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DYNAMIC RESPONSE OF CYLINDRICAL TANKS STORING A VISCOELASTIC MATERIAL

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ABSTRACT

Making use of a relatively simple, approximate but reliable method of analysis, a study is made of the responses to horizontal base shaking of vertical, circular cylindrical tanks filled with a uniform viscoelastic material. Both rigid and flexible tanks are considered. The method of analysis is highlighted, and numerical data are presented that elucidate the underlying response mechanisms and the effects and relative importance of the major parameters of the problem. In addition to the characteristics of the ground motion, the parameters examined are the ratio of tank-height to tank-radius and a dimensionless measure of the relative flexibility of the tank wall and the contained material. The response quantities studied are the maximum values of the total force on or base shear in the tank wall and the corresponding base moment.

KEYWORDS

Tanks; silos; radioactive wastes; viscoelastic solid; dynamic response.

INTRODUCTION

The study reported here is motivated by the need for improved understanding of the effects of earthquakes on vertical cylindrical tanks in nuclear facilities storing high-level radioactive wastes. The responses of these systems are normally evaluated on the assumption that the waste may be modeled as an incompressible, inviscid liquid. Although the mechanical properties of the wastes in such tanks cannot accurately be defined at this time, their representation as ideal liquids may not be appropriate, and it is desirable to consider other idealizations. In this paper, the tank content is modeled as a uniform viscoelastic solid that is free at its upper surface and is bonded to a rigid base undergoing a uniform horizontal motion.

The objective of the paper is twofold: (a) To highlight a simple, approximate, yet reliable method of analysis; and (b) through the study of comprehensive numerical solutions, to elucidate the underlying response mechanisms, and the effects and relative importance of the more important parameters of the problem. The emphasis is on the presentation and interpretation of the results rather than on the method of analysis.

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The problem considered is also of interest in the evaluation of the seismic response of grain-storage silos. To the authors' knowledge, the most comprehensive study of the latter problem is the one reported by Rotter and Hull (1989). Although of great value, the information presented therein is limited, however, to the static effects of the rigid-body inertia forces and does not provide for the true dynamic aspects of the problem.

SYSTEM CONSIDERED

The system considered is shown in Fig. 1. It is a vertical, circular cylindrical tank of radius R and height H that is filled with a homogeneous viscoelastic solid. The tank is presumed to be fixed to a rigid base undergoing a space-invariant, uniform horizontal motion, the acceleration of which at any time t is $\ddot{x}_g(t)$. The contained solid is considered to be free at its upper surface and bonded in the horizontal plane both at the base and along its cylindrical boundary.

The properties of the medium are defined by its mass density ρ, the shear modulus of elasticity G, Poisson's ratio ν, and the damping factor δ, which is considered to be frequency-independent and the same for both shearing and axial deformations. The latter factor is the same as the $\tan\delta$ factor used by one of the authors and his associates in studies of foundation dynamics and soil-structure interaction (e.g., Veletsos and Verbic, 1973; Veletsos and Dotson, 1988) and twice as large as the percentage of critical damping used by other authors in related studies (e.g., Roesset *et al.*, 1973; Pais and Kausel, 1988). The corresponding properties of the tank wall are denoted by ρ_w, G_w, ν_w and δ_w, and the thickness of the wall, considered to be uniform, is denoted by t_w. Points in the contained medium are defined by the cylindrical coordinate system, r, θ, z, the origin of which is taken at the center of the tank base, as shown in Fig. 1.

