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BAYESIAN ESTIMATION OF STRAIN DEPENDENCE OF DYNAMIC SOIL PROPERTIES FROM THE STRONG MOTION ACCELEROGRAMS RECORDED BY A VERTICAL ARRAY

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ABSTRACT

The strain dependence of dynamic soil properties was investigated from the strong motion accelerograms recorded by a vertical array. A Bayesian approach to nonlinear inversion was proved to be very useful to identify stably the changes of shear modulus and damping factor caused by the nonlinearity of soil. It was clearly shown that the shear modulus ratio in the near-surface layers decreases with increasing effective shear strain. On the other hand the increase in damping factor with increasing effective shear strain is not obvious. The relations between shear modulus ratio and effective shear strain obtained from the strong motion accelerograms are in good agreement with those from the laboratory tests of soil samples.

KEYWORDS

Nonlinear inversion; Bayesian approach; nonlinearity of soil; shear modulus; damping factor; spectral ratio; vertical array; one-dimensional wave propagation theory.

INTRODUCTION

The effect of the nonlinearity of soil is important for seismic response analysis when the peak acceleration of an input motion becomes high. An equivalent linear procedure proposed by Schnabel *et al.* (1972) is widely used as a practical method for the nonlinear response analysis of soil. Although the relations between dynamic soil properties and shear strain obtained from the laboratory tests of soil samples are used in the calculation by the equivalent linear procedure, studies showing the validity of them from observed strong motion records are not sufficient.

The validity of the relations of strain dependence obtained from the laboratory tests can be examined using the strong motion records observed by a vertical array (Satoh *et al.*, 1993). It is not easy,however, to obtain the changes of dynamic soil properties from array data by a classical least squares method. An instability in the solution may occur because there exist many solutions which can fit the data almost equally well. The solution minimizing the sum of the squared residuals may be physically unreasonable. To obtain a physically reasonable solution an additional constraint is needed. A Bayesian approach to nonlinear inversion proposed by Jackson and Matsu'ura (1985) can be used for this purpose.

The objective of the present study is to apply the Bayesian approach to an inverse problem for determining the changes of the dynamic soil properties caused by the nonlinearity of soil from the strong motion accelerograms recorded by a vertical array. In this approach a prior information is used as the additional constraint.

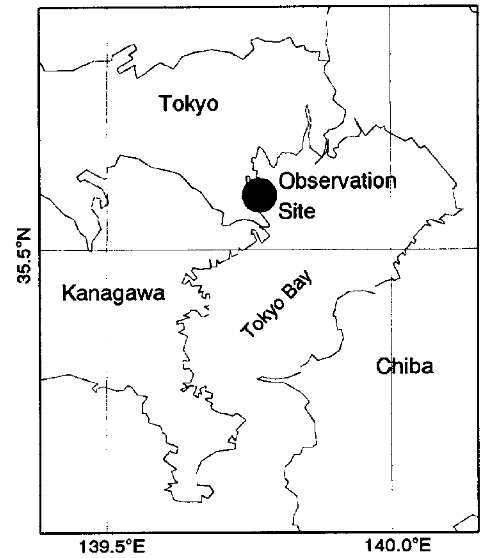

Fig.1 Location map of observation site

VERTICAL ARRAY

The location of a vertical array is shown in Fig.1. The array is installed on artificially reclaimed land in Tokyo along Tokyo Bay. Detailed soil investigations including *in-situ* PS logging were performed. Soil profile and locations of seismometers are shown in Table 1. A bore-hole type seismometer with three components is installed at each location. The relative orientation errors of the three underground seismometers with reference to the orientation of the near-surface seismometer (G4) are determined from the rotation deviations of the two particle orbits formed by low-pass filtered accelerograms. The estimated orientation errors of G1, G2, and G3 are 7°, 9°, and 5° in clockwise direction, respectively. The corrected accelerograms obtained by the coordinate transformation were used in the analysis. The strongest peak ground acceleration (PGA) of 113 cm/s^2 was recorded at G4 during the M_J 6.7 Chibaken-Tohou-Oki earthquake of Dec.17, 1987.

