11 WCEE
Paper No. 619. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

DETERMINATION OF MAXIMUM STRUCTURAL RESPONSE TO TWO HORIZONTAL GROUND MOTION COMPONENTS APPLIED ALONG ANY ARBITRARY DIRECTIONS FOR APPLICATION TO BUILDING CODES

O.A. López and R. Torres

IMME, Facultad de Ingeniería, Universidad Central de Venezuela
Apartado 50.361, Caracas 1050-A, Venezuela

ABSTRACT

A simple method which can be applied in building codes, is proposed to calculate the critical angle of incidence of one or two horizontal seismic ground motions and the associated maximum structural response. The ground motion components are given in terms of response spectra which may have different or identical spectral shape. The method requires the solution of standard cases of linear dynamic analysis and therefore can be easily implemented in existing computer programs. The method gives results that are more accurate than other existing methods. For the special case of identical spectra along the principal ground motion directions, the maximum structural response is independent on the angle of incidence. The accuracy of the method is numerically demostrated by applying it to the case of symetric and asymetric buildings.

KEYWORDS

Earthquake direction, critical angle, maximum structural response, dynamic analysis, building codes.

INTRODUCTION

The determination of the critical direction of seismic incidence that yields the maximum structural response, has been presented in several papers. Wilson and Button (1982) presented a simple method to determine the critical angle, but did not take into account the correlation of ground motion components when they act along the strutural principal directions. Smeby and Der Kiureghian (1985) using random vibration theory developed a formula to determine the critical angle for the case of two horizontal ground components with identical spectral shape that takes into account the proper correlation between seismic components. González (1992) presented an approximate method to include the effect of seismic direction in the dynamic analysis of buildings. The main objective of this paper is to develop a simple method which can be applied in building codes to determine the critical angle and the associated maximum structural response, for the general case of two horizontal ground motion components that may have different or identical spectral shape.

MAXIMUM STRUCTURAL RESPONSE TO TWO GROUND MOTION COMPONENTS ACTING ALONG AN ARBITRARY DIRECTION

Figure 1 illustrates the case of a building subjected to the simultaneous action of two orthogonal horizontal ground accelerations in directions 1 and 2. The component a_1 (t) forms an angle θ with the X axis; X and Y are the reference axis of the building. Directions 1 and 2 are the principal ground acceleration directions (Penzien and Watabe, 1975); therefore, ground components a_1 and a_2 are uncorrelated and are associated to the direction of maximum and minimum intensity, respectively, in terms of variances. Sa_1 and Sa_2 are the acceleration response spectra for the ground components a_1 and a_2 , respectively.

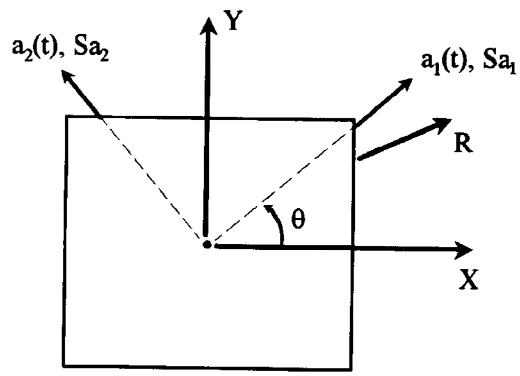

Fig. 1. Building subjected to two ground acceleration components

Let R be the maximum probable dynamic response to the simultaneous actions of spectra Sa_1 and Sa_2, of any response parameter such as displacement, stress, force, etc. For linear systems, the maximum dynamic modal response (R_i^1) to spectrum Sa_1 in the i^{th} mode of vibration, can be written as (López and Torres, 1993):

$$R_i^1 = R_i^{1x} \cos\theta + R_i^{1y} \sin\theta \qquad (1)$$

where R_i^{1x} and R_i^{1y} are the modal response values calculated when the spectrum Sa_1 acts along the reference axis of the building, X and Y, respectively. It should be pointed out that the algebraic sum comes from the fact that ground components along directions X and Y are totally correlated.