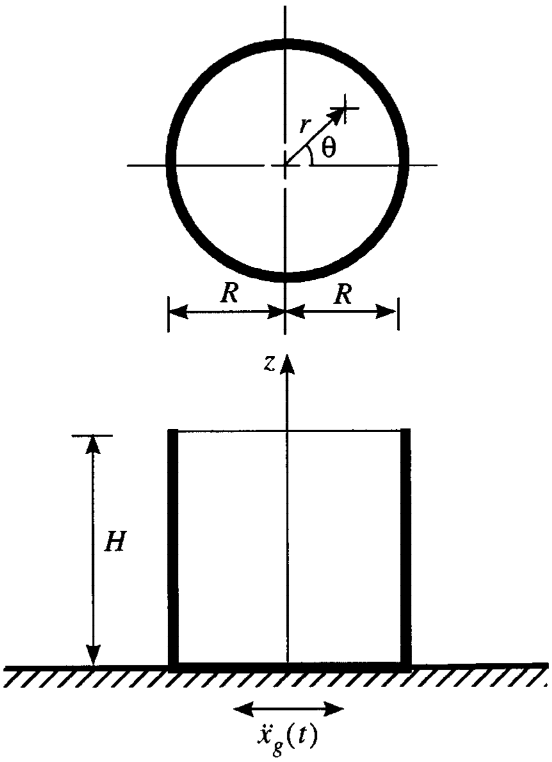

Fig. 1 System Considered

METHOD OF ANALYSIS

The method of analysis employed is similar to that used by Veletsos and Younan (1994) for the evaluation of the dynamic soil pressures induced by horizontal base shaking on a vertical rigid cylinder embedded in a viscoelastic stratum. It assumes the absence of any dynamic vertical normal stresses and that the horizontal variations of the vertical displacements of the contained material are negligible, so that the components of the shearing stresses on the top and bottom faces of an infinitesimal horizontal element, τ_{zr} and $\tau_{z\theta}$, may be expressed as

$$\tau_{zr} = G^*\frac{\partial u}{\partial z} \qquad \text{and} \qquad \tau_{z\theta} = G^*\frac{\partial v}{\partial z} \tag{1}$$

where u and v are the radial and circumferential displacement components of the contained material relative to the moving base; $G^* = G(1 + i\delta)$ is the complex-valued shear modulus; and $i = \sqrt{-1}$. The reliability of these assumptions has been confirmed for long, rigid rectangular tanks (Veletsos *et al.*, 1995) by comparing the results obtained by the present general approach with those obtained by Wood's exact solution (1973).

In the analysis of flexible tanks, it is further assumed that the tank wall responds as a uniform cantilever shear-beam, with its cross-section remaining circular during vibration.

For a harmonic input motion of acceleration $\ddot{x}_g(t) = \ddot{X}_g e^{i\omega t}$, in which $\ddot{X}_g$ is the acceleration amplitude and ω is the circular frequency of the motion, the steady-state radial and circumferential displacements of the medium, u and v, may be expressed as

$$u(\xi, \theta, \eta, t) = \sum_{n=1}^{\infty} U_n(\xi) \sin\left[\frac{(2n-1)\pi}{2}\eta\right] \cos\theta\, e^{i\omega t} \tag{2}$$

$$v(\xi, \theta, \eta, t) = \sum_{n=1}^{\infty} V_n(\xi) \sin\left[\frac{(2n-1)\pi}{2}\eta\right] \sin\theta\, e^{i\omega t} \tag{3}$$

where $\xi = r/R$ and $\eta = z/H$ are dimensionless position coordinates; the sine functions represent the natural modes of vibration of the contained solid when it is considered to act as a laterally unconstrained, vertical cantilever shear-beam; and U_n and V_n are the amplitudes of the modal components of the radial and circumferential displacements. Note that these expressions automatically satisfy the boundary conditions of zero displacements along the bottom surface and zero normal and shear stresses along the top surface.

The displacement amplitudes $U_n(\xi)$ and $V_n(\xi)$ are determined by substituting (2) and (3) into the governing partial differential equations of motions of the contained material, and solving the resulting ordinary equations subject to the appropriate continuity conditions along the cylindrical boundary. For a rigid tank, $U_n(1) = V_n(1) = 0$, whereas for a flexible tank, $U_n(1) = -V_n(1) = U_n^w$, where U_n^w is the amplitude of the nth component of the tank wall displacement, $u_w(\eta, t)$, when the latter is expressed in the form

$$u_w(\eta, t) = \sum_{n=1}^{\infty} U_n^w \sin\left[\frac{(2n-1)\pi}{2}\eta\right] e^{i\omega t} \tag{4}$$

