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SPECTRAL SUPERPOSITION UNDER TWO-DIRECTIONAL EXCITATION

JORGE VASQUEZ

Department of Structural and Geotechnical Engineering, Catholic University of Chile,
P.O. Box 306, Santiago 22, CHILE

ABSTRACT

A rational method for modal spectral analysis of structures under simultaneous action of two horizontal components of motion is presented. The method, based in the construction of a family of parametric spectral curves, can also be used to obtain the most unfavorable orientation of two-directional excitation. Construction of parametric spectral families is discussed. Spectral principal directions in an approximate sense were found. Applications show the significance of two-directional excitation. Results compared with standard code recommendations for orthogonal effects show unconservatisms by factors of over 1.5.

KEYWORDS

Two-directional excitation; orthogonal effects; spectral superposition; parametric spectra; design spectra; principal directions of motion.

INTRODUCTION

Some specialized codes require the use of multi-component design earthquake excitation (USNRI, 1976, API, 1979). Eventually all codes might require at least a two-directional approach. A multi-directional time-history analysis can be readily done, and maximum values of any response component can be easily computed (Vasquez, 1987). In modal spectral superposition, however, the problem is by no means trivial. Attempts have been made (Vasquez, 1987; Gupta, 1990; Yamamura and Tanaka, 1990) to use certain *principal directions* of the strong motion phase, as defined by Penzien and Watabe (1975), to justify a square root of the sums of the squares type of superposition estimator. However, the Penzien and Watabe principal directions are based in a low correlation of the corresponding components, so that interpreting their existence as an argument for invoking a sort of statistical independence or orthogonality of the two horizontal components of motion, is questionable. In one-directional excitation, the problem of finding the excitation orientation for maximum value of a given response variable is relatively straightforward (Vasquez, 1987), and can be solved analytically. However, extensions to two-directional excitation that have been proposed are not rational, and can be justified only through analogy with the results for the one-directional case. In a previous work (Vasquez et al., 1993) the concept of parametric spectra was introduced, as a means to obtain spectral superposition estimates of response under the simultaneous action of the two horizontal components of earthquake excitation. In the present paper the use of this concept is extended to the important problem of finding the most unfavorable direction of earthquake excitation Also some of the difficulties in the choice of appropriate signs of spectral readings from the parametric curves are addressed.

RESPONSE UNDER A TWO-DIRECTIONAL EXCITATION

The equations of motion of a linear model of an n degree of freedom structure having a mass matrix **M**, a damping matrix **C,** and a stiffness matrix **K**, under the simultaneous action of two components of motion in the X and Y directions, is

$$\mathbf{M}(\ddot{\mathbf{q}} + \ddot{u}_g \mathbf{e}_x + \ddot{v}_g \mathbf{e}_y) + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{1}$$

in which **q** is the degree of freedom (DOF) displacement vector, and $\mathbf{e}_x$ and $\mathbf{e}_y$ are the excitation incidence vectors for the X and Y directions, respectively.

Through the modal transformation to normal coordinates, χ_i, uncoupling of these equations can be achieved in the same way as in ordinary one-directional excitation response. The transformation is given by the linear expression

$$\mathbf{q}(t) = \sum_i \chi_i(t)\boldsymbol{\phi}_i \tag{2}$$

in which the $\boldsymbol{\phi}_i$ are the modal shapes. As in all summations in this paper, the range of the summation indices runs from mode 1 to mode n. The resulting uncoupled n modal equations are

$$\ddot{\chi}_i + 2\beta_i\omega_i\dot{\chi}_i + \omega_i^2\chi_i = -\mathcal{L}_{xi}\ddot{u}_g - \mathcal{L}_{yi}\ddot{v}_g \tag{3}$$

in which $\mathcal{L}_{xi}$ and $\mathcal{L}_{yi}$ are the X and Y direction modal participation factors. The definitions of the participation factors are

$$\mathcal{L}_{xi} = \frac{\boldsymbol{\phi}_i \cdot \mathbf{M}\mathbf{e}_x}{\boldsymbol{\phi}_i \cdot \mathbf{M}\boldsymbol{\phi}_i} \quad ; \quad \mathcal{L}_{yi} = \frac{\boldsymbol{\phi}_i \cdot \mathbf{M}\mathbf{e}_y}{\boldsymbol{\phi}_i \cdot \mathbf{M}\boldsymbol{\phi}_i} \tag{4}$$

