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Paper No. 572. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

STOCHASTIC ANALYSIS OF A SLENDER RIGID BLOCK UNDER EARTHQUAKE EXCITATION

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ABSTRACT

This paper describes a stochastic approach to the analysis of stability of a slender block, standing free on a plane subject to a seismic motion: the stability threshold is identified with the attainment of an appropriately defined "energy level". This approach is applied to a block which at first is not allowed to slide; then, an extended model is introduced, which takes into account the energy dissipation due to the short-duration slidings subsequent to impacts. The results of an extensive numerical investigation, that made use of artificial accelerograms compatible with the Eurocode 8 spectra, are presented. The most significant numerical results are shown in the block *size* vs. *slenderness* plane by means of the level curves of the probability that the total energy - hence the maximum rotation - remains below each given value (*isoprobability lines).*

KEYWORDS

Rigid blocks, seismic response, numerical simulation, probabilistic approach, stability, energy, rocking, sliding.

INTRODUCTION

The dynamic behaviour of systems of rigid blocks in unilateral contact with each other and with the base surface under a given motion of the support can give useful informations about the seismic response of many structural or non-structural systems. Indeed, the first studies on the seismic response of structures that can be modeled as single slender blocks standing on a horizontal moving plane, date back more than a century: they were mainly intended to get an estimate of the intensity of earthquakes from the study of the response of obelisks, tombstones, etc., but also of columns built for this specific purpose. During the last decade the problem has been extensively studied of the seismic safety of monumental structures made of simply supported stone blocks without mortar (e.g. obelisks, ancient temples). Typical systems like a single block, a multiblock column or a *trilith* (two columns and a lintel) have been studied, and the relevance has been underlined of the possible slidings between the blocks and with respect to the ground, hence of a correct treatment of frictional effects (e.g. Sinopoli, 1987; Augusti and Sinopoli, 1992; Sinopoli and Sepe, 1993). More recently also the seismic behaviour of objects contained in buildings and modelled as rigid bodies (like instrumentations, art objects in Museums, etc.) is receiving an increasing attention (Augusti and Ciampoli, 1996).

All these studies show that, however refined the mechanical model, the response to a specific input accelerogram is by itself of little significance with regard to the actual seismic response and safety of the considered structure. In fact, the dynamics of single-block or multi-block systems of this kind is governed by non-linear and discountinous differential equations, due to large displacements, changes of the contact points and axes of rotation at each impact, slindings in presence of friction; therefore slightly differences in the initial conditions or in the mechanical or geometrical parameters, or also a little perturbation of a given accelerogram, might turn out a completely different motion. It seems therefore quite interesting to try and develop stochastic analyses of the problem.

From this point of view, it can be useful to describe the response not by means of the state variables, but through some appropriate functions of such variables, able to give in a straightforward way a measure of the *probability of collapse* and easier to obtain from a given input, so that their statistics can be evaluated, either directly or by simulation.

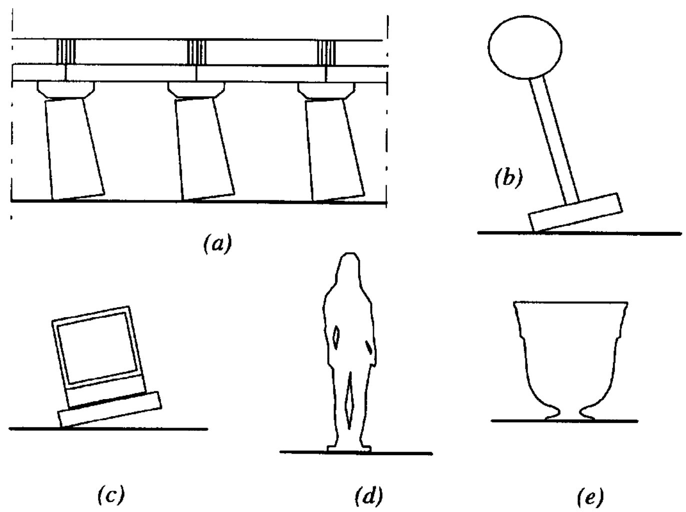

Fig.1 - *Typical examples of "rigid blocks":*
a) monumental structures; b) overhead tanks; c) instruments; d),e) art objects (statues, vases).