The radial normal stresses, σ_r, and the circumferential shearing stresses, $\tau_{r\theta}$, are then determined from

$$\sigma_r = \frac{2}{1-\nu} G^* \frac{\partial u}{\partial r} + \frac{2\nu}{1-\nu} G^* \left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right) \tag{5}$$

$$\tau_{r\theta} = G^* \left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}\right) \tag{6}$$

It should be noted that both the displacement and stress components for a system with a flexible wall are functions of the as yet undetermined displacement amplitudes U_n^w. The latter are obtained by evaluating the stress components along the cylindrical boundary, determining the resulting force per unit of wall height and satisfying the governing differential equation of motion for the tank wall on the assumption that it responds as a cantilever shear-beam. The shear and bending moment in the wall are then determined by statics from the forces acting on it.

For a rigid tank, the solution depends on Poisson's ratio ν, the corresponding damping factor δ, and the frequency ratio ω/ω_1 where ω_1 represents the fundamental circular frequency of the material when it is considered to act as an unconstrained, vertical cantilever shear-beam. The latter frequency is given by

$$\omega_1 = \frac{\pi}{2}\frac{v_s}{H} \tag{7}$$

where $v_s = \sqrt{G/\rho}$ is the shear-wave velocity of the material. In addition to these parameters, the solution for a flexible tank, depends on: (a) the relative flexibility of the tank wall and contained material, defined by the ratio $GR/G_w t_w$; (b) their relative mass, defined by the ratio $\rho_w t_w/\rho R$; and (c) the damping factor for the wall, δ_w.

Because of the assumption of vanishing dynamic vertical normal stresses, the component of the foundation moment induced by the pressures acting on the tank base cannot be evaluated directly. However, this difficulty can be overcome by first computing the overturning base moment induced by the inertia forces of the tank and the contained material, and substracting from it the corresponding moment induced by the dynamic wall pressures. A more detailed account of the method of analysis will be presented elsewhere.

With the harmonic response of the system established, the response to an arbitrary transient excitation is evaluated by Fourier transform techniques.

PRESENTATION OF RESULTS

It is desirable to begin by examining the responses obtained for excitations the dominant frequencies of which are small compared to the fundamental frequency of the tank and its contents. Such excitations and the resulting effects will be referred to as static, a term that should not be confused with that normally used to represent the effects of gravity forces. The static effects in the following sections will be identified with the subscripts st.

Static Effects

In Fig. 2(a), the static value of the base shear in the tank wall, $(Q_b)_{st}$, is plotted as a function of the slenderness ratio of the system, H/R, for fixed values of the relative flexibility parameter $d_w = GR/G_w t_w$. A zero value of the latter parameter defines a rigid tank. The mass of the tank wall in these and all other solutions that follow is considered to be negligible, and Poisson's ratio for the contained material is taken as $\nu = 1/3$. The results are normalized with respect to the product of the total retained mass, m, and the maximum ground acceleration, $\ddot{X}_g$. This product is clearly equal to the total inertia of the retained material when it is considered to act as a rigid body.

It is observed that the base shear, and hence the proportion of the retained mass contributing to this shear, is highly dependent both on H/R and on the relative flexibility parameter d_w. For rigid tanks with values of H/R greater than about 3, the inertia forces for all the retained material are effectively transmitted to the walls by horizontal extensional action, and practically the entire mass of the tank content may be considered to be effective. With decreasing H/R, a progressively larger portion of the inertia forces gets transferred by horizontal shearing action to the base, and the portion of the retained mass that contributes to the wall forces is reduced.

The effect of wall flexibility is to reduce the horizontal extensional stiffness of the contained material relative to its shearing stiffness, and this reduction, in turn, reduces the magnitudes of the resulting pressures on and associated forces in the tank wall. The reduced response of the flexible tanks is in sharp contrast to the well