Interesting insight on the nature of two-directional excitation response is gained by rewriting equation (3) as

$$\ddot{\chi}_i + 2\beta_i\omega_i\dot{\chi}_i + \omega_i^2\chi_i = -\mathcal{L}_{\alpha i}(\ddot{u}_g \cos\alpha_i + \ddot{v}_g \sin\alpha_i) \tag{5}$$

in which a modal direction, α_i, is defined, as well as an associated directional modal participation factor, $\mathcal{L}_{\alpha i}$. These directional modal properties are quantities defined as

$$\alpha_i = \tan^{-1}\left\{\frac{\mathcal{L}_{yi}}{\mathcal{L}_{xi}}\right\} \quad ; \quad \mathcal{L}_{\alpha i} = \sqrt{\mathcal{L}_{xi}^{\,2} + \mathcal{L}_{yi}^{\,2}} \tag{6}$$

The modal direction is defined through an angle ranging from 0 to 360°. Equation (5) can be regarded as implying that mode i is responding exactly as if it were excited by the component of ground motion in the direction α_i, with a modal participation factor that was $\mathcal{L}_{\alpha i}$.

THE SPECTRAL APPROACH

In time history analyses the uncoupled equations of motion can be easily integrated, and the time responses of the DOF displacements readily obtained from equation (2). And, of course, any response variable r(t) can be directly expressed in terms of the normal coordinates. In fact, all response variables, and in particular r(t), are related to the DOF displacements through an expression of the form

$$r(t) = \mathbf{S}\mathbf{q}(t) \tag{7}$$

in which **S** is a structural analysis computations matrix, originating in geometric compatibility, force-deformation relationships, static equilibrium, or in a combination of all three sources. Substituting from the modal transformation equation (2), the following expression is obtained

$$r(t) = \sum_i \chi_i(t)\mathbf{S}\boldsymbol{\phi}_i \tag{8}$$

in which the product $\mathbf{S}\boldsymbol{\phi}_i$ can be recognized, according to the same equation (7), as the value of the response variable $r(t_0)$ for a $\mathbf{q}(t_0)$, i.e., a deformed shape, coincident with the modal shape. Defining this value as

$$r_i = \mathbf{S}\boldsymbol{\phi}_i \tag{9}$$

the time-history of $r(t)$ can be written as

$$r(t) = \sum_i \chi_i(t)\, r_i \tag{10}$$

The time history of $r(t)$ is then available, and if needed, the maximum value of r can be easily obtained. In a spectral superposition context, only an estimate of that maximum value can be established through some estimation formula. In general those estimators have the form

$$R = \sqrt{\sum_i \sum_j \rho_{ij}\chi_i(t=t_i)\chi_j(t=t_j)r_i r_j} \tag{11}$$

in which t_i and t_j are the times at which, respectively, the normal coordinates χ_i and χ_j have their maximum values. The ρ_{ij} are modal coupling coefficient, specific to the estimation formula being used. The values of the $\chi_i(t=t_i)$ needed could, of course, be obtained from integration of the uncoupled modal equation (5). But that, certainly, is not the purpose of spectral superposition techniques. It is supposed that the maximum values are available from previous computations that have been performed once and for all, and will be used in the computation of the estimate R. This *previous* work is actually the origin of spectra in one-directional excitation, and it can be extended to the two-directional case, if allowance is made for a parameter in the definition of the spectra. Indeed, the differential equation

$$\ddot{z} + 2\beta\omega\,\dot{z} + \omega^2 z = -(\ddot{u}_g \cos\alpha + \ddot{v}_g \sin\alpha) \tag{12}$$

can be readily integrated to produce a spectral curve for a given value of the directional angle α, namely, for $\alpha = \alpha_i$. Thus, the value of $\chi_i(t=t_i)$ can be read from that *parametric* spectrum as

$$\chi_i(t=t_i) = \pm\mathcal{L}_{\alpha i} S_d(\omega = \omega_i; \beta = \beta_i; \alpha = \alpha_i) \tag{13}$$

