11 WCEE

Copyright © 1996 Elsevier Science Ltd
Paper No. 577. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

IMPULSE SERIES METHOD IN THE DYNAMIC ANALYSIS OF STRUCTURES

ADRIAN I. DANILIU

Romanian Association for Earthquake Engineering
Aleea Crizantemelor 6, Ap. 3, 6600 Botosani, Romania

ABSTRACT

The impulse series method has been proposed by researchers from Romania where it has been called Bukovine method. By means of the first mean value theorem from integral calculus, any Duhamel integral may be transformed into a sum. By this way all the relationships in linear analysis may be rewritten into another form.

KEY WORDS

Impulse, Duhamel integral, energy, probabilistic analysis, mathematical model, nonlinear.

INTRODUCTION

In linear dynamic analysis of structures, Duhamel integrals have to be computed. An analytic solution of these integrals may be obtained only in the case of some particular types of excitations (harmonic input, pulses). Step-by-step integration leads to a great complexity of calculus. Using the first mean value theorem from integral calculus one may seek an integration formula for Duhamel integrals.

LINEAR SYSTEMS

SDOF Systems

The following equations of motion that govern the behaviour of a single-degree-of-freedom system and a multi-degree-of-freedom system are taken into account :

$$\ddot{x} + 2\vartheta\omega_0\dot{x} + \omega_0^2 x = -\ddot{u} \quad (1)$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\}\ddot{u} \quad (2)$$

where x, $\dot{x}$, $\ddot{x}$ are respectively the relative displacement, velocity and acceleration of the SDOF system, ω_0 = angular frequency, ϑ = viscous damping factor, $\ddot{u}$ = base acceleration. $[M]$, $[C]$, and $[K]$ are respectively the diagonal mass matrix, the damping matrix and the stiffness matrix of the MDOF system. The vectors of relative displacements, velocities and accelerations of MDOF system are denoted as $\{x\}$, $\{\dot{x}\}$, $\{\ddot{x}\}$.

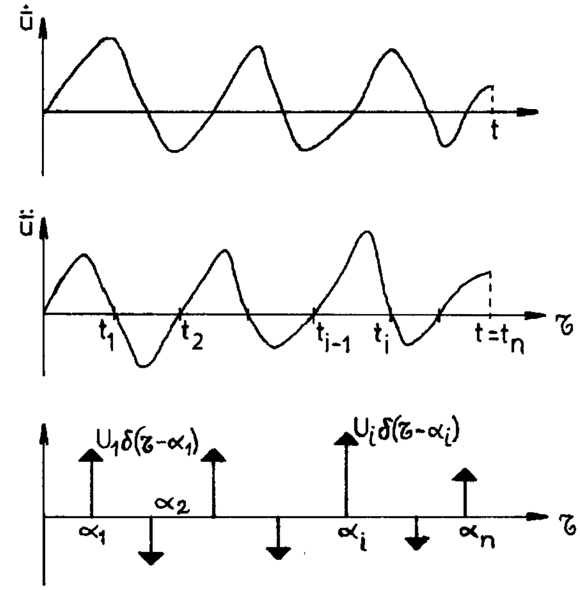

Fig.1. Correlation between u̇, ü and Dirac impulses

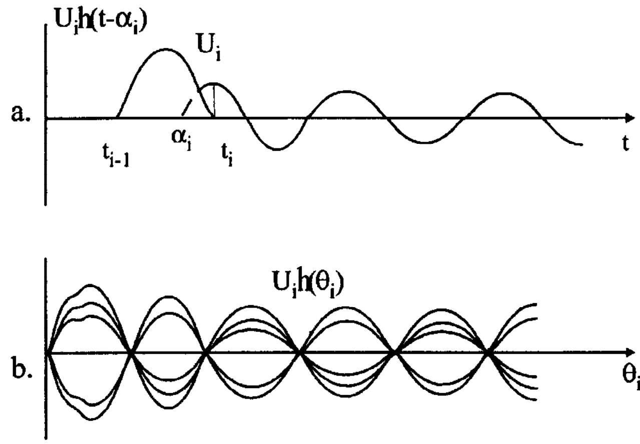

Fig.2. Free vibrations ;

If the SDOF system is initially at rest, $\dot{x}(0) = x(0) = 0$, then the solution of Eq. (1) is Duhamel integral :

$$x(t) = -\int_0^t \ddot{u}(\tau)h(t-\tau)d\tau \qquad (3)$$

h denoting the impulse response of system, $h(t-\tau) = \exp[-\vartheta\omega_0(t-\tau)]\sin\omega_a(t-\tau)/\omega_a$, $\omega_a = \omega_0\sqrt{1-\vartheta^2}$, t a fixed time instant, τ = variable of integration..