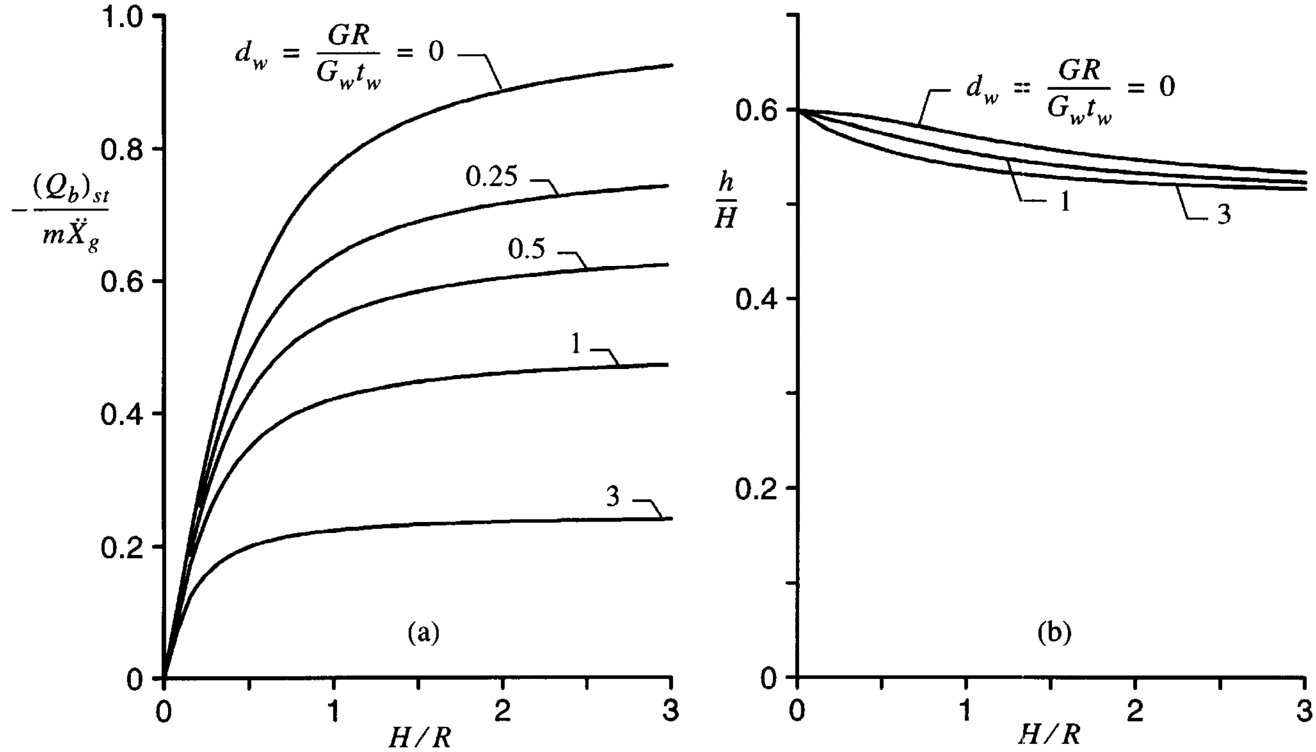

Fig. 2 Effects of tank wall flexibility on (a) static value of base shear in tank wall, $(Q_b)_{st}$, and (b) effective height, h; systems with $\rho_w = 0$ and $\nu = 1/3$

established behavior of liquid-containing tanks, for which the effect of wall flexibility is to increase rather than decrease the dominating, impulsive components of the wall pressures and forces. This matter is considered further in a later section.

The static value of the overturning base moment induced by the wall pressures, $(M_b)_{st}$, may conveniently be expressed as the product of the base shear and an appropriate height, h, to be referred to as the effective height. The ratio h/H is displayed in Fig. 2(b). It is observed that, almost independently of the relative flexibility parameter d_w, its value varies from 0.6 for broad tanks with values of H/R tending to zero to 0.5 for the rather tall, slender tanks. For the broad tanks, the wall pressures increase from the base to the top approximately as a quarter-sine wave, whereas for the rather slender tanks, the distribution is practically uniform.