There is an obvious physical interpretation of a parametric spectra: it is the ordinary one-directional spectra for the earthquake motion component in the direction corresponding to the parameter angle α. All parametric spectra for a given seismic event constitute a family of directional spectral curves. Of course, for practical use, the construction of the curves of the family for a number of reasonably close directions is sufficient. Furthermore, the numerical computations needed for the whole spectral family are limited to the same two time-history integrations required to obtain the one-directional spectra for each of the X and Y directions. This is quite apparent from the fact that the time response $z(t)$ for a given α can be written as

$$z(t) = u(t)\cos\alpha + v(t)\sin\alpha \tag{14}$$

where the response variables $u(t)$ and $v(t)$ are the X and Y displacements, resulting from the integration of

$$\ddot{u} + 2\beta\omega\,\dot{u} + \omega^2 u = -\ddot{u}_g \quad ; \quad \ddot{v} + 2\beta\omega\,\dot{v} + \omega^2 v = -\ddot{v}_g \tag{15}$$

respectively. It should also be quite apparent that the values of the directional parameter need not span a range greater than 180°, since for directional excitations at a 180° angle, the two responses only change signs. The parametric spectral curves for the α and α+180° directions are actually the same.

There is a plus or minus sign involved in equation (13). It appears as a problem in reading the spectral value of a modal coordinate χ_i when some of the coupling coefficients with other mode, j for instance, is significant, i.e., when ρ_{ij} is not negligible compared with 1. It should be noted that the choice of a minus sign is related to the above mentioned change in sign of the values of $z(t)$ in equation (12) when α is replaced by

$\alpha+180°$. The problem can be solved by continuing to consider the signs of the maximum values of the normal coordinate as all positive, and by replacing the coupling coefficient ρ_{ij} by

$$\rho'_{ij} = \rho_{ij} \cos(\alpha_i - \alpha_j) \tag{16}$$

This modified coefficient takes into account the sign reversal for a 180° angular difference, while leaving unaltered the coefficient when the angle between the two modal directions is 0. Quite reasonably, the orthogonal case is modified so as to render the modal responses effectively uncoupled.

An important characteristic of the families of parametric spectra is that, for almost the entire range of periods, an angle α_m can be detected, for which the spectral ordinates are smaller than the ordinates of all other parametric curves. This can be verified by observing the spectral family shown in Figures 1 and 2, for the records of the Viña del Mar station during the March 3, 1995 Central Chile Earthquake. It can also be noted that there is a second parameter α_M for which spectral ordinates are larger than the ordinates of all other parametric curves. The two directions are at right angles to each other. In an approximate sense, α_m and α_M can be regarded as principal directions of ground motion. The specific principal directions of the case shown can be taken as α_m = 75° and α_M = 165°. It will be seen that the spectral principal directions are indeed important in the construction of average parametric spectral families, for design purposes.

AVERAGE SPECTRAL FAMILIES

The spectral modal superposition technique can be considered from two different points of view. On the one hand, it can be regarded as a method to obtain estimators of response variables when analyzing the response to a specific seismic event. On the other hand, it can be regarded as a method to estimate the average values of the response variables under action of a set of earthquakes associated to a smoothed average design spectra.