For spectrum Sa_2, the maximum dynamic response (R_i^2) in the vibrational mode i^{th} is given by:

$$R_i^2 = R_i^{2y} \cos\theta - R_i^{2x} \sin\theta \qquad (2)$$

where R_i^{2x} and R_i^{2y} are the modal response values calculated when spectrum Sa_2 acts along the building reference directions X and Y, respectively.

The maximum probable dynamic response (R^1) to the seismic component 1 is obtained combining the modal responses including modal correlation (Rosemblueth and Elorduy, 1969; Der Kiureghian, 1981):

$$R^1 = \left[\sum_i \sum_j C_{ij} R_i^1 R_j^1 \right]^{1/2} \qquad (3)$$

and for seismic component 2:

$$R^2 = \left[\sum_i \sum_j C_{ij} R_i^2 R_j^2 \right]^{1/2} \qquad (4)$$

For not being correlated the seismic components 1 and 2, the maximum probable dynamic response (R) to the simultaneous actions of both components is given by:

$$R = \left[(R^1)^2 + (R^2)^2\right]^{1/2} \tag{5}$$

From equations (1) to (5), the response R is obtained as a function of the angle of incidence (θ):

$$R = \Big\{\left[(R^{1x})^2 + (R^{2y})^2\right] \cos^2\theta + \left[(R^{1y})^2 + (R^{2x})^2\right] \sin^2\theta + 2 \sin\theta \cos\theta \left[\sum_i\sum_j C_{ij} R_i^{1x} R_j^{1y} - \sum_i\sum_j C_{ij} R_i^{2y} R_j^{2x}\right] \Big\}^{1/2} \tag{6}$$

where,

$$R^{1x} = \sum_i\sum_j C_{ij} R_i^{1x} R_j^{1x} \tag{7}$$

$$R^{1y} = \sum_i\sum_j C_{ij} R_i^{1y} R_j^{1y} \tag{8}$$

$$R^{2x} = \sum_i\sum_j C_{ij} R_i^{2x} R_j^{2x} \tag{9}$$

$$R^{2y} = \sum_i\sum_j C_{ij} R_i^{2y} R_j^{2y} \tag{10}$$

The function R(θ) in equation (6) is periodical, with period equal to 180°. For $\theta \neq 0$, the equation (6) includes the effect of correlation of the seismic components acting along the reference structural directions (X, Y).

Critical Angle

The critical angle (θ_c) is defined by the value of θ that renders the maximum value of R in equation (6):

$$\theta_c = \frac{1}{2} \tan^{-1} \left\{ \frac{2 \sum_i\sum_j C_{ij} \left[R_i^{2y} R_j^{2x} - R_i^{1x} R_j^{1y}\right]}{(R^{1y})^2 + (R^{2x})^2 - (R^{1x})^2 - (R^{2y})^2} \right\} \tag{11}$$

Therefore, equation (11) gives two roots for θ_c, separated 90° between each other, that define the maximum and the minimum values of the structural response R in equation (6).

Proposed Method of Analysis

Next we present a summary of the steps to follow in order to determine the critical angle of incidence and the corresponding maximum structural response to two different horizontal spectra applied along any arbitrary directions 1 and 2 that form an angle θ with the building reference directions X and Y (Figure 1): 1) Solve the four basic cases of dynamic analysis: applied spectrum 1 along direction X to calculate the response value R^{1x}, and then along direction 2 to obtain the response R^{1y}. Similarly, apply spectrum 2 to obtain R^{2x} and R^{2y}. Modal response values for all four cases are kept for the modes of interest; 2) the two critical values of the angle θ are determined from equation (11); 3) for each of the two angles obtained, the maximum probable response is determined from equation (6). The greatest value is the maximum response for all possible angles of incidence of the ground motion components.