Dynamic Effects

Figure 3 shows the dynamic amplification factor, AF, for base shears in rigid tanks subjected to the N-S component of the 1940 El Centro, California earthquake ground motion record, for which the maximum acceleration is $\ddot{X}_g = 0.312\,g$. The AF is defined as the ratio of the maximum dynamic base shear, $(Q_b)_{max}$, to its corresponding static value, $(Q_b)_{st}$. As before, the tank wall in these solutions is considered to be massless, and Poisson's ratio and the damping factor of the retained material are taken as $\nu = 1/3$ and $\delta = 0.1$. The results are plotted as a function of the fundamental period of the tank-solid system, $T_{11} = 2\pi/\omega_{11}$, where ω_{11} is the corresponding circular frequency, given by

$$\omega_{11} = \sqrt{1 + 4.589(H/R)^2}\,\omega_1 \tag{8}$$

Three values of H/R in the range between zero and 3 are considered. Equation (8) applies only to rigid tanks storing materials with $\nu = 1/3$. Clearly, the effect of wall flexibility will be to decrease the natural frequency of the system and increase its period.

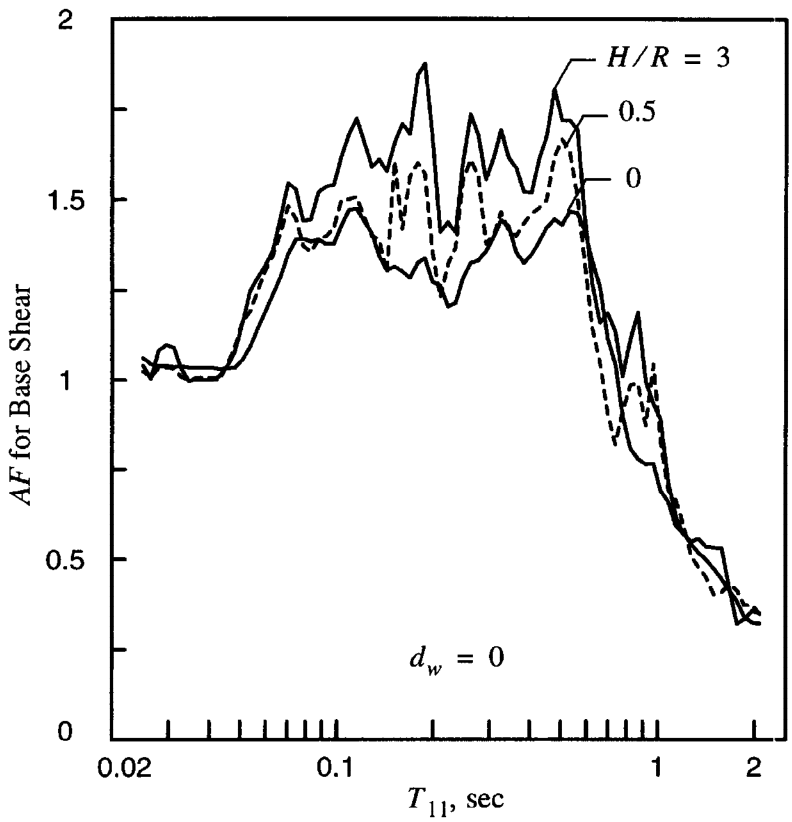

Fig. 3 Amplification factors for base shear in wall of rigid tanks subjected to El Centro record; systems with $\rho_w = 0$, $\nu = 1/3$ and $\delta = 0.1$

The plots in Fig. 3 are similar to, but by no means the same as, the response spectra for similarly excited, viscously damped single-degree-of-freedom systems. Specifically, for low-natural-period, stiff materials, the maximum values of the dynamic base shear are equal to their static values, and the amplification factors are unity. With increasing flexibility of the contained material or increasing natural period of the system, the amplification factors increase, and after attaining nearly horizontal plateaus, they reach values that may be substantially less than unity.

The absolute maximum amplification factor for the conditions considered ranges from 1.47 to 1.87, the larger value corresponding to the taller, more slender systems. As the tank radius is decreased relative to its height, the waves in the retained material must travel progressively shorter distances before they get reflected by the rigid boundary; accordingly, they are not affected as much by material damping as would be the case for the broader systems with the larger radii.

Comparable trends have also been found for flexible tanks, although the peak amplification factors for such tanks tend to be substantially larger than for the corresponding rigid tanks. This is demonstrated in Fig. 4 which refers to systems with a relative flexibility factor $d_w = 1$. It is important to note that the period T_{11} in this figure refers to the fundamental period of the particular flexible tank under consideration, not of the corresponding rigid tank. The evaluation of the natural periods of flexible tanks is beyond the scope of this paper but will be considered in another publication.