Table 1 Soil profile and locations of seismometers

Layer No.	Thickness (m)	Vs (m/s)	Density (g/cm^3)	Seismometers
1	5.0	100	1.85	● G.L.-1.5m G4
2	5.0	100	1.50	
3	4.0	170	1.80	
4	3.0	420	1.70	
5	5.0	220	1.60	
6	9.5	420	1.80	● G.L.-22.3m G3
7	1.5	520	1.80	
8	6.0	320	1.72	
9	7.0	420	1.85	● G.L.-38.5m G2
10	16.0	340	1.85	
11	12.0	500	2.05	G1
12	-	560	1.80	● G.L.-80.0m

INVERSION METHOD

The problem is to determine the optimal dynamic soil properties by fitting the amplitude of transfer function (spectral ratio) calculated from the soil model by one-dimensional wave propagation theory to that from the observed records. The thickness and mass density of every layer given in Table 1 are assumed as known parameters. The unknown parameters determined by an inversion are S-wave velocities and damping factors of all layers. Usually the optimal solution is obtained by minimizing the sum of the squared residuals between observed and calculated spectral ratios. However an additional constraint is needed to obtain a physically reasonable solution as stated previously. A Bayesian approach to nonlinear inversion proposed by Jackson and Matsu'ura (1985) was applied to the problem.

Suppose a nonlinear observation equation is represented by $\mathbf{y}^0=\mathbf{f}(\mathbf{x})+\mathbf{e}$, where $\mathbf{y}^0$ is a vector of observed data, $\mathbf{f}$ is a vector of known functions depending on the arguments $\mathbf{x}$, $\mathbf{x}$ is a vector of unknown parameters, and $\mathbf{e}$ is a vector of random errors. In this study the components of $\mathbf{y}^0$ consist of the natural logarithms of spectral ratios at specified frequencies for specified combinations of seismometers. The function $\mathbf{f}(\mathbf{x})$ is theoretically calculated from soil model by one-dimensional wave propagation theory. The unknown parameters consist of S-wave velocities and damping factors. The errors $\mathbf{e}$ are Gaussian, with zero mean and covariance $\mathbf{E}$. The prior information is written as $\mathbf{x}^0=\mathbf{x}+\mathbf{d}$, where $\mathbf{x}^0$ are the prior estimates of the unknown parameters. The errors $\mathbf{d}$ are Gaussian, with zero mean and covariance $\mathbf{D}$. According to Bayes' theorem the joint probability density function (pdf) of unknown parameters posterior to observed data is proportional to the likelihood function multiplied by the prior pdf. The best solution of the problem is given by $\mathbf{x}^*$ which maximize the posterior pdf. The maximum of the posterior pdf is realized by minimizing the following function:

$$s(\mathbf{x})=(\mathbf{y}^o-\mathbf{f}(\mathbf{x}))^t\mathbf{E}^{-1}(\mathbf{y}^o-\mathbf{f}(\mathbf{x}))+(\mathbf{x}^o-\mathbf{x})^t\mathbf{D}^{-1}(\mathbf{x}^o-\mathbf{x}). \tag{1}$$

The solution $\mathbf{x}^*$ which minimizes $s(\mathbf{x})$ can be obtained by an iterative procedure.

When the functions $\mathbf{f}(\mathbf{x})$ are nearly linear within some reasonable confidence region about $\mathbf{x}^*$, the asymptotic covariance matrix **C** will give a good approximation to exact covariance. The matrix C is given by the following equation:

$$\mathbf{C}=(\mathbf{A}^t\mathbf{E}^{-1}\mathbf{A}+\mathbf{D}^{-1})^{-1}. \tag{2}$$

The matrix **A** is called a Jacobian matrix: (i,j) element of **A** is given by $(\partial f_i/\partial x_j)_{\mathbf{x}=\mathbf{x}^*}$.

If the covariance matrices **E** and **D** are completely given, the solution can be strictly determined. However it is very difficult to determine the non-diagonal elements of **E** and **D** reasonably. Therefore only diagonal elements are considered in this analysis. Instead of neglecting the non-diagonal elements, a parameter β to adjust the relative weight of the two terms of the right-hand side of equation (1) is introduced as follows:

$$s(\mathbf{x})=(\mathbf{y}^o-\mathbf{f}(\mathbf{x}))^t\mathbf{E}^{-1}(\mathbf{y}^o-\mathbf{f}(\mathbf{x}))+\beta^2(\mathbf{x}^o-\mathbf{x})^t\mathbf{D}^{-1}(\mathbf{x}^o-\mathbf{x}). \tag{3}$$

The resultant solution depends on the value of β. If the value of β is too large, the solution cannot fit the data well. On the other hand if it is too small, the solution becomes unstable. Although the value of β is subjectively determined in this study from the trial results, it is necessary to develop a method for determining the proper value of β objectively.