In this paper the treatment will be limited to single blocks, standing free on a plane subject to a seismic motion. In this case, collapse can be identified with overturning, and an adequate, albeit approximate, measure of the *probability of collapse* can be obtained from the ratio between the total energy attained during the motion and an appropriately defined *critical value*. This energy approach, described in detail in a subsequent Section, has been applied to two mechanical models, the first of which is not allowed to slide on the supporting plane, while the second model can slide and thus dissipate further energy.

MODELS OF BLOCK BEHAVIOUR

The first model studied (Fig.2) was proposed by Housner (1963) to describe the plane motion of a rigid slender parallelepiped of mass M, subjected to a given horizontal acceleration $a(t)$, and can be applied to blocks of any shape, indicating by $2b$ the width of the base and by h the height of the centroid on the supporting plane. The geometry of the block is described by the size R and the slenderness ratio $\lambda = h/b$, or equivalently by the critical angle $\alpha = arctg(b/h)$ corresponding to the configuration of unstable statical equilibrium. It is assumed that the block cannot slide on the supporting surface but can only rotate about either edge of its base, O and O'. The minimum (*threshold*) value a_s of the horizontal acceleration of the support able to initiate the rocking motion of a block is given by the *West formula*,

$$a_s = \frac{b}{h} g \qquad (1)$$

in which g is the acceleration of gravity. However, it is well known that under an imposed alternate support motion like a seismic motion the block can survive, rocking without overturning, to peak accelerations significantly larger than a_s. In fact, the block *rocks*, i.e. rotates alternatively about O and O', impacting on the supporting plane every time the angle of rotation θ becomes nil: the motion is described by a nonlinear second order differential equation, in which a very peculiar type of nonlinearity appears: in fact, the restoring moment is due to the weight of the block and plotted versus θ exhibits a sharp discontinuity at $\theta = 0$, because of the alternative rotation axis (O or O'); then, for either $\theta > 0$ or $\theta < 0$, the moment decreases with increasing absolute values of the rotation, and this decrease may be significantly nonlinear for stocky blocks near the overturning condition $\theta = \pm \alpha$. In this model, in which no sliding occurs, energy is dissipated only because of the inelastic impacts between block and supporting plane: the law of conservation of angular momentum leads to smaller velocities of rotation after each impact, and this decrement depends only on the slenderness, larger energy dissipations corresponding to less slender blocks.

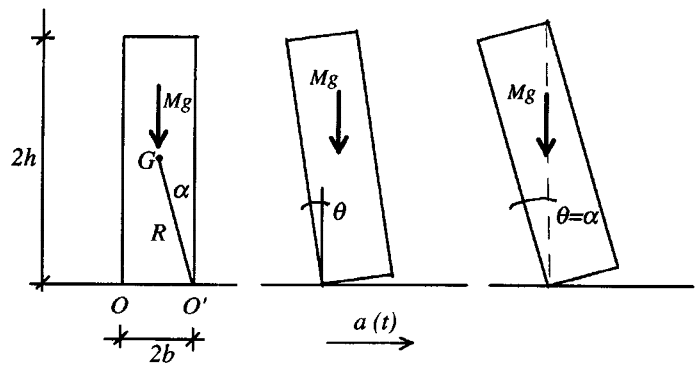

Fig. 2 - *Housner Model*
before the impact: rotation about O;
after the impact: rotation about O'.

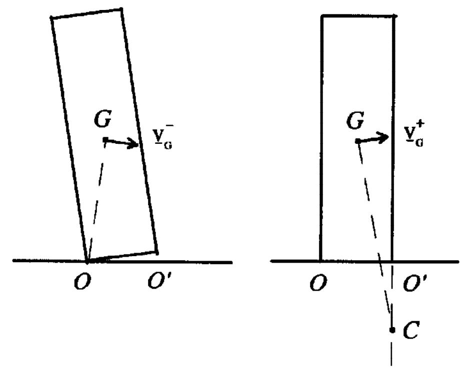

Fig. 3 - *Extended model* (after Sinopoli, 1989)
before the impact: assume rotation about O;
after the impact: rotation about O' plus translation.