Considering that the fundamental period of many systems falls in the amplified region of the plots presented in Figs. 3 and 4, it is of special interest to examine in some detail the values of the amplification factors in this region. Figure 5(a) shows the average amplification factor for base shear in the wall of systems with fundamental periods in the range of 0.1 to 0.5 sec. The periods referred to are, again, those of the flexible system under consideration. The results are plotted as a function of the relative flexibility parameter d_w for four different values of H/R. The damping factor for the tank wall in these solutions is $\delta_w = 0.04$ or 2 percent of critical damping.

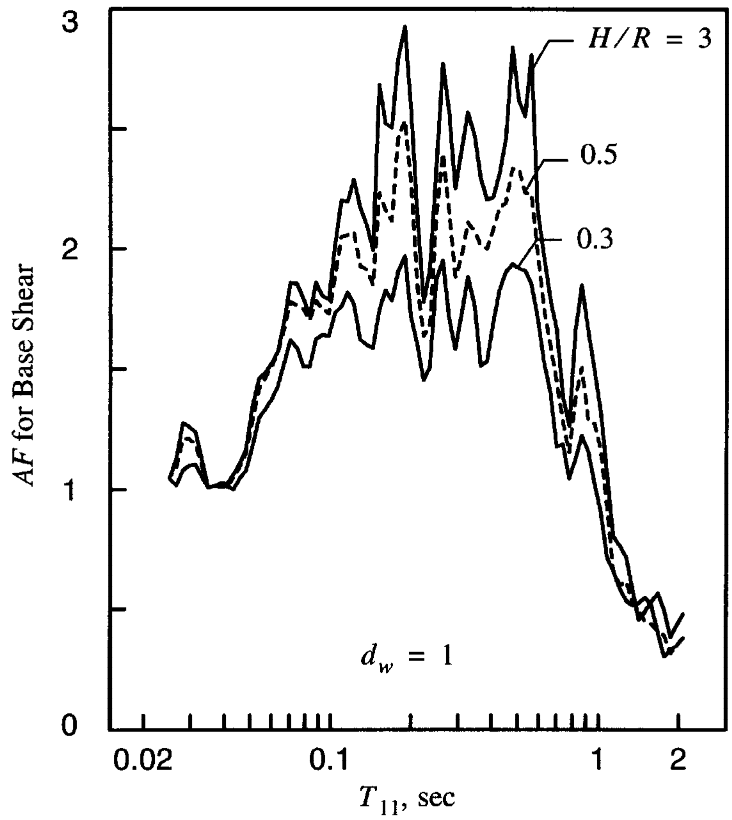

Fig. 4 Amplification factors for maximum base shear in wall of flexible tanks subjected to El Centro record; systems with $\rho_w = 0$, $d_w = 1$, $\delta_w = 0.04$, $\nu = 1/3$ and $\delta = 0.1$

