen .0001 and .00005/(length unit) for $d\varepsilon$ and $d\phi$, respectively. The related computer program is written in *C* on *IBM* compatible *PC*. A special graphical representation of building deflections can produce animation of its movements. This problem only takes a few minutes to run on a 486-*DX*2 / 66 *Mhtz*.

CONCLUSION

A new second order analysis method called *Geometric Load* Method is introduced to facilitate accurate inelastic analysis of structures. The objective has been to enhance modelling of non-linear inelastic behavior of steel structures subjected to earthquakes. The results of static elastic second order analysis are verified for the shallow arch problem. The results are in excellent agreement with the results given elsewhere. The process of loading and unloading (to the original no load case) was used to determine the error, which is recommended to be cautiously used to compare performances of different methods of second order analysis. The method is straight forward, and easy to implement. In addition, it may be extended to any other finite element structural system. In the unlikely case of extreme loading, before collapse of the building, solutions are available to prevent divergence during correctional steps, which are not discussed in this paper. Other relevant issues such as stress/strain modelling, treatment of flexible joints (panel zones), effect of shear strains in member deflections, etc. are being investigated.

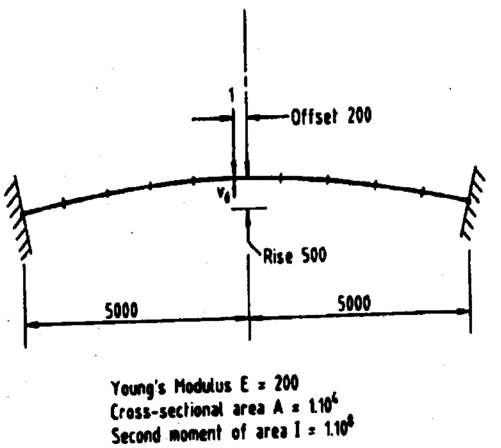

Fig. 5a. Geometry and loading. Both ends of the arch are pinned.

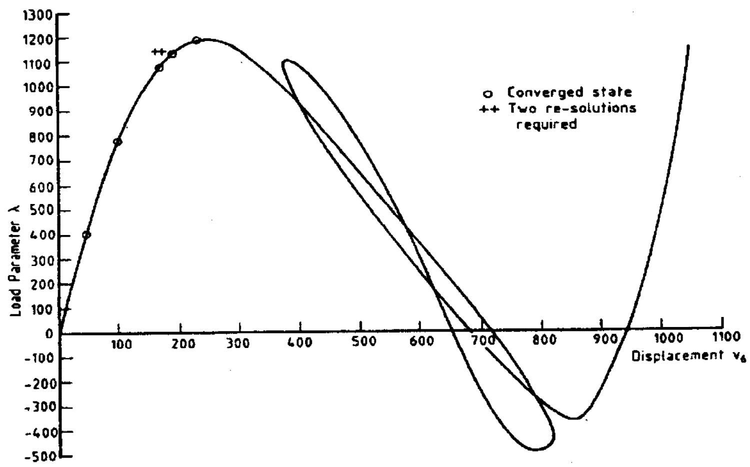

Fig.5b. Computer converged states-Solution 1.

Fig. 5. Shallow circular arch subject to near central point load from (Clarke and Hancock, 1990).

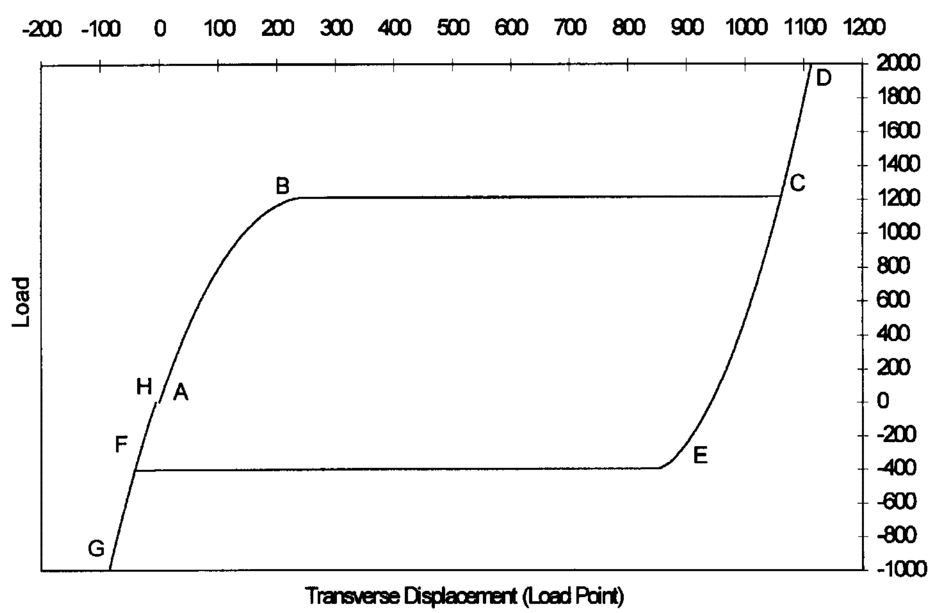

Fig. 6 Load-Deflection curve produced by the proposed method for the example problem (shallow circular arch).

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