The Housner model so far described, in which the friction between the block and the supporting plane is considered large enough to prevent any sliding, is exact in limit case of infinite slenderness λ, and is all the more acceptable the larger is λ. Recent papers (Sinopoli, 1989; Sinopoli, 1996) have shown that the impact always gives rise to some sliding; if before the impact the block rotates about the bottom edge O (Fig.3) , the motion after the impact is a rotation around the other edge O' plus a horizontal translation of the edge itself, that is a rotation about a centre C below O'. C is closer to O' the larger is the slenderness λ, and coincides with O' in the limit $\lambda \rightarrow \infty$. Viceversa, when $b/h \geq \sqrt{2}/2$ (*stocky blocks*), the distance CO' is infinite, and the motion subsequent to the impact is a pure traslation. In any case, for a given angular velocity of the rocking block before the impact, the angular velocity after the impact is smaller than in the Housner model, because a part of the kinetic energy goes into the traslational motion; however, for usual values of the friction coefficients, the translational kinetic energy is rapidly dissipated and the sliding dies out very soon. This kinematic approach (Sinopoli, 1989), which assumes that the friction forces do not work during the impact and therefore do not influence at all the subsequent motion, represents a limit assumption opposite to the Housner model: a more refined model is at the moment being developed (Sinopoli, 1996).

If one excludes that the amount of slidings is so large to produce other unacceptable effects, as for example is the case of art objects allowed to slide (Augusti and Ciampoli, 1996), also for this model collapse can be identified with overturning, and it can be assumed that the sliding caused by the impact has the only consequence of reducing the angular velocity, due to the larger dissipation.

ENERGY APPROACH

As well known, (see e.g. Uang and Bertero, 1990) seismic excitations introduce energy in a structural system; a part of this energy is stored as potential, kinetic or elastic energy, while another part is dissipated: for the Housner model of block, the dissipation is due only to the impact; for the extended model, dissipation is also due to sliding. Denoting by $E(t)$ the total energy of the system at a given time t, sum of potential and kinetic energy, $E(t) = U(t) + T(t)$ (as it is sometimes done for elastoplastic oscillators, it is convenient to consider *relative energies*, i.e. energies referred to the moving support) and by $D(t)$ the energy dissipated from the beginning of the support motion, the input energy is

$$I(t)=U(t)+T(t)+D(t) \tag{2}$$

For the free block, the overturning condition coincides with crossing the configuration of static unstable equilibrium ($\theta = \alpha$, Fig.2), and can be expressed as $\theta > \alpha$. Under external excitation, the expression

$$E(t) = U(t) + T(t) > U^* \quad \Rightarrow \quad \textit{overturning} \tag{3}$$

where U^* is the *potential energy* in $\theta = \alpha$ appears a good approximation of the collapse condition. This hypothesis is confirmed by the fact that the numerical investigation has seldom shown motions without overturning with $E(t)$ larger than U^*, and in these few cases only slightly larger. More generally, a correlation has been found between the total energy and the largest rotation during the motion, so that the probabilities

$$P = Prob\{E \leq KU^*\} \quad ; \quad K \leq 1 \tag{4}$$

give significant estimates of the probability of attaining each prescribed maximum rotation and, for K=1, of collapsing.

With a similar energy approach, and describing the seismic action through the pseudovelocity response spectrum, Housner (1963) estimated that, for a given pseudovelocity S_V, a block with size R and critical angle α has a survival probability equal to 50% if

$$\alpha = arctg\frac{1}{\lambda} = \frac{S_V}{\sqrt{gR}}\sqrt{\frac{MR^2}{I_0}} = \frac{S_V}{\sqrt{gR}}\sqrt{\frac{3}{4}} \tag{5}$$

I_0 being the inertia moment around a bottom edge. Note in particular the size effect shown by eq. (5): for a given slenderness λ, the seismic intensity corresponding to an overturning probability of 50% increases with the block size R.