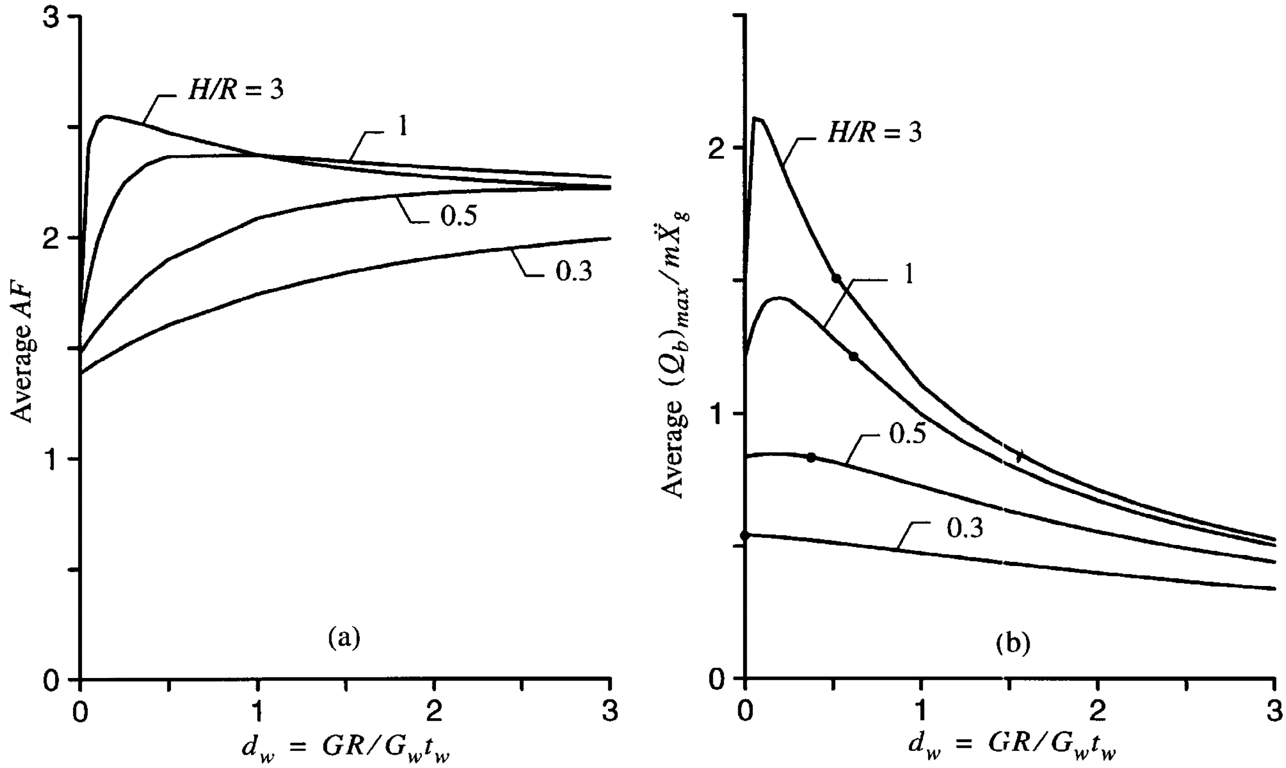

Fig. 5 Effects of tank wall flexibility on (a) average amplification factor of base shear in wall of systems with $0.1 \le T_{11} \le 0.5$ sec and (b) on corresponding value of $(Q_b)_{max}/m\ddot{X}_g$; systems with $\rho_w = 0$, $\delta_w = 0.04$, $\nu = 1/3$ and $\delta = 0.1$ subjected to El Centro record.

It is observed that, with minor deviations, the average AF increases with H/R and that, for all but the very slender systems, it also increases with increasing values of the relative flexibility factor d_w. Expressed differently, the larger the H/R or d_w, the lower is the effective damping capacity of the system. The absolute maximum amplification factor for the conditions considered is about 2.5.

In Fig. 5(b), the average value of the base shear within the period range of 0.1 to 0.5 sec is replotted normalized with respect to the product of the total contained mass and the maximum ground acceleration. Note that for the combination of parameters represented by points to the right of the heavy dots, the effect of wall flexibility is to reduce the response to levels that may be substantially lower than those applicable to rigid tanks. This reduction, which is due to the increasing capacity of the material in flexible tanks to transfer the inertia forces to the base by horizontal shearing action, is, as already noted, in sharp contrast with the response of liquid-containing tanks for which the effect of wall flexibility is to increase rather than decrease the response. Only for extremely slender tanks for which the relative shearing stiffness of the contained material becomes negligible, as for a liquid, does the wall flexibility increase the response. For the range of parameters normally encountered in practice, the dynamic forces for tanks storing a viscoelastic solid can be expected to decrease with increasing wall flexibility.

CONCLUSIONS

A study of the response to horizontal base shaking of a vertical cylindrical tank storing a viscoelastic material has been reported. Both the long-period, quasi-static effects and the dynamic effects induced by an actual earthquake ground motion have been examined. The numerical data presented and their analysis provide not only valuable insights into the effects and relative importance of the principal parameters involved, but also a conceptual framework for the planning, implementation and interpretation of the results of solutions for more involved systems as well.

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