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SEISMIC RISK ANALYSIS AND ASSESSMENT USING EXTENDED ß-FACTOR METHODS

A. VULPE [1] and A. CARAUŞU [2]

[1] Dept. of Structural Mechanics, Technical Univ. of Iaşi, str.Ralet 7, RO-6600 Iaşi, Romania
[2] Dept. of Mathematics, Technical Univ. of Iaşi, str. Dr.V.Babeş 8, RO-6600 Iaşi, Romania

ABSTRACT

The Alpha Factor Model and the Multiple Greek Letter method, developed for the probabilistic risk assessment of nuclear structures, are adapted for general structures subjected to various types of loads including sesmic excitations, including the seismic ones.

KEYWORDS

Probabilistic risk assessment; Common Cause Failure; α-Factor model; Multiple Greek Letter method.

INTRODUCTION AND PRELIMINARIES

The so-called *common cause failure* (CCF) in redundant systems of nuclear power plants (NPPs) has been intensively investigated since the mid 70s. Various types of models and methods were developed in connection with this concept, mainly with the US nuclear program (Fleming, 1975; Vesely, 1977; NUREG/CR-4780, 1988-1989). We mention, among them, the *ß-Factor method* (ßFM), the *α-Factor model* (αFM), the *Multiple Greek Letter* (MGL) method, which are extensively presented in NUREG/CR-4780 (1988, 1989). The latter two are extensions of the ßFM. All these methods (and other ones, too) fall in what is called *dependent failure analysis* (Ballard, 1989). On another hand, the concept of *seismic fragility* was also developed in the same area of PRA of NPPs. The power and elegance of the fragility models determined us to propose some approaches for using them in the PRA of more general structures (Vulpe *et al.*, 1990, 1991). The problem of taking into account dependence relations among component seismic fragilities was less considered in the literature, e.g. by Yamaguchi (1991), but we also proposed an approach to it in (Vulpe *et al.*, 1995). In this paper we develop some stochastic dependence models - presented in (Carauşu *et al.*, 1991; Vulpe *et al.*, 1993) - with more attention paid to the CCF concept. We have found as the most appropriate of the CCF methods to be applied in reliability and seismic risk evaluation of general structures the αFM and MGL. Some preliminaries on these methods are unavoidable, and we are going to recall them briefly, according to the report NUREG/CR-4780 (1988, 1989).

The reliability / seismic risk assessment of (sub)systems in NPPs based on the analysis of common cause (CC) events involves the assumption that several *similar components* behave in a similar way. Probabilistically speaking, all the components in a CC group can fail due to a common cause, hence they share a common *total unavailability* denoted by Q_t. If such a CC group consists of m components, a *basic event* occurs when k out of the m components fail $(1 \le k \le m)$. Clearly, it is possible that no component fails : $k = 0$. The probability of such an event is denoted by Q_k. It follows that the unavailability of k components in a CC group is given by

$$Q_t = \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k . \tag{1}$$

The *multiple Greek letter* (MGL) model is an indirect multiparameter nonshock model whose m parameters are $Q_t,\ \beta = \rho_2,\ \gamma = \rho_3, \ldots, \ldots = \rho_m$. By convention, $\rho_1 = 1,\ \rho_{m+1} = 0$. In this model, the unavailablity of k components is given by

$$Q_k = \frac{1}{\binom{m-1}{k-1}} \left(\prod_{i=1}^{k} \rho_i \right) (1 - \rho_{k+1})\, Q_t . \tag{2}$$

The unavailabilty of the whole group (or subsystem) depends on the failure criterion for it. For example, if $m = 3$ and the group becomes unavailable if at least two components fail, then

$$Q_S = 3\,(1-\beta)^2 Q_t^2 + \tfrac{3}{2}\beta\,(1-\gamma)\,Q_t + \beta\gamma Q_t . \tag{3}$$

In fact, Eq.(3) gives only an approximation for Q_S obtained by neglecting the terms of order ≥ 3 in ß and Q_t.