a_g = peak ground acceleration

$$a = \beta_s(T) a_g$$

$$\frac{S_V}{a_g} = \frac{\beta_s}{2\pi / T}$$

$$\frac{S_V}{a_g} = 0.238 \text{ s} \quad \text{for } T > 0.6 \text{ s}$$

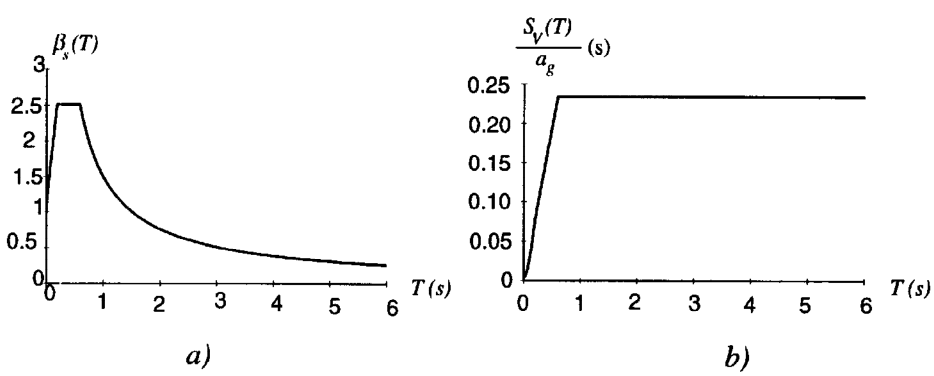

Fig. 4 - *Eurocode 8, type B soil: a) acceleration response spectrum, b) pseudovelocity response spectrum*

NUMERICAL INVESTIGATION

An extensive numerical investigation has been performed (Augusti and Sepe, 1995; Sepe and Augusti, 1995; Cattaruzza, 1996): only the main results can be reported here. As already indicated, two models have been considered, namely, i) the Housner model, ii) the extended model in which, due to a rapidly dying traslational motion, the kinetic energy of the rotational motion after the impact is reduced. The seismic excitation (assumed horizontal) has been simulated by 50 artificial accelerograms in accord with the response spectrum B of Eurocode 8 (medium soil) (Fig.4), with durations of 30, 40 or 50 seconds and peak ground acceleration equal to 0.35 or 0.45g. The amplitude of the accelerograms is modulated by a deterministic function, that rises linearly from zero to the maximum in the first 5 seconds, and decreases to zero in the last 15 seconds.

The response of about 60 blocks, with slenderness λ and size R respectively in the ranges (3 - 7) and (0.5 - 6.0 m), has been obtained by numerical integration. In particular, in accord with physical evidence and previous well known results (Housner, 1963; Aslam *et al.* 1980; Yim *et al.* 1980), it is confirmed that the overturning probability increases with increasing slenderness and decreasing size, and of course increases also with the duration of the accelerogram.

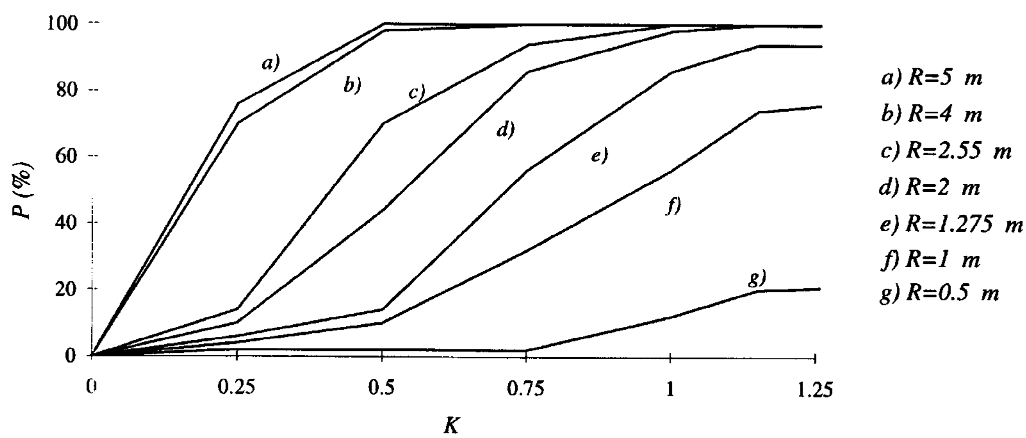

Fig.5 - *Housner Model:* $P = Prob\{E \leq KU^*\}$ *in function of size R;* $\lambda = 3.5$, $a_g = 0.45g$; *duration* $t_d=50$ *s*