Let $\Delta: \{0 = t_0 < t_1 < \ldots t_{n-1} < t_n = t\}$ be a division of [0,t] with t_i chosen so that $\ddot{u}(t_i) = 0$ for $i=\overline{1,n}$ (Fig. 1). Neglecting the sign in the front of integral (3) one may write :

$$x(t) = \int_0^{t_1} \ddot{u}(\tau)h(t-\tau)d\tau + \int_{t_1}^{t_2} \ddot{u}(\tau)h(t-\tau)d\tau + \ldots + \int_{t_{n-1}}^{t} \ddot{u}(\tau)h(t-\tau)d\tau \qquad (4)$$

Using the first mean value theorem (Siretchi, 1985) it result the existence of $\alpha_i \in [t_{i-1}, t_i]$ so that :

$$\int_{t_{i-1}}^{t_i} \ddot{u}(\tau)h(t-\tau)d\tau = h(t-\alpha_i)\int_{t_{i-1}}^{t_i} \ddot{u}(\tau)d\tau \qquad (5)$$

where α_i is not unique determined in the interval $[t_{i-1}, t_i]$. As a consequence of the equality $\int_{t_{i-1}}^{t_i} \ddot{u}(\tau)d\tau = \dot{u}(t_i) - \dot{u}(t_{i-1})$, denoting the difference of base velocities $\dot{u}(t_i) - \dot{u}(t_{i-1}) = U_i$ it follows :

$$x(t) = \sum_{i=1,n} U_i h(t-\alpha_i) \qquad (6)$$

The relative velocity may be obtained in a similar manner :

$$\dot{x}(t) = \int_0^t \ddot{u}(\tau)\dot{h}(t-\tau)d\tau = \sum_{i=1,n} U_i \dot{h}(t-\beta_i) \qquad (7)$$

where $\dot{h}(t-\tau)$ is the first derivative of h(t-τ) with respect to t and $\beta_i \in (t_{i-1}, t_i]$. It should be noted that α_i, n, β_i depend on variable t, $\alpha_i = \alpha_i(t)$, n = n(t), $\beta_i = \beta_i(t)$. The values U_i are constant for $i = \overline{1, n-1}$ but $U_n = U_n(t) = \int_{t_{n-1}}^{t} \ddot{u}(\tau)d\tau$. Taken separately one term from the sum (6) it follows :

$$x_i = U_i h(t-\alpha_i) = \frac{U_i}{\omega_a} e^{-\vartheta\omega_0(t-\alpha_i)} \sin\omega_a(t-\alpha_i) \qquad (8)$$

It may be seen that $x_i(t)$ represents the response of the SDOF system to a free damped vibration, with the initial conditions $\dot{x}(\alpha_i) = U_i$, $x(\alpha_i) = 0$ (see Fig. 2.).

The total response of the system subjected to ground acceleration may be evaluated as a superposition of (n-1) free vibrations contributions :

$$x(t) = \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} U_i h(\theta_i) \tag{9}$$

in which $\theta_i = t - \alpha_i(t)$, (see Fig. 2b). Only the last term is a forced vibration $x_n(t)=U_n(t)h(t-\alpha_n)$. The values α_i are not known. Practical computations have been shown α_i lies in the vicinity of the time instants where $\ddot{u}(\tau)$ attains its maximum values on the interval $[t_{i-1}, t_i]$.

MDOF Systems

Turning to Eq. (2), using the modal transformation $\{x(t)\} = [v]\{q\}$, in which $[v]$ is the modal matrix of the N-degrees of freedom system and $\{q\}$ the vector of normal coordinates, one obtains the displacement corresponding to the l-th generalised coordinate as :

$$x_l(t) = \sum_{j=1}^{N}\sum_{i=1}^{n} m_j U_i g_{lj}(t-\alpha_i^j) \ , \quad l = \overline{1,N} \tag{10}$$

where $g_{lj} = \sum_{r=1,n} v_l^r v_j^r h_r(t)$, $h_r(t) = \exp(-\vartheta_r \omega_r t)\sin\omega_r t$, $r = \overline{1,N}$, m_j=mass at the level j, ω_r and ϑ_r are respectively the angular frequency and damping ratio in the r-th mode.