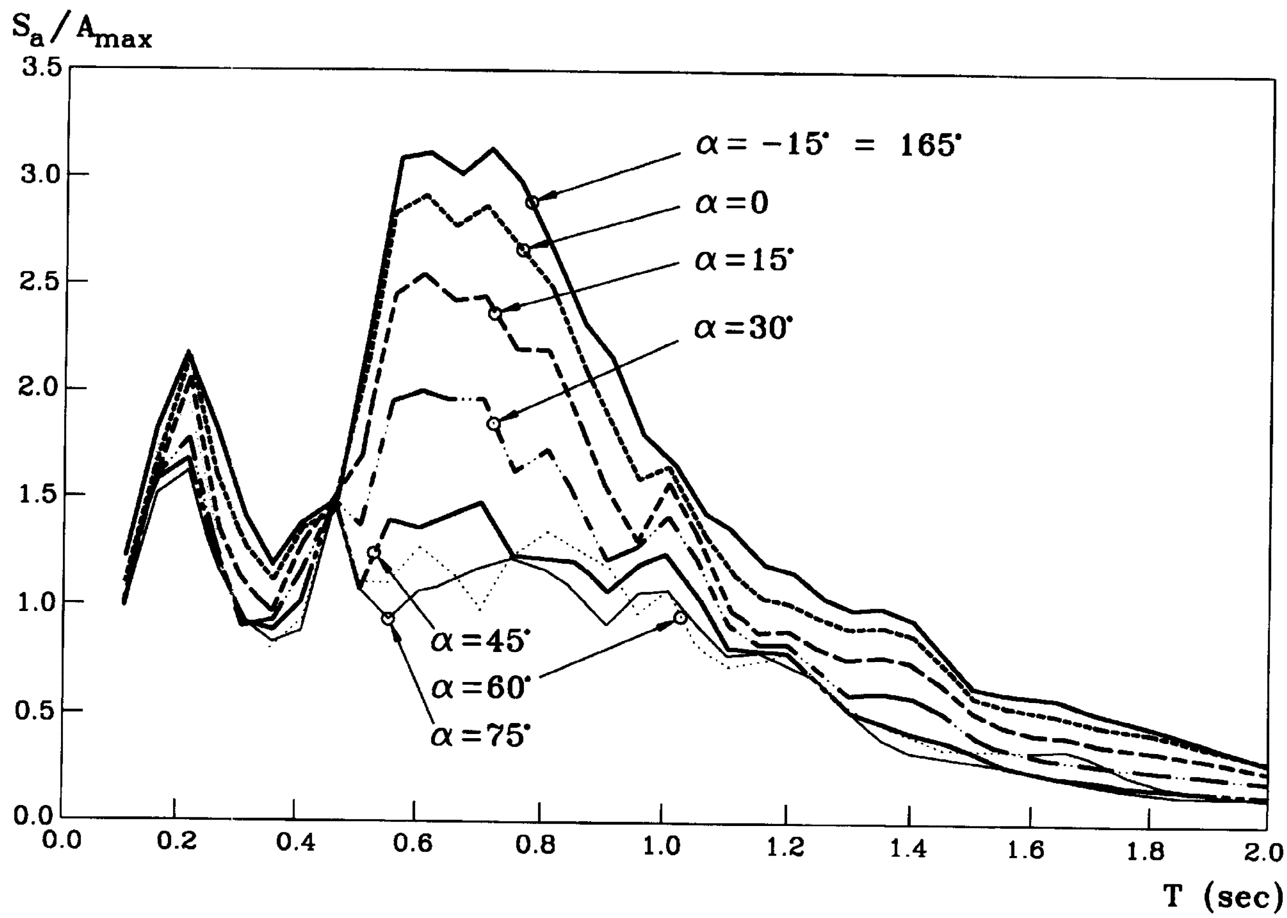

Figure 1. Parametric Spectra: Viña del Mar 3-3-1985 Earthquake (Range α = -15° to α = 75°)