APPLICATION TO SYNTHETIC DATA

In order to examine the accuracy of obtained results, the inversion method was applied to synthetic test data first. The results of the tests are shown in Tables 2 and 3. The frequency dependence of damping factor is represented by the form of $h=h_o/f^p$. The value of p is 0.0 for the case of Table 2, whereas it is 1.0 for the case of Table 3. The p is treated as the known parameter.

The time histories of the test data for each case were synthesized from the target soil model given in

Table 2 Results of the test using the synthetic data in case of p=0.0.

Layer No.	Shear Wave Velocity (m/s)			Damping Factor ho(%)		
	Initial model	Optimal model	Target model	Initial model	Optimal model	Target model
1	100	91	90	5.0	8.7	10.0
2	100	90	90	5.0	9.7	10.0
3	170	162	160	5.0	7.7	7.0
4	420	420	420	5.0	5.3	5.0
5	220	218	220	5.0	5.9	5.0
6	420	421	420	5.0	5.6	5.0
7	520	520	520	5.0	5.0	5.0
8	320	320	320	5.0	5.4	5.0
9	420	420	420	5.0	5.0	5.0
10	340	340	340	5.0	5.2	5.0
11	500	500	500	5.0	5.1	5.0
12	560	560	560	5.0	4.9	5.0

Table 3 Results of the test using the synthetic data in case of p=1.0.

Layer No.	Shear Wave Velocity (m/s)			Damping Factor ho (%)		
	Initial model	Optimal model	Target model	Initial model	Optimal model	Target model
1	100	90	90	30.0	57.9	60.0
2	100	90	90	30.0	58.7	60.0
3	170	160	160	30.0	44.5	42.0
4	420	420	420	30.0	30.4	30.0
5	220	220	220	30.0	32.0	30.0
6	420	420	420	30.0	30.1	30.0
7	520	520	520	30.0	30.0	30.0
8	320	320	320	30.0	31.9	30.0
9	420	420	420	30.0	30.1	30.0
10	340	340	340	30.0	29.9	30.0
11	500	500	500	30.0	29.1	30.0
12	560	560	560	30.0	30.4	30.0

each Table by one-dimensional wave propagation theory. The dynamic soil properties of the target model are slightly changed from those of the initial model: the S-wave velocities of the layers from No.1 to No.3 decrease, whereas the damping factors of them increase. This change is natural for the nature of the strain dependence of soil properties. Using the NS component of the strong motion observed at G4 during the 1987 Chibaken-Tohou-Oki earthquake as an input motion for the target model, the underground time histories at G1, G2, and G3 were obtained. The duration time of the time histories is 100.0 sec and the sampling interval is 0.01 sec. This set of the four time histories for each case was used as the test data.

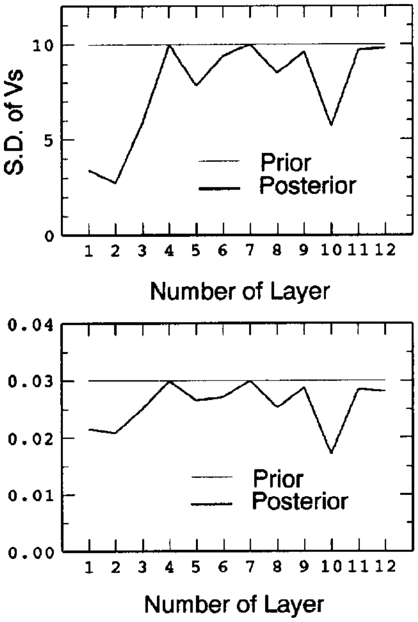

Fig.2 Comparison of standard deviation (S.D.) of errors between prior and posterior estimations

Fourier amplitude spectra were calculated for the selected time histories with a duration time of 40.96 sec including the major part of S-wave portion by the Fast Fourier Transform algorithm. Using the Fourier spectra smoothed by Parzen window with a band width of 0.1 Hz, the spectral ratios were calculated for three combinations between near-surface and underground seismometers: G1/G4, G2/G4, and G3/G4. The three combinations of the spectral ratios in the frequency range from 0.5 to 15.0 Hz were simultaneously used to determine the optimal soil model.