NONLINEAR SDOF SYSTEMS

Consider a SDOF system defined by its mass m, damping constant c, initial stiffness k_0. The total displacement x(t) is divided into two parts : an elastic instantantaneous displacement $x_e(t)$ and a plastic instantaneous displacement $x_p(t)$. The stiffness at time t is given by the function k(x,t). Thus it resulted a mathematical model that was used in author's own papers (Daniliu 1986,1987) :

$$m\ddot{x} + c\dot{x}_e + k(x,t)x_e = -m\ddot{u} \tag{11}$$

Upon substitutions of $x_e=x - x_p$, $\dot{x}_e = \dot{x} - \dot{x}_p$, $k(x,t)=k_0 - \Delta k(x,t)$, into Eq.(1), rearranging terms and dividing through by m, the equation of motion of the nonlinear system becames :

$$\ddot{x} + 2\vartheta\omega_0\dot{x} + \omega_0^2 x = -\ddot{u} + R(t,x(t)) \tag{12}$$

where $c\dot{x} + k_0 x_p - \Delta k(t,x)(x - x_p) = R(t,x(t))$, $\omega_0^2=k_0/m$, $\vartheta=c/2\omega_0 m$. By comparing Eq. (12) with Eq. (1), the solution of Eq. (12) is :

$$x(t) = \sum_{i=1,n} U_i h(t-\alpha_i) + \int_0^t R(\tau,x(\tau))h(t-\tau)d\tau \tag{13}$$

Theoretically, the motion of the nonlinear system can viewed as the motion of the initial linear system subjected to a modified excitation $\ddot{u}(t) + R(t,x)$. Evidentely R(t,x) is not known in advance.

ENERGY ASPECTS

The energy equations will be formulated with respect to the relative frame of reference, where the inertia force $m\ddot{u}$ is treated as an external force. The following symbols are used : E_c= kinetic energy; E_p= potential energy; E_d= viscous damping energy; E_h= hysteretic dissipated energy; E_m= mechanical energy due to the sum of kinetic and potential energies; E_{in}= input energy due by the action of extenal force; E_a= absorbed energy equals to the work of external force; k= stiffness of the linear system; c= viscous damping coefficient; c= $2m\vartheta\omega_0$; m= mass.

Linear systems

The total motion of the linear SDOF system consists of (n-1) free vibration and a forced vibrations, see Eqs. (6), (8), (9). For one of these vibrations it may be written :

$$E_c^i = \frac{m\dot{x}_i^2}{2} = \frac{mU_i^2 h'^2(t-\beta_i)}{2}; \qquad E_p^i = \frac{kx_i^2}{2} = \frac{kU_i^2 h^2(t-\alpha_i)}{2} \qquad (14)$$

$$E_d^i = c\int_0^t \dot{x}_i dt = c\int_0^t U_i^2 h'^2(t-\beta_i)dt; \qquad E_a^i = E_{in}^i = \frac{mU_i^2}{2} = E_c^i + E_p^i + E_d^i$$

where $h'^2(t-\beta_i)$ is the square of the derivative of the impulse response function, included in Eq. (7). The total displacement is $x(t) = x_1(t)+x_2(t)+...+x_n(t)$ therefore at a certain time instant the energies of the total motion are :

$$E_c(t) = \frac{m}{2}\sum_{i=1}^{n}\dot{x}_i^2 + \frac{m}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\dot{x}_i\dot{x}_j; \qquad E_p(t) = \frac{k}{2}\sum_{i=1}^{n}x_i^2 + \frac{k}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i x_j; \qquad (15)$$

$$E_d(t) = c\sum_{i=1}^{n}\int_0^t \dot{x}_i^2 dt + c\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^t \dot{x}_i\dot{x}_j dt; \qquad i \neq j$$

where $n= n(t)$, x_i and $\dot{x}_i$ result from Eq. (8). The absorbed energy at time t_e corresponding to the end of the ground motion can be expressed as :

$$E_a(t_e) = E_c(t_e)+E_p(t_e)+E_d(t_e) = \sum_{i=1}^{n_e}(E_c^i + E_p^i + E_d^i) + \sum_{i=1}^{n_e}\sum_{j=1}^{n_e}\left(\frac{m}{2}\dot{x}_i\dot{x}_j + \frac{k}{2}x_i x_j + c\int_0^t \dot{x}_i\dot{x}_j dt\right) \qquad (16)$$

$$\sum_{i=1}^{n_e}(E_c^i + E_p^i + E_d^i) = m\sum_{i=1}^{n_e}\frac{U_i^2}{2}$$

in which n_e= total number of impulses of the considered earthquake. The increment in absorbed energy during the interval $[t_{i-1},t_i]$, $\Delta E_a = E_a(t_i) - E_a(t_{i-1})$ is given by (17), if $\Delta\dot{x}_i\dot{x}_j = \dot{x}_i(t_i)\dot{x}_j(t_i) - \dot{x}_i(t_{i-1})\dot{x}_j(t_{i-1})$:

$$\Delta E_a = \frac{mU_i^2}{2} + \sum_{1}^{i-1}\sum_{1}^{j-i}\left(\frac{m}{2}\Delta\dot{x}_i\dot{x}_j + \frac{k}{2}\Delta x_i x_j + c\int_{t_{i-1}}^{t_i}\dot{x}_i\dot{x}_j dt + \sum_{j=1}^{i-1}\left(\frac{m}{2}(\dot{x}_i\dot{x}_j) + \frac{k}{2}(x_i x_j) + \int_{t_{i-1}}^{t_i}\dot{x}_i\dot{x}_j dt\right)\right) \qquad (17)$$

On the other hand integrating over x Eq. (1) one may be written $E_a(t) = \int_0^t m\ddot{u}\dot{x}\,dt = E_c + E_p + E_d$ and $\Delta E_a = \int_{t_{i-1}}^{t_i} m\ddot{u}\dot{x}\,dt = \Delta E_c + \Delta E_p + \Delta E_d$. Applying the first mean value theorem, two formulas are found :

$$E_a(t_e) = \frac{m}{2}\sum_{i=1}^{n_e}U_i^2\,\dot{h}(b_i^i) + m\sum_{i=1}^{n_e}\sum_{j=i+1}^{n_e}U_i U_j\,\dot{h}(b_i^j) \qquad \Delta E_a = \frac{mU_i^2}{2}\dot{h}(b_i^i) + m\sum_{j=1}^{i-1}U_i U_j\,\dot{h}(b_i^j) \qquad (18)$$

where $b_i^j = t_i^j - \beta_i(t_i^j)$, $t_i^j \in [t_{j-1},t_j]$, $\beta_i \in [t_{i-1},t_i]$, $h'(b_i^j) = \exp(-\vartheta\omega_0 b_i^j)(\cos\omega_a b_i^j - \vartheta_1\sin\omega_a b_i^j)$, $\vartheta_1 = \vartheta\sqrt{1-\vartheta^2}$.

The time instants b_i^j, t_i^j are not exactly known. If the whole energy induced in a system is not damped or dissipated it cumulates to the next cycles. Actually, the increment of the absorbed energy ΔE_a contains the input energy in the excursion i, denoted $mU_i^2/2$ and the mechanical energy $E_m(t_{i-1}) = E_c(t_{i-1}) + E_p(t_{i-1})$ that has not consumed in the previous semi-cycle i-1, (see Fig. 3). The increment in input energy ΔE_{in} may be included in the equality $\Delta E_{in} + E_c(t_{i-1}) + E_p(t_{i-1}) = E_c(t_i) + E_p(t_i) + \Delta E_l$ where ΔE_l denotes the increment in lost energy. Further it results :

$$\Delta E_{in} = \frac{mU_i^2}{2} = \Delta E_c + \Delta E_p + \Delta E_l \qquad (19)$$

where the sign "Δ" denotes the change in kinetic energy, potential energy and lost energy ($\Delta E_c = E(t_i) - E_c(t_{i-1})$)

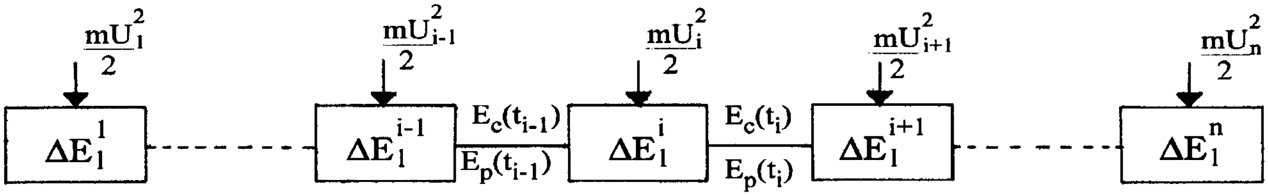

Fig.3. Input- output diagram

In general ΔE_d differs from ΔE_l which is equivalent to the presence of a damping that is different from the viscous damping. The increment in input energy ΔE_{in} is the same for all the structures subjected to a certain earthquake. The increment in absorbed energy is different and $\Delta E_{in} \neq \Delta E_a$. The absorbed energy depends on the structure characteristics ϑ, ω. Physical considerations suggest when k tends to infinity E_a approaches zero and for $k\to 0$ it results $E_a \to 0$. In Eqs. (18) for $\vartheta = 0$, $h'(b_j^i) = \cos\omega_0 b_j^i$ therefore $\Delta E_a \neq \Delta E_{in}$. The total input energy on the whole duration of the design earthquake is :