THE CASE OF SPECTRA WITH IDENTICAL SHAPE

Let $Sa_2 = \alpha\, Sa_1$ where the spectral ratio α is a number between 0 and 1. Then, the maximum resonse values defined before are simplified as follows: $R^{2x} = \alpha\, R^{1x}$, $R^{2y} = \alpha\, R^{1y}$, and also the modal response values, $R_i^{2y} = \alpha\, R_i^{1y}$ and $R_i^{2x} = \alpha\, R_i^{1x}$. Substituting in equation (6), the maximum probable dynamic response R is given by López and Torres (1993):

$$R = \Big\{\left[(R^{1x})^2 + (\alpha R^{1y})^2\right]\cos^2\theta + \left[(\alpha R^{1x})^2 + (R^{1y})^2\right]\sin^2\theta + 2\sin\theta\cos\theta(1-\alpha^2)\sum_i\sum_j C_{ij} R_i^{1x} R_j^{1y}\Big\}^{1/2} \tag{12}$$

The critical angle for $\alpha \neq 1$ is given by:

$$\theta_c = \frac{1}{2}\tan^{-1}\left\{\frac{2\sum_i\sum_j C_{ij} R_i^{1x} R_j^{1y}}{(R^{1x})^2 - (R^{1y})^2}\right\} \tag{13}$$

In the particular case of horizontal spectra of equal intensity ($\alpha = 1$), the critical angle is undefined.

Equation (13) gives two critical angles, separated 90° between each other, that substituted in equation (12) lead to the maximum and the minimum structural response. It should be pointed out that the critical angle does not depend on α; this is, whenever spectra in directions 1 and 2 have identical shape, the critical directions of incidence is the same, whether it is considered one or two simultaneous seismic components.

It is possible to demonstrate that the equations (12) and (13) are identical to the ones previously found by Smeby and Der Kiureghian (1985) from the use of concepts of ramdom vibrations. The equations (12) and (13) differ from the ones presented by Wilson and Button (1982) in that we have incorporated here the effect that in each mode has the simultaneous action of two seismic components that are totally correlated. The proposed formulas of the named reference can be obtained from equation (12) if only the effect of one mode of vibration is considered.

Proposed Method of Analysis

The steps to determine the critical angle of incidence of the corresponding maximum structural response when spectra along directions 1 and 2 have identical shapes, are: 1) solve two basic cases of dynamic analysis, with spectrum Sa_1 acting first in direction X and later in direction Y; the corresponding maximum probable dynamic responses are denoted by R^{1x} and R^{1y}, respectively. Modal response values R_i^{1x} and R_i^{1y} are kept for the modes of interest; 2) the two critical values of the angle of incidence are determined from equation (13); 3) determine two maximum probable responses from equation (12) for each of the two critical angles. The greatest value is the maximum response for all of the incidence angles.

The Case of Equal Spectra Applied in the Directions 1 and 2

The maximum probable response for the particular case in which the same spectrum acts in the two horizontal directions 1 and 2, is obtained making $\alpha = 1$ in equations (12):

$$R = \left\{ (R^{1x})^2 + (R^{1y})^2 \right\}^{1/2} \tag{14}$$

from which we can observe that the maximum response does not depend on the value of the angle of incidence; any value of θ is a critical angle. For that reason, it is sufficient to analyze the typical case of $\theta = 0°$ to determine the maximum structural response to any angle of incidence. This outcome, surprising in some way, has application in seismic design because the value of R given by equation (14) is the maximum of all possible responses. If a more detailed analysis is required to avoid being so conservative, then a value of the spectral ratio α should be adopted and equations (13) and (12) should be used to obtain the critical angle and the associated maximum structural response as indicated before.