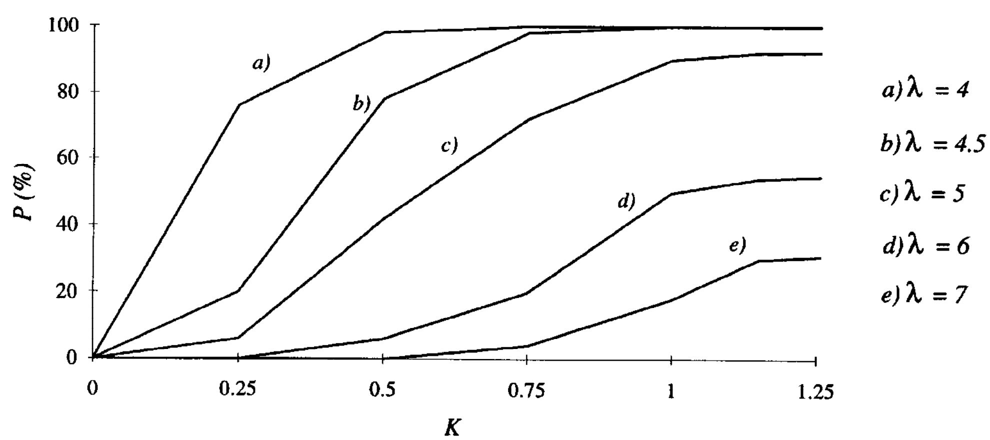

Fig.6 - *Housner Model:* $P = Prob\{E \leq KU^*\}$ *in function of slenderness* λ; $R = 4$ *m*, $a_g=0.35g$; $t_d=50$ *s*

The probabilities (4), referred to four values of K, namely

$$K\,U^* = 0.25\,U^* \;;\; 0.5\,U^* \;;\; 0.75U^* \;;\; U^*$$

have been obtained by considering the response of a given block to the whole set of simulated accelerograms as a statistical sample. Quite obviously, for a given geometry of the block the *survival probability*, i.e. the probability that the maximum total energy remains below the critical value U^* (K=1), decreases with increasing peak acceleration and increasing duration of the support motion; the probability that $E \leq KU^*$, with K<1, varies in a similar way. Fig.5 and Fig.6, which refer to the Housner model, underline respectively the influence of the size for a block with given slenderness, and of the slenderness for given size.

Note that sometimes accelerograms with $E \geq U^*$, i.e. satisfying condition (3), do not lead to overturning: in fact, for particular action histories, the total energy can reach a maximum value slightly greater than U^* and then decrease. But whenever the total energy is significantly larger than U^* ($E > 1.1 - 1.2\ U^*$), the motion leads eventually to overturning. Therefore, eq.(3) is a safe *hypotesis*, not too far from numerical evidence.

ISOPROBABILITY LINES

It has deemed interesting to describe the results in the plane of the geometrical parameters λ and R, by means of the lines corresponding to costant values of the probability (4) for given values of K (*isoprobability lines*). The lines shown in the following plots have been obtained by linear regression of the numerical results.

Some isoprobability lines pertaining to K=1 (*survival probability*) are shown in Fig.7 and 8, both for the Housner model (broken lines) and the extended model (solid lines): the imposed motion lasts 50 seconds, the peak acceleration is equal respectively to 0.45 or 0.35g.