$$E_{in}(t_e) = m\sum_{i=1}^{n_e}\frac{U_i^2}{2} \tag{20}$$

When ΔE_a-ΔE_d>0, the mechanical energy of the system increases, $\Delta E_m = \Delta E_c + \Delta E_p > 0$. It is an unfavourable situation since the maximum displacement in the next cycle i+1 will increase. Inversely if ΔE_a-ΔE_d<0 then the system damps more energy that it absorbs, the amplitudes of the response will decrease (Hangan and Crainic 1980).

Nonlinear Systems

By integrating over x Eq. (11), using the substitution $x = x_e + x_p$, $dx = dx_e + dx_p$, one obtains :

$$\underbrace{\frac{m\dot{x}^2}{2}}_{E_c} + \underbrace{\int_0^t c\dot{x}_e\,\dot{x}\,dt}_{E_d} + \underbrace{\frac{k(x,t)x_e^2}{2}}_{E_p} \underbrace{- \int_0^t \frac{x_e^2}{2}\frac{dk}{dt}dt + \int_0^t k\dot{x}_e\,\dot{x}_p\,dt}_{E_h} = \underbrace{-\int_0^t m\ddot{u}\dot{x}\,dt}_{E_a} \tag{21}$$

where the last two integrals in the left member side of the equation represent hysteric energy E_h and the potential (elastic) energy is given as $E_p(t) = kx_e^2/2$.

SEISMIC RESPONSE AS A GENERALISED FUNCTION

The concept of generalized function (or distribution) is more comprehensive than the one of the ordinary function. Any classical function may be treated as a generalized function. In the domain of generalized functions seismic acceleration and impulse response function have to be defined on the whole real line :

$$\ddot{u}_t(\tau) = \begin{cases} 0 & \text{if } \tau < 0 \text{ or } \tau > t \\ \ddot{u}(\tau) & \text{if } 0 \le \tau \le t \end{cases} \tag{22}$$

where $t \ge 0$ is viewed as a parameter and τ is the variable of the functions. Function $h_t(\tau)$ is called test function. Using the relationship between physical systems and distributions and Eqs.(3), (6) the result of applying $\ddot{u}_t(\tau)$ to $\bar{h}_t(\tau)$ is expressed in two symbolic forms :

$$\bar{x}(t) = \langle \ddot{u}_t(\tau), \bar{h}_t(\tau) \rangle = \int_{-\infty}^{+\infty} \ddot{u}_t(\tau) h_t(\tau) d\tau = \sum_{i=1}^{n} U_i h(t-\alpha_i) \tag{23}$$

$$\bar{x}(t) = \langle \sum_{i=1}^{n} U_i \delta(\tau-\alpha_i), \bar{h}_t(\tau) \rangle = \int_{-\infty}^{+\infty} \sum_{i=1}^{n} U_i \delta(\tau-\alpha_i) h_t(\tau) d\tau = \sum_{i=1}^{n} U_i h(t-\alpha_i)$$

in which $\bar{x}(t)$ is the structure response, $\delta(\tau-\alpha_i(t))$ denote Dirac generalized functions concentrated at the points $\tau=\alpha_i(t)$. In both above equations t is constant. Hence for a certain t, in computation of the seismic response, the continuous base acceleration may be replaced by a series of Dirac impulses applied at the time instants $\alpha_i(t)$. Only for a specified impulse response function $\bar{h}_t(\tau)$ one may be written :

$$\ddot{u}_t(\tau)=\sum_{i=1}^{n} U_i\delta(\tau-\alpha_i(t)) \tag{24}$$

the impulse magnitudes being $U_i=\int_{t_{i-1}}^{t_i}\ddot{u}(\tau)d\tau$ and $n=n(t)$, $U_n=U_n(t)$, U_i=constant $(i=1,n-1)$, (see Fig.1).