NUMERICAL VERIFICATION

The one story reinforced concrete building shown in Figure 2 is asymmetrical in both directions with excentricities $e_x = 0.33\ B_x$ and $e_y = 0.38\ B_y$. Beam and column sections are 40 x 60 cm^2 and 60 x 60 cm^2, respectively. Story weigth is 65 tons. Damping ratio is 5 % for all modes. Story heigth is 3 meters. Natural periods for the three modes are 0.20, 0.16 and 0.07 sec. Spectrum in direction 1 is a flat spectrum with acceleration equal to 0.66 g. The spectrum in direction 2 has identical shape than in direction 1, with spectral ratios α equal to 0, 0.5 and 1. All of the required dynamic analysis were performed with the computer program SAP-90. The structral response (R) is determined in terms of the shear force at the selected column indicated in Figure 2.

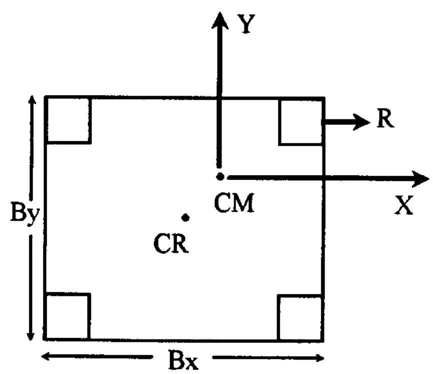

Fig. 2. One story building. CM=center of mass, CR=center of rigidity. B_x =6 m, B_y=8 m

The building was analyzed for each possible angle of seismic incidence in the interval [-90°, + 90°] according to the three methods that follow : a) Exact; a complete dynamic analysis was performed for each angle of incidence, with intervals of 10°. The modes were combined according to the CQC criteria (Wilson et al, 1981); b) WB; analysis with the method given by Wilson and Button (1982); c) Proposed: Analysis with equation (12). The correlation coefficients C_{ij} are given by Wilson et al (1981).

In the proposed method, only two dynamic analysis were performed to determined the shear force at the column: one with the given spectra applied along the X direction ($\theta = 0°$) and the other applied along the Y direction ($\theta = 90°$). The modal response vlues (Kg) are: $R_1^{1x} = 11540$, $R_2^{1x} = 4494$, $R_3^{1x} = -41$, $R_1^{1y} = -9311$, $R_2^{1y} = 5735$ and $R_3^{1y} = 24$. The modal correlation coefficients are : $C_{12} = 0.155$, $C_{13} = 0.007$ and $C_{23} = 0.013$. The maximum probable dynamic responses are $R^{1x} = 13003$ and $R^{1y} = 10150$. According to equation (13), the two critical angles are -33,5° and 56,5°.

The maximum probable dynamic shear force at the selected column is shown in Figure 3 for each angle of incidence. It is observed that the results obtained with the proposed method are identical to the ones of the exact method, and differ from the WB method. Furthermore it is convenient to emphasize that the numerical results given by the exact method confirm the independence of the maximum response from the angle of incidence for the case of identical spectra (α = 1) in both horizontal ground motion components. Also the numerical results confirm that the critical angle is independent of the spectral ratio α; this aspect has been pointed out previously by Smely and Der Kiureghian (1985). Additional and detailed numerical verifications of the proposed method are given by Torres (1996).