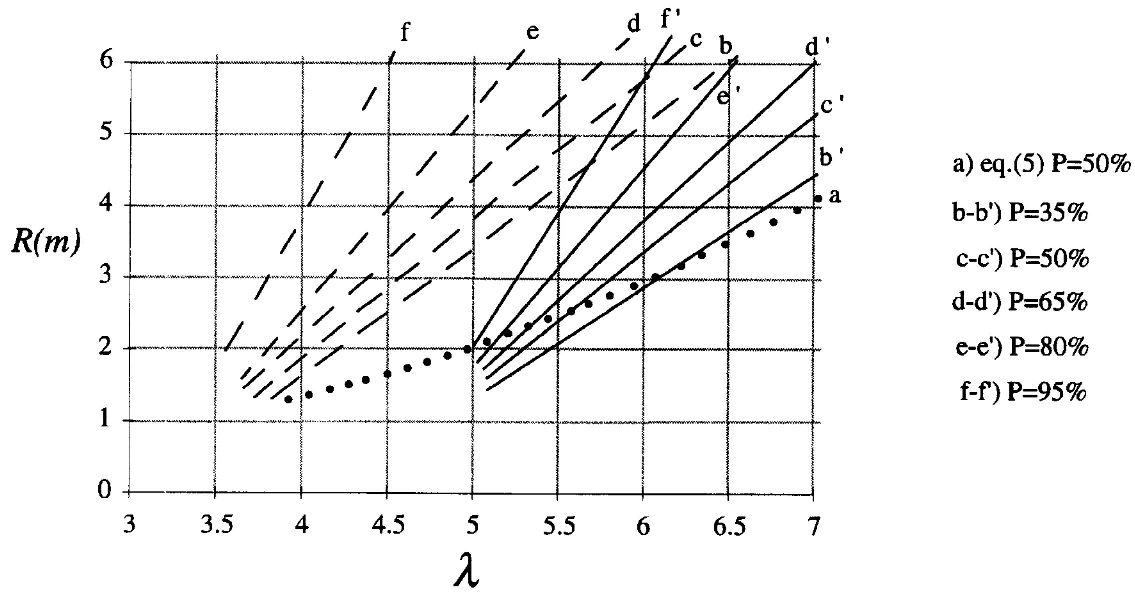

Fig.7 - *Survival probability: P = Prob{E ≤U*}*, a_g = 0.45g, *duration* 50 *s*
b)-f) *Housner model;* b')-f ') *extended model;* a) *Housner's eq.(5)*

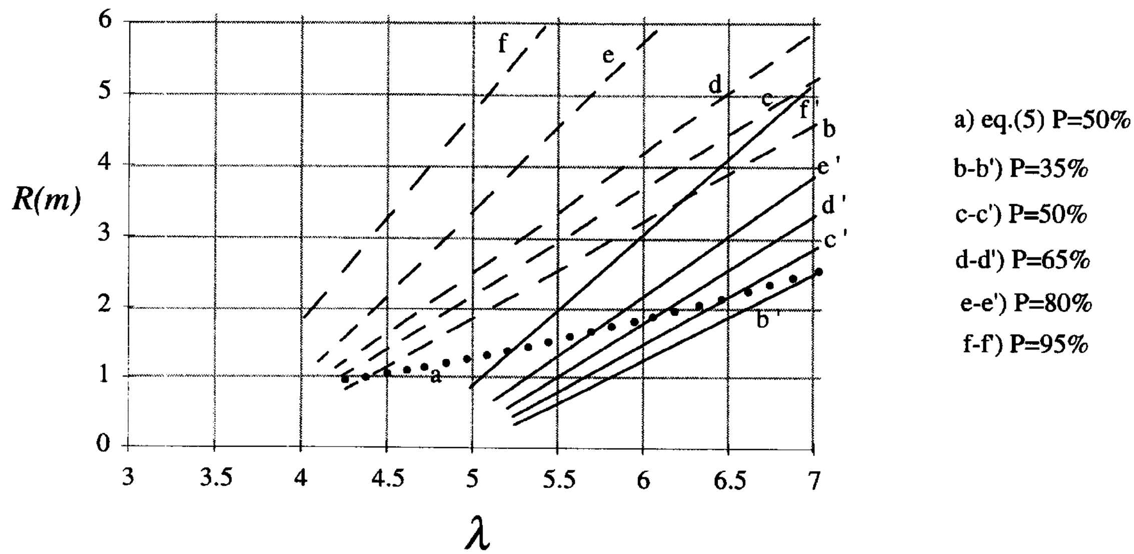

Fig.8 - *Survival probability: P = Prob{E ≤U*}*, a_g = 0.35g, *duration* 50 *s*
b)-f) *Housner model;* b')-f ') *extended model;* a) *Housner's eq.(5)*