The Fourier transforms of $\bar{x}(t)$, $\bar{h}_t(\tau)$, $\ddot{u}_t(\tau)$ are given by the following integrals :

$$X(\omega)=\int_{-\infty}^{+\infty}\bar{x}(\tau)e^{-j\omega\tau}d\tau\ ;\ H(\omega)=\int_{-\infty}^{+\infty}\bar{h}_t(\tau)e^{-j\omega\tau}d\tau\ ;\ U(\omega,t)=\int_{-\infty}^{+\infty}\sum_{i=1}^{n}U_i\delta(\tau-\alpha_i(t))e^{-j\omega\tau}d\tau=\sum_{i=1}^{n}U_ie^{-j\omega\alpha i} \tag{25}$$

The Fourier transform of the translated impulse response function $h(t-\tau)=\exp[-\vartheta\omega(t-\tau)]\sin\omega_a(t-\tau)/\omega_a$ is :

$$F(h(t-\tau))=e^{j\omega t}H(\omega) \tag{26}$$

where F denotes the Fourier transform and the receptance $H(\omega)=1/[\omega_0^2-\omega^2+j(2\vartheta\omega_0\omega)]$. By means of Parseval's equality (Gelfand and Shilov 1983) a similar relationship to Eq. (23) may be derived in the frequency domain :

$$\bar{x}(t)=<\ddot{u}_t(\tau),h_t(\tau)>=\frac{1}{2\pi}<U(\omega,t),H(\omega)e^{i\omega t}>=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U(\omega,t)H(\omega)e^{j\omega t}d\omega \tag{27}$$

Passing to the probalistic approach, $\ddot{u}$ is defined as $m[\ddot{u}]=\sum_{i=1}^{\bar{n}}E[U_i]\delta(\tau-\alpha_i)$ where E denotes the mean operator. The generalized autocorrelation function of the seismic acceleration is obtained from Eq. (24) :

$$R_{\ddot{u}}(t_1,t_2)=E[\ddot{u}_{t1}\times\ddot{u}_{t2}]=E\left[\sum_{i=1}^{n_1}U_i\delta(\tau_1-\alpha_i^1)\times\sum_{l=1}^{n_2}U_l\delta(\tau_2-\alpha_l^2)\right] \tag{28}$$

where the sign "x" denotes the direct product between two generalized functions, $n_1=n_1(t_1)$, $n_2=n_2(t_2)$, $n_2\geq n_1$. According to the definition of the direct product (Gelfand and Shilov 1983) and inverting the operator with the sums, it results :

$$R_{\ddot{u}}(t_1,t_2)=\sum_{i=1}^{n_1}\sum_{l=1}^{n_2}E[U_iU_l]\delta(\tau_1-\alpha_i{}^1,\tau_2-\alpha_l{}^2) \tag{29}$$

where $\delta(\tau_1-\alpha_i{}^1,\tau_2-\alpha_l{}^2)$ is the Dirac generalized function of two variables. If the impulses are uncorrelated, i.e. $E[U_iU_l]=0$ if $i\neq l$, one may be written :

$$R_{\ddot{u}}(t_1,t_2)=\sum_{i=1}^{n_1}E\left[U_i{}^2\right]\delta(\tau_1-\alpha_i{}^1,\tau_2-\alpha_i{}^2)\ ;\quad R_{\ddot{u}}(t)=\sum_{i=1}^{n}E\left[U_i{}^2\right]\delta(\tau_1-\alpha_i{}^1,\tau_2-\alpha_i{}^2)\ \text{ for } t_1=t_2=t \tag{30}$$

PROBABILISTIC APPROACH

Turning to the classical random functions the sum (6) is understood in the probabilistic sense. In general the magnitudes U_i are considered normal random variables with zero mean, except $U_n(t)$ which is a random function. The random functions $\alpha_i(t)$ depend on the lenghts of the subintervals $[t_{i-1},t_i]$. Hence $h(t-\alpha_i(t))$ are deterministic functions of random functions. The number of impulses is also a random function but with positive integer values. The mean and the autocorrelation of x are found as :

$$m_x(t)=\sum_{i=1}^{\bar{n}}E[U_ih(t-\alpha_i)] \qquad R_x(t_1,t_2)=E\left[\sum_{i=1}^{n_1}\sum_{l=1}^{n_2}U_iU_lh(t-\alpha_i^1)h(t-\alpha_i^2)\right] \tag{31}$$

where $\bar{n}$ is the mean number of impulses at time t, E=mean operator, $n_k=n_k(t_k)$, and $\alpha_i^k=\alpha_i^k(t_k)$, k=1,2. As an approximation if U_i are uncorrelated $E[U_iU_l]=0$, for $i\neq l$, and α_i are taken as deterministic functions, then it follows

$$m_x(t)=\sum_{i=1}^{\bar{n}}E[U_i]h(t-\alpha_i) \qquad E\left[x^2(t)\right]=\sum_{i=1}^{\bar{n}}E\left[U_i^2\right]h^2(t-\alpha_i)=\sum_{i=1}^{\bar{n}}D_ih^2(t-\alpha_i) \tag{32}$$