The prior estimates of the unknown parameters were given by the initial model for each Table. The prior estimation errors of the unknown parameters can be defined by standard deviations. The standard deviations of estimation errors for S-wave velocity and damping factor are denoted by σ_V and σ_h, respectively. The assumed prior estimation errors were $\sigma_V = 10.0$ and $\sigma_h = 0.03$ for the case of Table 2 and $\sigma_V = 30.0$ and $\sigma_h = 0.54$ for the case of Table 3. The standard deviation of errors for the observed spectral ratios was assumed as 1.0 for all frequencies and the value of β was set to 1.0. Using these prior informations and the observed data the optimal models in Tables 2 and 3 were obtained by the Bayesian inversion. For both Tables the optimal model is almost the same as the target model. The changes of damping factors are correctly evaluated as well as those of S-wave velocities.

The estimation errors of the unknown parameters given by the asymptotic covariance matrix **C** are compared with those by the prior information **D**. The comparison of the standard deviations is shown in Fig.2. The thin and thick lines denote the prior and posterior standard deviations of the estimation errors respectively. The decrease in the standard deviation is relatively large at the layers 1, 2, 3, and 10. In these layers the reliability of the soil parameters are improved by the observed data. However the standard deviations at the layers 4 and 7 are not changed. The reliability of the soil parameters in these layers are not affected by the observed data.

APPLICATION TO OBSERVED DATA

The records for 19 earthquakes with horizontal peak ground accelerations higher than 10 cm/s^2 were used in the analysis. First, the spectral ratios obtained from weak motion data were analyzed to determine the optimal model for low-strain level. Next the spectral ratios for each earthquake were analyzed individually to determine the changes of the dynamic soil properties from the optimal model for low-strain level. The relations between soil properties and shear strain obtained from the array records were compared with those from the laboratory tests of the soil samples.

Optimal Model for Low-Strain Level

The weak motion data with horizontal PGAs from 10 to 25 cm/s^2 were selected for eight earthquakes. It is thought that the effect of the nonlinearity of soil can be neglected for these earthquakes. Fourier amplitude spectra were calculated for the selected time histories with a duration time of 40.96 sec including the major part of S-wave portion as same as the case of the synthetic data. The spectral ratios were also obtained by the same process as the synthetic data. The means and standard deviations of the spectral ratios were used in the inversion as the observed data and their errors, respectively.

Although the prior estimate of S-wave velocity can be easily obtained from the results of soil investigation, that of damping factor is rather difficult. Therefore the prior estimate of damping factor is determined from a preliminary analysis. In the preliminary analysis it was assumed that the values of p and h_0 are the same for all layers. Using the S-wave velocities given in Table 1 as the known parameters, the best-fit solutions of p and h_0 were determined from the spectral ratios by the classical least squares method. The solutions are $p=0.315$ and $h_0=7.9\%$ for the NS component, and $p=0.330$ and $h_0=7.1\%$ for the EW component.

If the parameters h_0 and p of damping factor are both included in the unknown parameters in the inversion, the tendency of the changes of damping factor caused by the nonlinearity of soil will depend on frequency. In order to recognize the tendency of the changes easily, the value of p is fixed to 0.32 from the above results. The degree of the fitting was hardly affected by this constraint.

The optimal models for the EW and NS components obtained by the Bayesian inversion are shown in Table 4 and the comparison of the spectral ratios obtained from the observed records with those calculated from the optimal model for the NS component is shown in Fig.3. The prior estimates of the unknown parameters were given by the initial model in Table 4. The S-wave velocity structure is the same as that in Table 1. The damping factor of 7.5 % is determined from the preliminary analysis. The assumed prior estimation errors were $\sigma_V=10.0$ and $\sigma_h=0.06$. The value of β was set to 7.0.