In Figs. 7 and 8 Housner's eq. (5) is also plotted (dotted lines): it represents the geometry of blocks with a 50% survival probability under seismic actions with a given spectral pseudovelocity S_V. In particular (cfr. Fig.4) $S_V = 1.05$ m/s or $S_V = 0.82$ m/s respectively for peak accelerations equal to 0.45 or 0.35 g. It is evident that for seismic action with the assumed spectra and duration and for the geometry of the blocks under consideration, eq. (5) turns out to be unsafe if compared with the numerical results given by Housner model (broken lines), because it predicts a survival probability (or equivalently an overturning probability) equal to 50% for blocks that show a smaller survival probability.

Comparing the results obtained for the extended and the Housner model, it is also evident that the sliding can reduce significantly the overturning probability of the block. On the other hand, the sliding itself could become unacceptably large; this aspect of the dynamic behaviour, already investigated in recent papers on the seismic behaviour of art objects (cf Augusti and Ciampoli, 1996 and the references quoted therein), has not yet been considered in the light of the energy approach.

In an analogous way, it is also possible to plot in the λ - R plane lines that correpond all to the same probability (4) but to different energy levels K: for example, Fig.9 refers to P = *95%*. If the block is represented in Fig.9 by a point below (above) the line K=1, its probability of survival is less (more) than 95%, i.e. its probability of overturning is more (less) than 5%.

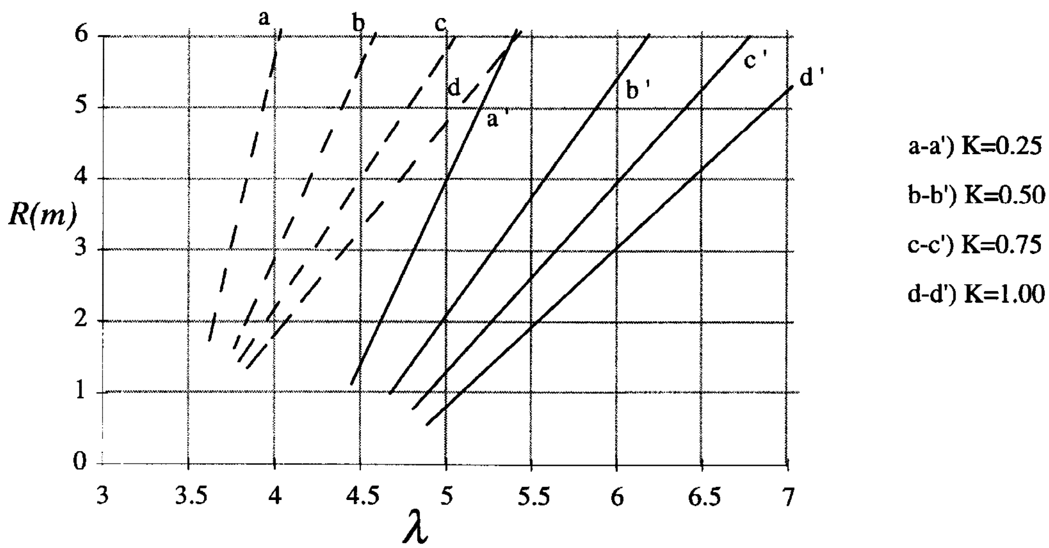

Fig.9 - $P = Prob\{E \leq KU^*\}$=95%; a_g = 0.35g, *duration* 50 *s*
a)-d) *Housner model;* a')-d') *extended model*

CONCLUDING REMARKS

This paper deals with the overturning probability under seismic actions of a single rigid block, described first by the classical Housner model and then by an extended model that takes into account the energy dissipation connected with the short-duration slidings that follow each impact between the block and the supporting plane. *Isoprobability lines* (i.e. lines corresponding to blocks with a given probability to have a total energy - and consequently a maximum rotation - below a prescribed reference level for given characteristics and duration of the seismic excitation) have been plotted in the size vs. slenderness plane.