where $D_i=E[U_i^2]$ and $h^2(t-\alpha_i)=\exp[-2\vartheta\omega(t-\alpha_i)]\sin^2[\omega_a(t-\alpha_i)]/\omega_a^2$. The maximum values of $h^2(t-\alpha_i)$ are reached when $\omega_a(t-\alpha_i)=2k\pi\pm\pi/2-\tan^{-1}\vartheta_1$, with $k=0,\pm1,\pm2,...$ and $\vartheta_1=\vartheta/\sqrt{1-\vartheta^2}$ (the points where $h'(t-\alpha_i)$ vanishes). Retaining the value $\pi/2-\tan^{-1}\vartheta_1=\pi/2-\gamma_1$, one obtains $\max[h^2(t-\alpha_i)]=\exp[-2\vartheta_1(\pi/2-\gamma_1)]\sin^2(\pi/2-\gamma_1)/\omega_a^2=B^2/\omega_a^2$. If x(t) is a zero mean process $\sigma_x^2(t)=E[x^2(t)]$. Presuming that the maximum of the variance will be attains in the vicinity of the maximum impulse in absolute value, an approximative formula is :

$$\sigma_x^{max}\cong\frac{B}{\omega_a}\sqrt{\sum_{i=1,M}D_i} \qquad i=\overline{1,M} \tag{33}$$

where $\max\{|U_i|\}=U_M$; $D_M=E\left[U_M^2\right]$; the values of the coefficient B are given in Table 1.

Table 1.

ϑ	B
0	1
0.02	0.969260
0.05	0.925532
0.07	0.897837
0.1	0.858276

Another formula is based on the observation that the most defavourable situation occurs when $\omega_a(t-\alpha_M)-\gamma_1=\frac{\pi}{2}$; $\omega_a(t-\alpha_{M-1})-\gamma_1=\frac{3\pi}{2}$ and so on. The derived formula is :

$$\sigma_x^{max}\cong\frac{e^{\vartheta_1\gamma_1}\sin(\frac{\pi}{2}-\gamma_1)}{\omega_a}\sqrt{\sum_{i=1,M}D_ie^{-\vartheta_1(M-i+1)\pi}} \tag{34}$$

where the sum stops at the maximum impulse taken on its absolute value. Formula (33) overstimates the standard deviation, formula (34) is more adequate although it referrs to a limit situation. Equation (5) implies that for a given interval $[t_{i-1}, t_i]$ may be aproximatively evaluated as $h(t-\alpha_i)\cong p_1h(t-\tau_1)+p_2h(t-\tau_2)+...+p_{n_i}h(t-\tau_{ni})$ where $\tau_1, \tau_2, ..., \tau_{ni}$ are n_i points belonging to $[t_{i-1}, t_i]$, $p_r=\ddot{u}(\tau_r)/U_i$ and $U_i=\int_{t_{i-1}}^{t_i}\ddot{u}(\tau)d\tau$. The values $p_1, p_2, ..., p_{ni}$ are bounded between zero and unity and their sum is $p_1+p_2+...+p_{ni}=1$. One may be established and analogy with the sum of probabilities that gives a distribution function of a random variable. Let τ be the random variable and $p_r=\ddot{u}(\tau_r)/U_i$ the values of its probability density function $f_\theta^i(\tau)$.
One may be written $f_\theta^i(\tau_r)=p_r$, $r=1, 2, ..., n_i$. If $n_i\to\infty$ then it follows :

$$h(t-\alpha_i)=\int_{t_{i-1}}^{t_i}f_\theta^i(\tau)h(t-\tau)d\tau \tag{35}$$

One has reached to an outcome similarly to the computation formula for the mathemathical expectation of a function $h(t-\tau)$ of a random variable τ (Elishakoff 1983). On the whole interval [0, t] substituting (35) in (6) :

$$x(t)=\sum_{i=1,n}U_i\int_{t_{i-1}}^{t_i}f_\theta^i(\tau)h(t-\tau)d\tau=\int_0^t U(\tau)f_\theta(\tau)h(t-\tau)d\tau \tag{36}$$

where $U(\tau)=U_i$ and $f_\theta(\tau)=f_\theta^i(\tau)$ when $\tau\in(t_{i-1}, t_i]$.
Comparing (36) and (3) the product $U(\tau)f_\theta(\tau)$ may be viewed as a random fictitious seismic acceleration $A(\tau)$. The mean and the autocorrelation functions take the forms :