The differences of the optimal models between the EW and NS components are relatively small. The

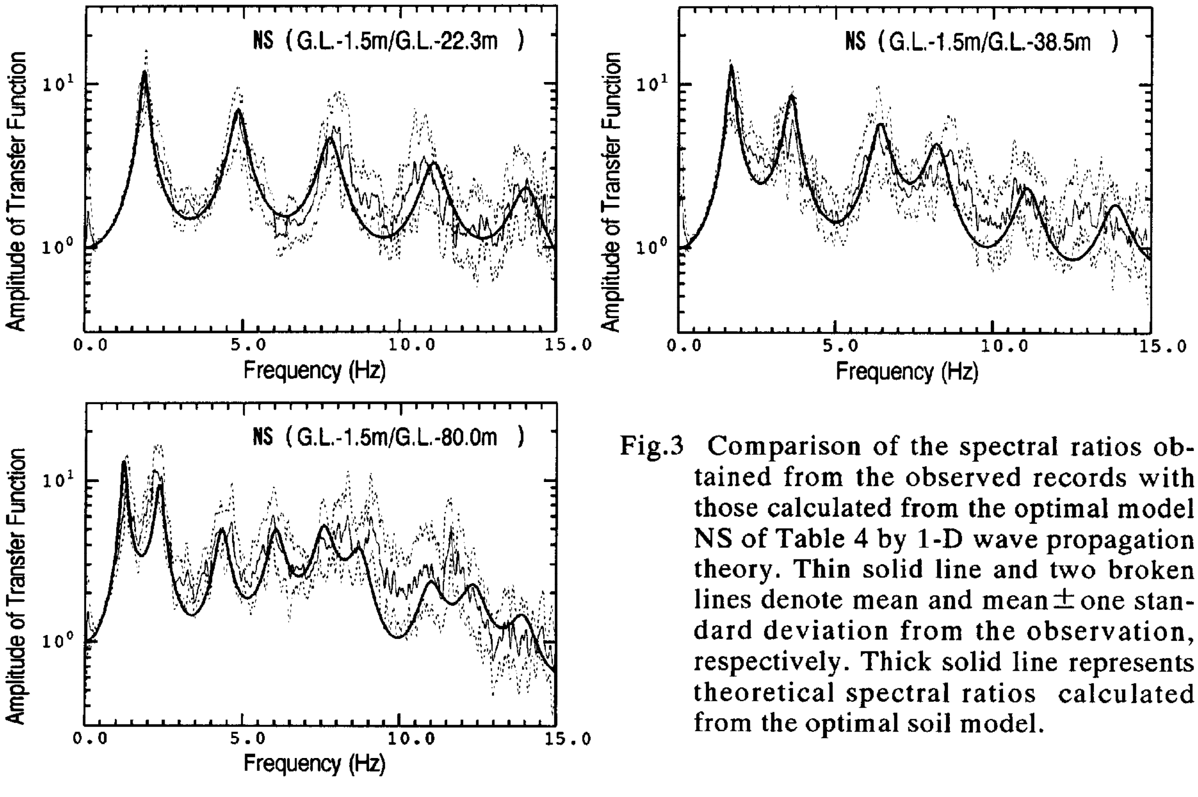

Fig.3 Comparison of the spectral ratios obtained from the observed records with those calculated from the optimal model NS of Table 4 by 1-D wave propagation theory. Thin solid line and two broken lines denote mean and mean ± one standard deviation from the observation, respectively. Thick solid line represents theoretical spectral ratios calculated from the optimal soil model.

changes of S-wave velocities are small except in the layer No.3. The damping factors of the upper layers are slightly larger than those of the lower layers for the EW component.

The agreement of the spectral ratios between observation (thin solid line) and calculation (thick solid line) is very well in Fig.3. The standard deviations of the observed spectral ratios are relatively small, especially in the lower frequency range. This means that the observed spectral ratios agree well each other.

Optimal Model for Each Earthquake

To investigate the strain dependence of dynamic soil properties the changes from the optimal models for low-strain level in Table 4 were determined for 19 earthquakes with horizontal peak ground accelerations greater than 10 cm/s^2. The eight earthquakes used for the weak motion analysis are also included.