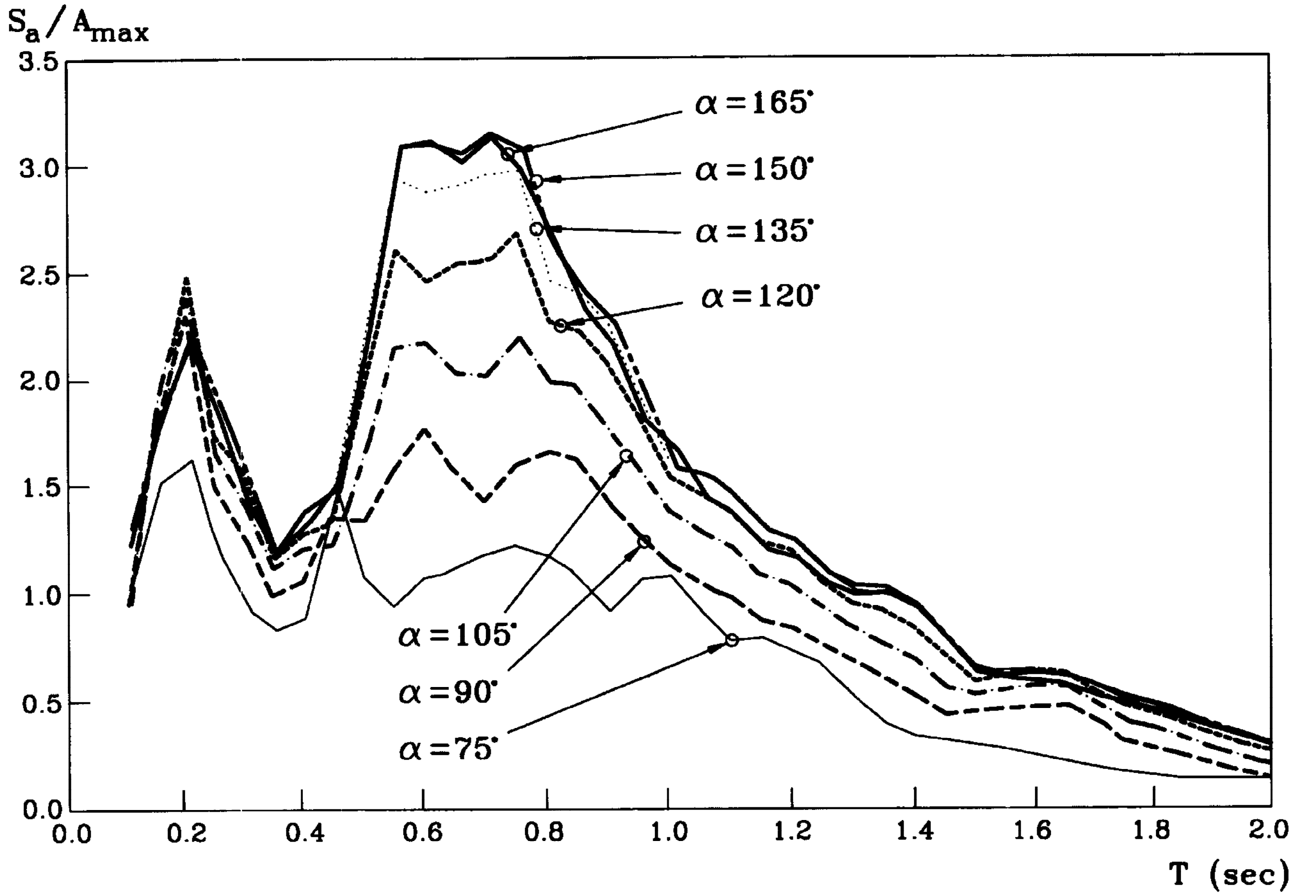

Figure 2. Parametric Spectra: Viña del Mar 3-3-1985 Earthquake (Range $\alpha = 75°$ to $\alpha = 165°$)

Actually only the second of this interpretations can be given, from stochastic considerations, a rational justification. The first of the interpretations can only be accepted as a source of reasonable estimations by extrapolation of the second point of view. Nevertheless, when compared with exact time-history results, the estimations are ordinarily found to be of good quality, with errors in the order of 10 percent, and seldom exceeding over twice that value. In applications of the proposed method to this type of problem, errors were found to be of this order, the same as in ordinary single-direction excitation cases (Vasquez et al., 1993).

However, the proposed method of parametric spectra gives rise to doubts with regard to the averaging procedure for the construction of smoothed design spectra. In fact, particular spectral families associated to a specific seismic event are parameterized through an angle α measured with respect to a somewhat arbitrary direction. This arbitrary direction is related to the position in which the instrument has been installed. Unless the records being averaged all belong to a local zone, in which a geographical effect can be important, the reference directions are quite meaningless. The spectral principal directions furnish a solution to this difficulty, when the angular parameter in all the records being averaged is measured starting from one of the principal directions, for instance, that of maximum spectral ordinates. Figure 3 shows the average spectrum family obtained from combining the parametric families of the two horizontal components recorded during the March 3, 1985 Central Chile Earthquake, at the stations in La Ligua, Talca, Melipilla, Iloca, Hualañé, San Fernando, Illapel, San Felipe and Llo-Lleo, all corrected for principal direction reference.

ARBITRARY ORIENTATION EXCITATION

In considering arbitrary orientation a in two-direction excitation, the X and Y directions of the structure will be rotated in a given orientation phase angle ψ with respect to the X and Y directions of the records. The components of motion of the structure's X and Y directions, $u_{g\psi}$ and $v_{g\psi}$, are given by the transformation

$$u_{g\psi} = u_g \cos\psi - v_g \sin\psi \quad ; \quad v_{g\psi} = u_g \sin\psi + v_g \cos\psi \tag{17}$$