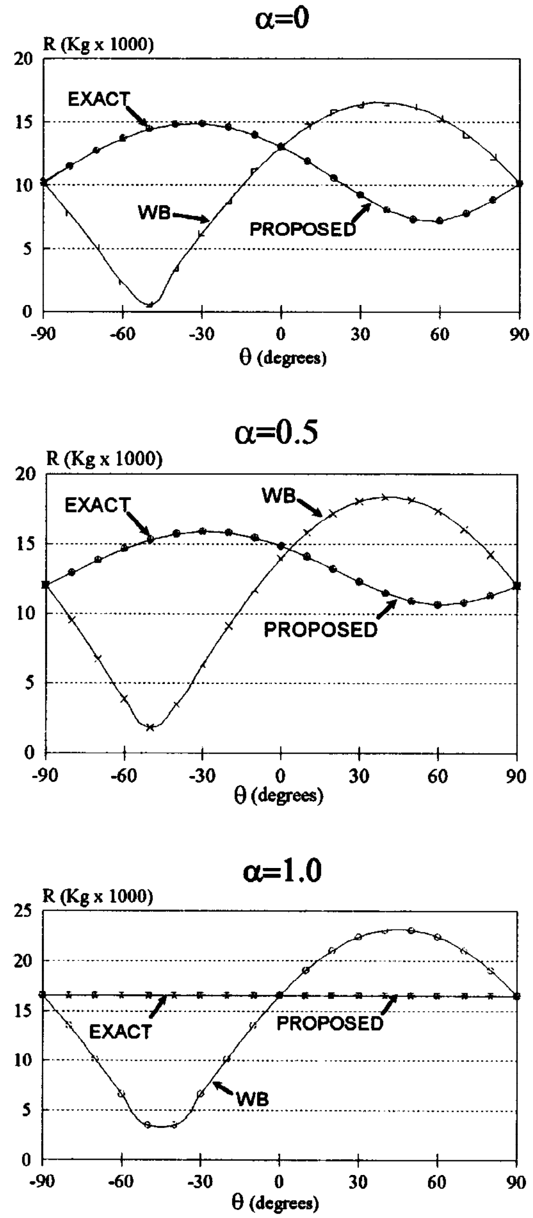

Fig. 3. Shear force at selected column of building for any angle of incidence for the spectral ratios α = 0, 0.5 and 1

CONCLUSIONS

The proposed method to calculate the maximum probable dynamic response associated to the critical angle of incidence of one or two seismic horizontal ground motion components, requires only the solution of standard cases of dynamic analysis with the specified spectra acting along the main (or reference) building directions. The spectra for both components may have different or identical shape. The method is accurate, its validity has been proved by numerical results, and simple to use; it can be easily implemented in existing computer programs for practical applications or included in seismic codes for the design of irregular structures where these effects are significant. For the special case of identical spectrum along the principal ground motion directions, the maximum structural response is independent of the angle of incidence; this response values is an upper bound to all of responses for any spectrum ratios and angle of incidence.

REFERENCES

Der Kiureghian A. (1981). A response spectrum method for random vibration analysis of MDF systems. Earthq. Eng.and Struct. Dyn., **9**: 419-435.

González P. (1992). Considering earthquake direction on seismic analysis. Earthquake engineering. Proc. Tenth world Conf. Earth. Eng., Madrid, Spain, VII: 3809-3813.

López O. A. y R. Torres (1993). Determinación del ángulo de incidencia del movimiento sísmico para el análisis estructural. VIII seminario latinoamericano de ingeniería sísmica. Mérida, Venezuela, A 88-A100.

Penzien J. and M. Watabe (1975). Characteristics of 3-dimensional earthquake ground motions. Earthquake engineering and structural dynamics, **3**:365-374.

Rosemblueth E. and J. Elorduy (1969). Response of linear systems to certain transient disturbances. Proc. 4th world Conf. on Eng. Chile, I:185-196.

Smeby W. and A. Der Kiureghian (1985). Modal combinations rules for multicomponent earthquake excitation. Earthq. Eng. and Struct. Dyn. **13**:1-12.

Torres R. (1996). Influencia de la direccionalidad del movimiento sísmico sobre la respuesta dinámica de estructuras. Trabajo de grado para optar al título de Magister Scientarium. IMME, Fac. de Ingeniería, Universidad Central de Venezuela.

Wilson E. L., A. Der Kiureghian and E. T. Bayo (1981). A replacement for the SRSS method in seismic analysis. Earthq. Eng. and Struct. Dynamics, **9**: 187-194.

Wilson E. and M. Button (1982). Three-dimensional dynamic analysis for multicomponent earthquake spectra. Earthq. Eng. and Struct. Dyn. **10**: 471-476.