Table 4 Optimal models determined from the spectral ratios obtained from the weak motion data

Layer	Shear Wave Velocity (m/s)			Damping, ho (%), p=0.32		
No.	Initial model	Optimal (EW)	Optimal (NS)	Initial model	Optimal (EW)	Optimal (NS)
1	100	98	100	7.5	8.4	7.9
2	100	104	101	7.5	8.4	6.7
3	170	155	157	7.5	7.8	6.7
4	420	419	419	7.5	7.5	7.4
5	220	223	224	7.5	6.7	7.6
6	420	419	417	7.5	7.7	6.9
7	520	520	520	7.5	7.6	7.5
8	320	324	322	7.5	8.8	7.5
9	420	420	419	7.5	7.2	7.0
10	340	343	341	7.5	7.6	7.1
11	500	502	501	7.5	7.2	6.6
12	560	561	560	7.5	6.7	6.6

The comparison of the spectral ratios from the 1987 Chibaken-Tohou-Oki earthquake with those from the weak motion data are shown in Fig.4. The PGAs of the EW and NS components observed at G4 during the 1987 Chibaken-Tohou-Oki earthquake are 77 cm/s^2 and 113 cm/s^2, respectively. The peak frequencies for the Chibaken-Tohou-Oki earthquake clearly shift to lower frequency side from those for the weak motion data. This shift is thought to be caused by the decrease in the shear modulus.

The prior estimates of the unknown parameters were given by the optimal models in Table 4. The assumed prior estimation errors were σ_V =10.0 and σ_h= 0.06. The value of β was set to 7.0. These values are the same as the weak motion data. Because it is impossible to determine the standard deviations of the spectral ratios from the data for single earthquake, those obtained from the weak motion data were used after being smoothed by the moving window averaging with a band width of 0.5 Hz.

As an example the result for the NS component of the Chibaken-Tohou-Oki earthquake is shown in Fig.5. The S-wave velocities in the layers 1 and 2 clearly decrease from those of the optimal model for low-strain level. The S-wave velocities of the other layers are scarcely changed. The damping factors of the near surface layers become greater than those of the optimal model for low-strain level. As shown

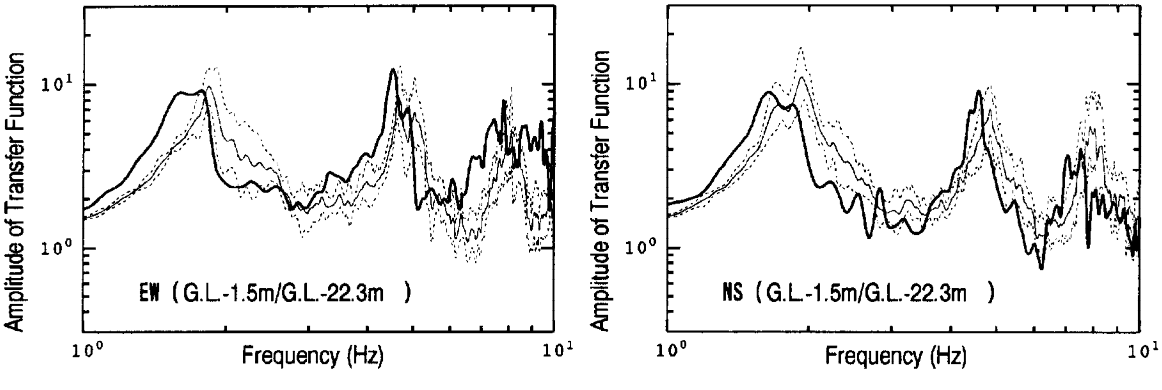

Fig.4 Comparison of the spectral ratio from the Dec.17,1987 earthquake (thick solid line) with that from weak motion data. The latter is the same as that in Fig.3.