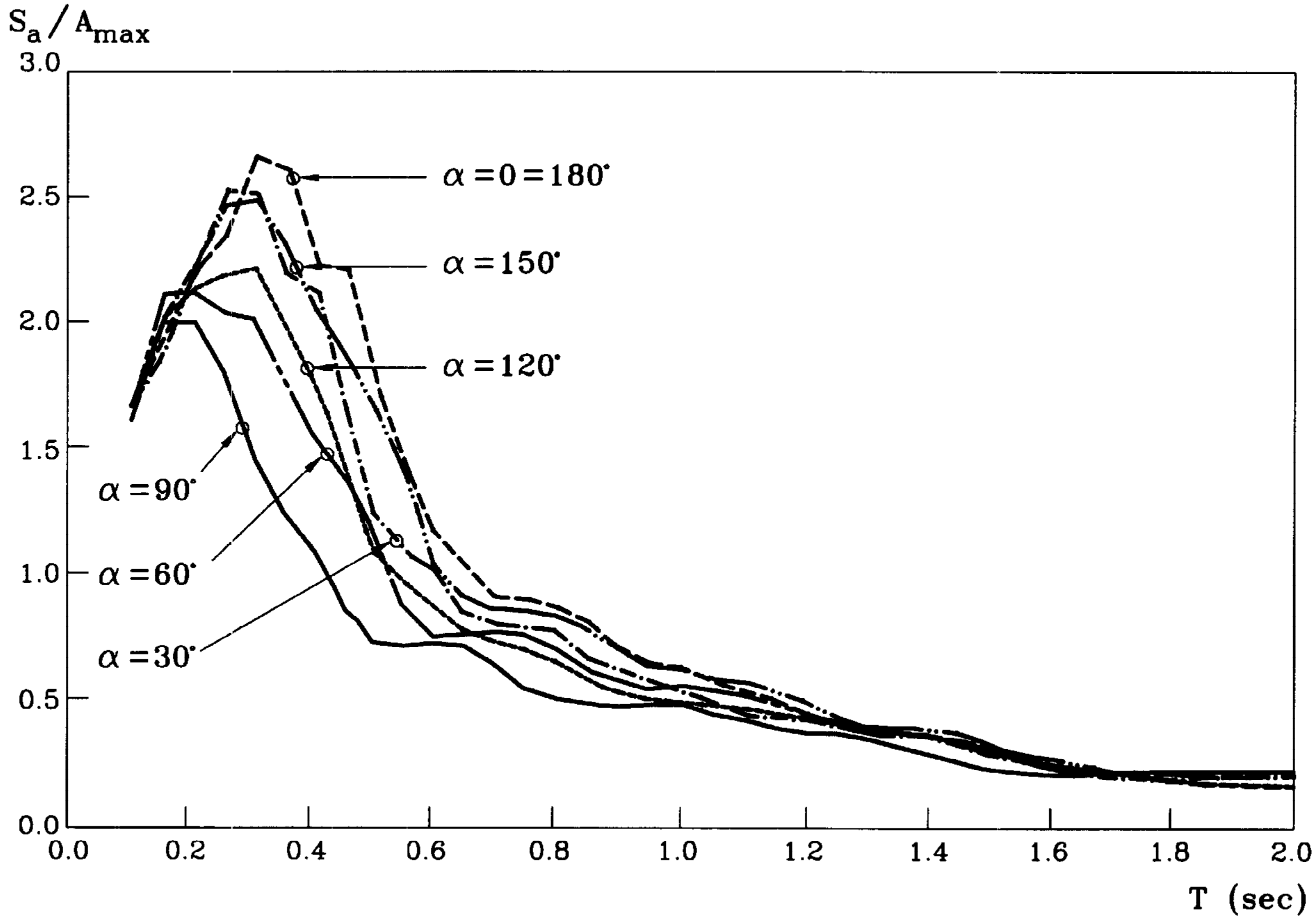

Figure 3. Average Parametric Spectra: central Chile 3-3-1985 Earthquake.

in which u_g and v_g are the recorded displacements of the previous discussion. The equations of motion of the structural model will have to be written as the following slight variation of equation (1)

$$\mathbf{M}(\ddot{\mathbf{q}} + \ddot{u}_{g\psi}\mathbf{e}_x + \ddot{v}_{g\psi}\mathbf{e}_y) + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{18}$$

The uncoupling of this matrix equation through transformation to normal coordinates in this case leads to

$$\ddot{\chi}_i + 2\beta_i\omega_i\dot{\chi}_i + \omega_i^2\chi_i = -\mathcal{L}_{xi}\ddot{u}_{g\psi} - \mathcal{L}_{yi}\ddot{v}_{g\psi} \tag{19}$$