These results have been obtained in a numerical investigation, simulating the seismic action by artificial accelerograms compatible with the response spectrum suggested by Eurocode 8 for medium soils, and considering the responses to the artificial accelerograms as a statistical sample: an analytical formulation is now under investigation. It is also well known that artificial accelerograms compatible with a given response spectrum can sometimes be inadequate to obtain reliable information, and perhaps the obtained results should be checked by analogous ones obtained with the use of a set of artificial accelerograms derived from earthquakes records.

In any case, the presented diagrams give a straightforward, easy-to-read and effective image of the survival probability of a single block with respect to overturning, in fuction of its geometrical parameters, which can be very useful in a first screening of the seismic risk. It would be very important to obtain analogous curves for other structural types so that detailed analyses can be limited only to systems prone to overturning. A detailed analysis, in fact, requires large computational efforts and presupposes a good knowledge not only of the geometrical parameters, but also of the mechanical properties of the materials and of the contact surfaces, in order to consider in the correct way every influence on the dynamical behaviour; sometimes also the hypotesis of plane motion so far assumed should be removed.

ACKNOWLEDGEMENT

Many numerical results presented in this paper have been developed by Daniele Cantalini and Aldo Cattaruzza during the preparation of their Degree Thesis in Civil Engineering at the University of Rome "La Sapienza".

REFERENCES

Aslam, M., W. Godden and T. Scalise (1980). Earthquake rocking response of rigid bodies, *Journal of Structural Division, ASCE*, **106**, 377-392

Augusti, G. and A. Sinopoli (1992). Modelling the dynamics of large blocks structures, *Meccanica*, **27** (3), 195-211

Augusti, G. and V. Sepe (1995). Energy dissipation in rigid-block structures under dynamic excitation, *Proc. ASCE Structures Congress XIII: "Restructuring: America and Beyond"*, Boston, 968-971

Augusti, G. and M. Ciampoli (1996). Guidelines for seismic protection of Museum contents, *Proc. 11th World Conference on Earthquake Engineering*, Acapulco, Mexico (in print)

Cattaruzza, A. (1996). Approccio energetico nella dinamica sismica di un blocco snello, *Degree Thesis in Civil Engineering*, Università di Roma "La Sapienza"

EUROCODE 8 (1988): Structures in Seismic Regions - Design - Part 1, General and Building; CEN: European Committee for Standardization, Bruxelles

Housner, W.G. (1963). The behavior of inverted pendulum structures during earthquakes, *Bull. Seism. Society of America*, **53**, 403-417

Sepe V. and G. Augusti (1995). Dinamica sismica di un blocco snello, *Ingegneria Sismica*, **12** (3)

Sinopoli, A. (1987). Dynamics and impact in a system with unilateral constraints: the relevance of dry friction, *Meccanica.*, **22**, 210-215

Sinopoli, A. (1989). Kinematic approach in the impact problem of rigid bodies, *Applied Mechanics Reviews, ASME*, **42** (11), Part 2, 233-244

Sinopoli, A. and V. Sepe (1993). Coupled motion in the dynamic analysis of a three block structure, *Appl. Mech. Rev.*, **46** (11), Part 2, 185-197

Sinopoli, A. (1996). Unilaterality and dry friction. A geometrical formulation for two dimensional rigid body dynamics, *submitted*

Uang, C.M. and V.V. Bertero (1990). Evaluation of seismic energy in structures, *Earthquake Eng. Struct. Dyn.*, **19**, 77-90

Yim, C., A.K. Chopra A.K. and J. Penzien J. (1980). Rocking response of rigid blocks to earthquakes, *Earthquake Engr. and Struct. Dynamics*, **8**, 565-587