$$m_x(t)=\int_0^t E[U(\tau)f_\theta(\tau)]h(t-\tau)d\tau \qquad R_x(t_1,t_2)=\int_0^{t_1}\int_0^{t_2}E[A(\tau_1)A(\tau_2)]h(t_1-\tau_1)h(t_2-\tau_2)d\tau_1d\tau_2 \tag{37}$$

at the time instants t_1, t_2. The simplest estimation for $f_\theta^i(\tau)$ would be the uniform density function $f_\theta^i(\tau)=1/\Delta t_i$ and $h(t-\alpha_i)=\frac{1}{\Delta t_i}\int_{t_{i-1}}^{t_i} h(t-\tau)d\tau$. Hence the random response is $x(t)=\sum_{i=1}^{n}\frac{U_i}{\Delta t_i}\int_{t_{i-1}}^{t_i} h(t-\tau)d\tau$.
After some calculations one may be written :

$$m_x(t)=\sum_{i=1}^{\bar{n}} E\left[\frac{U_i}{\Delta t_i}\right]\int_{t_{i-1}}^{t_i} h(t-\tau)d\tau \qquad E\left[x^2(t)\right]\cong\sum_{i=1}^{\bar{n}} E\left[\frac{(U_i)^2}{(\Delta t_i)^2}\right]\left(\int_{t_{i-1}}^{t_i} h(t-\tau)d\tau\right)^2 \qquad (38)$$

where the ipotesis $E[U_iU_j]=0$ for $i\neq j$ has been done. The integral $\int_{t_{i-1}}^{t_i}(\omega_a)^{-1}e^{-\nu\omega(t-\tau)}\sin\omega_a(t-\tau)d\tau$ may be calculated using the integral tables from literature (Siretchi 1985). However more adequate are the probability density function that have greater values at the middle points of subintervals $[t_{i-1}, t_i]$.

APPLICATIONS

The accelerograms of two earthquakes have been analysed : 1) the 1977 Bucharest earthquake NOOS record (17 seconds) 2) the 1940 El Centro earthquake SOOE records (7,8 seconds).
The following magnitudes have been computed : the impulse magnitudes U_i, the input energy per unit mass E_{in}/m; input energy per unit mass and unit time; the averages of the sums of positive and negative impulses $\sum U_i^+/n^+$, $\sum U_i^-/n^-$, the durations of impulses $\Delta t_i=t_i-t_{i-1}$. The results are presented in Table 2.

Table 2.

Record	U_i^{max} (cm/s)	U_i^{min} (cm/s)	$E_{in}/m(cm^2/s^2)$	$E_{in}/mt_e(cm^2/s)$	$\sum U_i^+/n^+$(cm/s)	$\sum U_i^-/n^-$(cm/s)	Δt_i (s)
Bucharest	+89.85	-99.71	10,466	615.65	4.870	7.156	0-0.85
El Centro	+41.6	-47.2	5,697.5	730.45	8.231	9.707	0-0.4

CONCLUSIONS

Any Duhamel integral may be transformed into a sum of products between the impulse magnitudes U_i and the corresponding impulse response functions $h(t-\alpha_i)$. By means of this approach all the relationships in linear analysis may be rewritten in another form. New formulas (6)-(10), (12), (13), (24)-(30), (31)-(34), (36)-(38) have been obtained. For time history analysis and spectrum computations a limitation of the impulse series method is due to the values α_i which are not known in advance. The first practical applications prove the simplicity of this method and its promising capability to be used especially in probabilistic analysis and energy methods.

REFERENCES

Bruneau, M. (1994). Normalisation of energy-based methods to predict ductile responseof SDOF structures. In: *Proceedings of the 10th ECEE*, Vol.2, pp. 919-925. Wien

Daniliu, A. (1987). Nou model matematic ïn proiectarea antiseismica. In: *Constructii*, Nr.1. Romania

Daniliu, A. (1994). Dynamic analysis of structures using impulse series method. In: *Proceedings of the 10th ECEE*, Vol.2, pp.1207-1209. Wien

Elishakoff, I. (1983). Probabilistic methods in the theory of structures. John Wiley and Sons, New York.

Gelfand, I. M., Shilov, G. E. (1983). Functii generalizate. Editura stiintifica si Enciclopedica, Bucharest.

Hangan, S., Crainic, L. (1980). Concepte si metode energetice , Editura Academiei, Bucharest

Siretchi, Gh. (1985). Calcul diferential si integral. Vol. 1. Editura Stiintifica si Enciclopedica, Bucharest.