When the components of motion in the rotated directions are substituted from equation (17), the following expression is obtained

$$\ddot{\chi}_i + 2\beta_i\omega_i\dot{\chi}_i + \omega_i^2\chi_i = -\mathcal{L}_{xi}(\ddot{u}_g\cos\psi - \ddot{v}_g\sin\psi) - \mathcal{L}_{yi}(\ddot{u}_g\sin\psi + \ddot{v}_g\cos\psi) \tag{20}$$

The right hand side of this differential equation can be regrouped so as to render to it the same form of equation (5), namely the form

$$\ddot{\chi}_i + 2\beta_i\omega_i\dot{\chi}_i + \omega_i^2\chi_i = -(\mathcal{L}_{xi}\cos\psi + \mathcal{L}_{yi}\sin\psi)\ddot{u}_g + (\mathcal{L}_{xi}\sin\psi - \mathcal{L}_{yi}\cos\psi)\ddot{v}_g \tag{21}$$

The coefficient of the ground acceleration components $\ddot{u}_{g\psi}$ and $\ddot{v}_{g\psi}$ can then be regarded, in the context of the transformation from equation (3) to equation (5), as generalized modal participation factors $\mathcal{L}_{xi}$ and $\mathcal{L}_{yi}$. This leads to the definition of the modal direction $\alpha_{\psi i}$ and its corresponding modal participation factor $\mathcal{L}_{\alpha\psi i}$,

$$\alpha_{\psi i} = \tan^{-1}\left\{\frac{-\mathcal{L}_{xi}\sin\psi + \mathcal{L}_{yi}\cos\psi}{\mathcal{L}_{xi}\cos\psi + \mathcal{L}_{yi}\sin\psi}\right\} \quad ; \quad \mathcal{L}_{\alpha\psi i} = \sqrt{(\mathcal{L}_{xi}\sin\psi + \mathcal{L}_{yi}\cos\psi)^2 + (\mathcal{L}_{xi}\cos\psi - \mathcal{L}_{yi}\sin\psi)^2} \tag{22}$$

A straightforward expansion of the sums of the squares appearing in the definition of $\mathcal{L}_{\alpha\psi i}$, shows clearly that it is equal to $\mathcal{L}_{\alpha i}$. This result is important, since it means that the participation factor is independent of the angle ψ. The expression for the modal direction can also be expanded, and recast as follows

$$\frac{-\mathcal{L}_{xi}\sin\psi+\mathcal{L}_{yi}\cos\psi}{\mathcal{L}_{xi}\cos\psi+\mathcal{L}_{yi}\sin\psi}=\frac{-\cos\alpha_i\sin\psi+\sin\alpha_i\cos\psi}{\cos\alpha_i\cos\psi+\sin\alpha_i\sin\psi}=\frac{\sin(\alpha_i-\psi)}{\cos(\alpha_i-\psi)}=\tan(\alpha_i-\psi) \quad (23)$$

leading to an identity from which it can be readily seen that $\alpha_{\psi i}=\alpha-\psi$. Hence, the directional angle, though not independent of ψ, is related to it through a very simple relationship. Negative directional angles are bound to appear when the whole 360° of the orientation plane are ranged. In such a case it should be clear that the circular complement, i.e., $\alpha_{\psi i}=360°+\alpha_i-\psi$, must be used.

Though the dependence is linear, it does not allow for an analytical solution of the problem of finding, for a given response variable, the most unfavorable direction. In this respect it differs from the case of single one-directional excitation, where a rather simple *Mohr circle* type of solution is available (Vasquez, 1987). This difference does not mean that a rational solution cannot be found. It only implies that that solution must necessarily be obtained by a numerical procedure. The process through which a set of orientation phase angles can be considered, spanning the range of all possible orientations in the plane so as to numerically establish the maximum value, is not excessively involved. It requires recomputation of the superposition formula (11) for each phase angle ψ, changing only the values of the spectral ordinates, to account in the modal direction $\alpha_{\psi i}$ for the subtraction of ψ. This may seem rather cumbersome if graphical spectral ordinate reading from the spectral family drawings is in mind. However, direct interpolation from the data that actually leads to the construction of the spectral graphs may make the computations quite manageable. Indeed, the procedure can be easily automated.