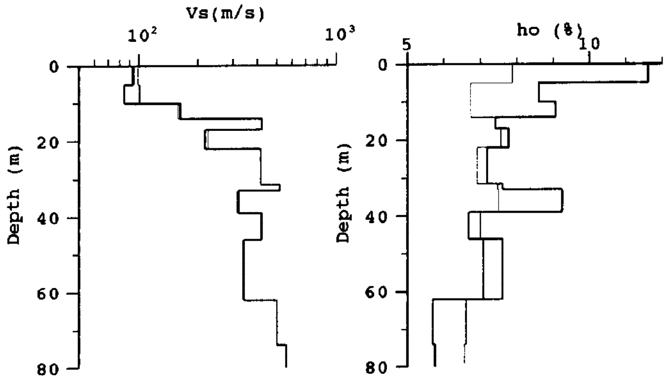

Fig.5 Comparison of the profiles of S-wave velocity, V_S, and damping factor at 1 Hz, h_0, for the NS component of the Dec.17, 1987 earthquake (thick line) with those for low-strain level (thin line)

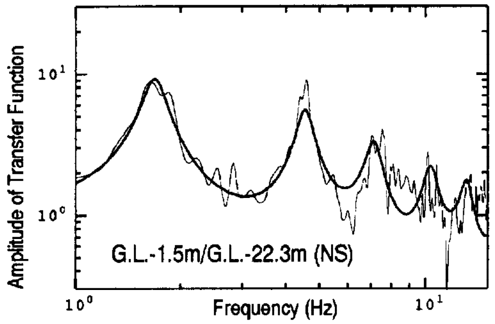

Fig.6 Comparison of the spectral ratios observed from the Dec.17,1987 earthquake (thin line) with those calculated from the optimal model for this earthquake (thick line)

in Fig.6 the calculated spectral ratio from the optimal model agrees very well with the observed spectral ratio.

The estimation errors of the unknown parameters given by the asymptotic covariance matrix **C** were compared with those by the prior information **D** for the two cases of the observed data.The comparison of the standard deviations is shown in Fig.7. The thin line denotes the prior standard deviations. The thick solid and broken lines represent the posterior standard deviations for the weak motion data and the strong motion data of the NS component of the 1987 Chibaken-Tohou-Oki earthquake, respectively. The pattern of the decreases in the standard deviations is almost similar to that of Fig.2 for the synthetic data. This shows that the the reliability of the estimated soil properties are mainly controlled by the array configuration and the soil structure.

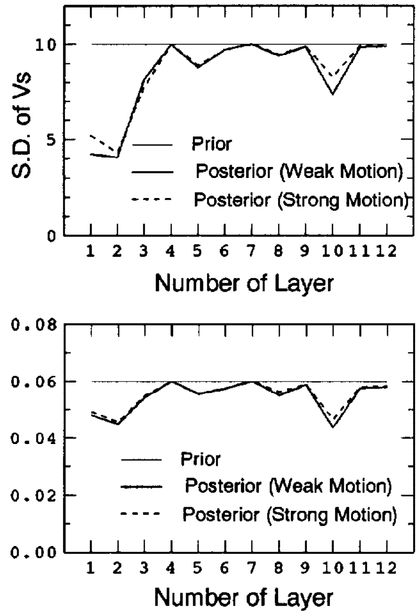

Fig.7 Comparison of standard deviations of errors between prior and posterior estimations

Strain Dependence of Dynamic Soil Properties

The maximum shear strains (γ_{max}) in all layers for each earthquake were calculated from the optimal model for each earthquake by one-dimensional wave propagation theory. The observed time histories at G4 were used as an input motion for the response analysis. To compare the laboratory tests of soil samples the effective shear strain is evaluated by 0.65 $\times\gamma_{max}$. The relations between soil properties (shear modulus ratio, G/G_0, and damping factor, h) and effective shear strain for the uppermost 3 layers obtained from the analysis of the strong motion records and the laboratory tests are shown in Fig.8. The shear modulus at low-strain level, G_0, were determined from the S-wave velocities in Table 4 and the mass densities in Table 1. For damping factor, the values at 9 Hz from the records are compared with the curves from the laboratory tests. This is because the damping factor obtained by assuming frequency independence is nearly equal to that at 9 Hz obtained by assuming frequency dependence for this array site. In the reclaimed clay layer No.2 the largest effective shear strain about 7×10^{-4} is generated. The decrease in G/G_0 is clearly shown for the layer No.2 from the strong motion data and its tendency agrees well with the curves from the laboratory tests. The