APPLICATIONS

A standard one story structural model with 3 DOFs associated to a rigid roof diaphragm has been studied. The eccentricity of the model was introduced through offsets of the center of mass e_x in the X direction and e_y in the Y direction. These eccentricities are expressed as percentages of the rotational radius of gyration in the mass matrix. Displacements of different points in the diaphragm were obtained, both through the directional method here proposed, as the maximum value of the directional estimates, R_ψ, and two of the well known *design code* recommendations for the consideration of the so-called *orthogonal effects*. Namely, the code estimators used were, R_2, the square root of the sum of the squares of the one-directional X and Y responses estimated independently, and R_1, the sum of the larger of the two one-directional responses plus a 30 percent of the other response. It was found that these code formula were in general unconservative.

The analyses reported here were made for the case in which the periods of the zero eccentricity structure were 0.5 sec in the X direction, 0.4 sec in the Y direction, and 0.25 sec in torsion. The design spectral family used was the one of Figure 3. Figure 4 shows the ratio of the directional estimate of a displacement in the X direction at a distance equal to the radius of gyration from the center of mass, in the case having eccentricity only along Y axis, of 10 percent, and in the case of having equal eccentricity, of 10 percent, in both directions. Specifically, the figures show the ratio between the directional estimate, R_ψ, and the R_2 estimate.

It can be seen that in the first case the directional estimate is in general less than the R_2 estimate, while the maximum value of R_ψ is only slightly larger than R_2. In the second case the directional estimate has a maximum value over 50 percent larger than R_2, for an orientation phase angle ψ of about 125°. The fact that for $\psi = 0$ the ratio always has a value close to 1 can be easily understood. In both cases the R_1 estimator is only slightly larger than the R_2 estimator, so if R_1 had been taken as a reference, the curves would have been almost identical. Actually, the variability of the directional effects shows that two component analysis is indeed important. It points to the need of reconsideration of the way in which excitation orientation and two component excitation are mingled under the name of orthogonal effects in current practice.

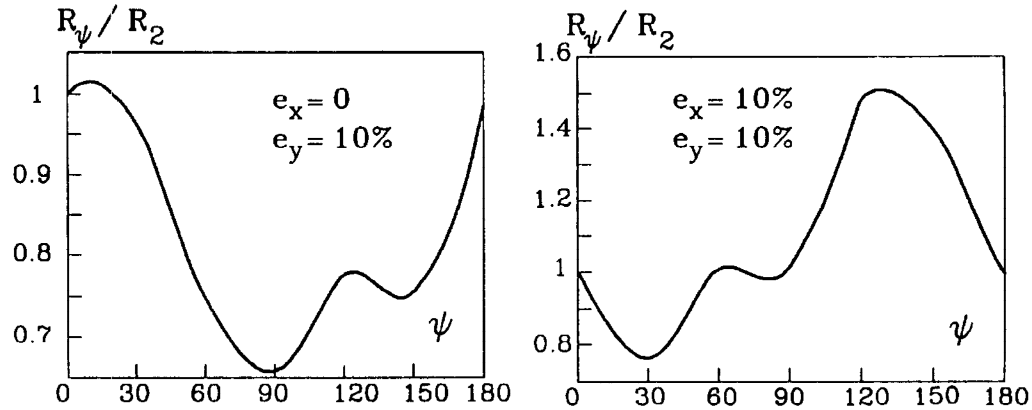

Figure 4. Displacement Estimator Ratio in Eccentric One-Story Model

CONCLUSIONS

A rational method for modal spectral analysis of structures under simultaneous action of two horizontal components of motion has been successfully devised. The method is based in the construction of a family of parametric spectral curves, in which the parameter is associated to directions in the horizontal plane. The spectral families were found to present *principal directions*, in the sense that the spectral curves for two particular orthogonal directions are, in an approximate sense, envelopes of all the curves in the family. The construction of average design spectral families was also discussed, for the purpose of which the concept of principal directions is most useful. The method was also used to obtain the most unfavorable orientation of the two-directional excitation. The solution of the orientation problem, though not analytical, is a rational one. It does require a numerical sweeping of the orientation plane, but it does not involve excessive numerical work. Actually. the procedure is not difficult to implement in computer programs. The application of the method shows the significance of two-directional excitation, particularly when used in the context of finding the orientation of maximum value of a response component. These results were compared with the standard code recommendations for orthogonal effects, and unconservatisms by factors of over 1.5 were found.

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