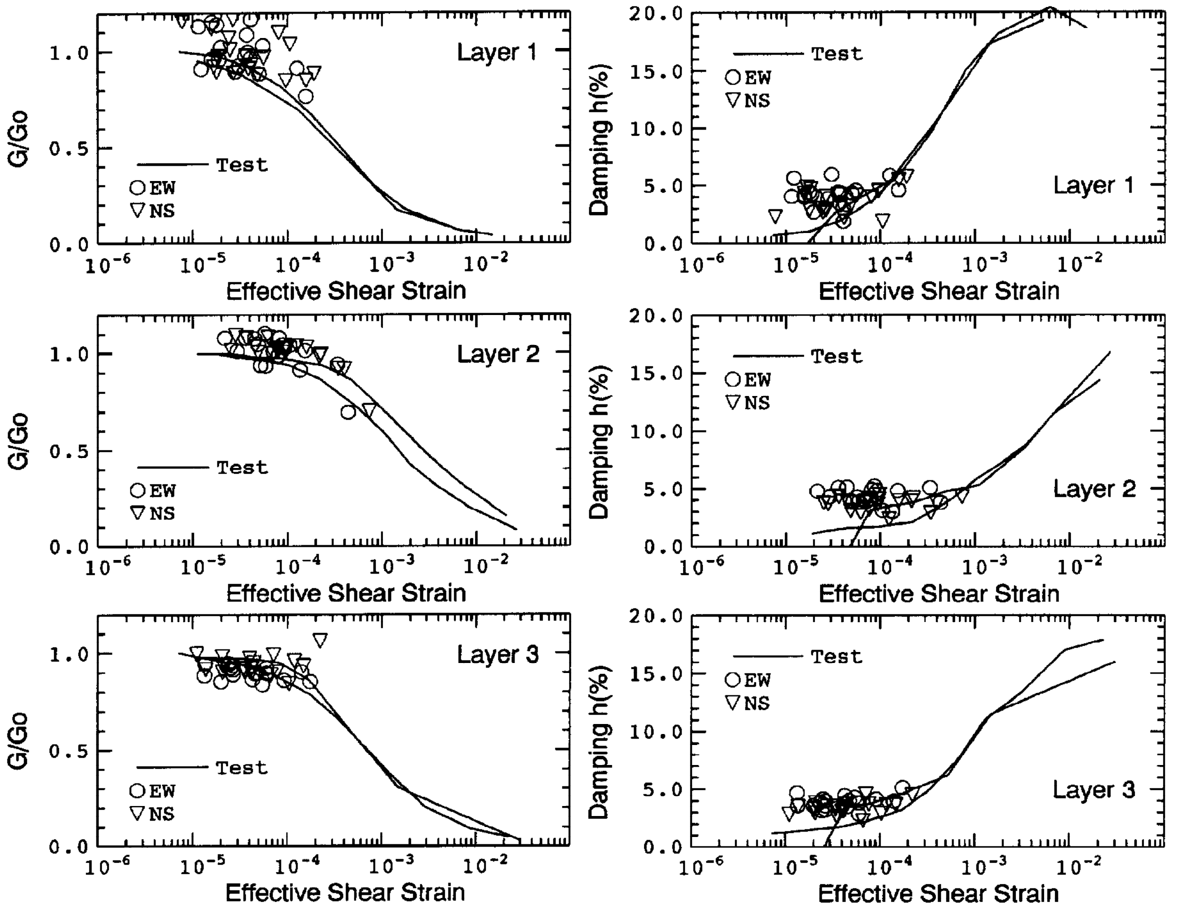

Fig.8 Relations between soil properties (shear modulus ratio, G/G_0, and damping factor, h) and effective shear strain from strong motion records and laboratory test

decrease in G/G_0 is also demonstrated from the strong motion data for the reclaimed sand layer No.1. For the layer No.3 the values of G/G_0 are almost constant. Although the damping factors from the strong motion data have a tendency to increase slightly in the range of effective shear strain above 10^{-4} for the layer No.1, the increase in damping factor is not so clear as the decrease in G/G_0.

CONCLUSIONS

The strain dependence of dynamic soil properties was investigated from the strong motion accelerograms recorded by a vertical array. A Bayesian approach to nonlinear inversion was proved to be very useful to identify stably the changes of shear modulus and damping factor caused by the nonlinearity of soil. The relations between shear modulus ratio and effective shear strain obtained from the strong motion accelerograms in good agreement with those from the laboratory tests of soil samples.

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