The αFM is also a multiparameter model with $m+1$ parameters, namely Q_t with the same meaning as in MGL method and $\alpha_1, \alpha_2, \ldots, \alpha_m$ where $\alpha_k =$ the fraction of the total frequency of failure events that occur in the system involving the failure o k components due to a common cause. The m parameters are clearly ≥ 0 and they also satisfy the condition $\alpha_1 + \alpha_2 + \ldots + \alpha_m = 1$. To illustrate the diference between MGL and αFM on the preceding example with $m = 3$, we give the expressions of the unaivailabilities $Q_k,\ k = \overline{1,3}$ (according to MGL in column 1 and to αFM in column 2, respectively):

$$\begin{aligned} Q_1 &= (1-\beta)\,Q_t & \qquad Q_1 &= \frac{\alpha_1}{\alpha_t} Q_t \\ Q_2 &= \tfrac{1}{2}\beta\,(1-\gamma)\,Q_t & \qquad Q_2 &= \frac{\alpha_2}{\alpha_t} Q_t \\ Q_3 &= \beta\gamma Q_t & \qquad Q_3 &= 3\,\frac{\alpha_3}{\alpha_t} Q_t \end{aligned} \tag{4}$$

where $\alpha_t = \alpha_1 + 2\,\alpha_2 + 3\,\alpha_3$ is a normalizing factor. In general, the partial unavailability of k components in the αFM is given by

$$Q_k = \frac{k}{\binom{m-1}{k-1}} \frac{\alpha_k}{\alpha_t} Q_t, \quad k = 1, 2, \ldots, m. \tag{5}$$

In the same case of "2 out of 3" system failure / success, denoted by $[2/3]$, the system unavailability under the αFM is

$$Q_S = 3\,\frac{Q_t}{\alpha_t}\left(\frac{\alpha_1^2}{\alpha_t} Q_t + \alpha_2 + \alpha_3\right). \tag{6}$$

Bayesian estimates for the parameters of the MGL model and the αFM are given in Vol.2 of (NUREG/CR-4780, 1989).

ADAPTING *αFM* AND *MGL* TO REDUNDANT STRUCTURAL SYSTEMS

Common Cause Failures in Structural Systems

As already mentioned in the Introduction, the CCF concept was considered mainly in redundant equipment (sub)systems of NPPs. However, several interpretations of this concept can be met in the literature. Roughly speaking, the components that can fail due to a common cause are *similar*, that is, they are equipment units of the same type (motor operated pumps, for example), of the same manufacturing and sharing the same technological function. If we attempt at introducing CCF-like models for general structural systems, we have to look for some analogies. For a frame structural system consisting of column and bar elements, we suggest that the elements of the same type (as regards the geometrical and material properties) placed in equivalent positions from the point of view of system topology and loading conditions could be grouped in a *CCF group* or *CCF class*. This is only a possible approach, maybe too simplist; but it is based upon the ways the *dependence* relationship is considered, e.g., by Kaplan *et al.* (1981), Kaplan (1985), Apostolakis (1989) a.o.

Let us consider a structural system S consisting of n components. The components will be labeled by a positive integer, hence the set of components is represented by $C = \{1, 2, \ldots, n\}$. Clearly, The system S itself cannot be identified with the set C of components, since the latter ones can differ as regards their properties and position/role in S. This remark induces the necessity of *partitioning* the set C in subsets of "similar" components:

$$C = \bigcup_{i=1}^{\kappa} C_i \quad \text{with} \quad i_1 \neq i_2 \Rightarrow C_{i_1} \cap C_{i_2} = \emptyset . \tag{7}$$

The system is assumed to fail according to m possible *failure modes* (or fundamental mechanisms). A failure mode M_j $(j = 1, 2, ..., m)$ is, in fact, a subset of m_j potentially failing components; hence

$$M_j = \{\ell_{1,j}, \ell_{2,j}, \ldots, \ell_{m_j,j}\} \subset C. \tag{8}$$

It will be necessary to consider the subclass of components of type i that enter in the failure mode M_j, that is, $C_{ij} = C_i \cap M_j$ with $\mathrm{card}\, C_{ij} = \kappa_{ij}$, $1 \le \kappa_{ij} \le \kappa_i = \mathrm{card}\, C_i$. An alternative (and more suitable) representation of a failure mode would be as an ordered k-tuple of nonnegative integers :

$$M_j = [k_{1,j}\ k_{2,j}\ \ldots\ k_{\kappa,j}] \text{ with } 0 \le k_{i,j} \le \kappa_i. \tag{9}$$

The difference between representations (8) and (9) of a failure mode is clear: the members of the set in the r.h.s. of Eq.(8) are *component labels* in the set C, while the entries in the κ-vector of Eq.(9) are numbers of the components from each similarity class C_i that enter in the failure mode M_j. It is clear that $k_{i,j}$ in Eq.(9) are equal to $\kappa_{ij} = \mathrm{card}\, C_{ij}$.

Let us illustrate the concept of CCF classes on the example of the (redundant) frame structure in the following figure:

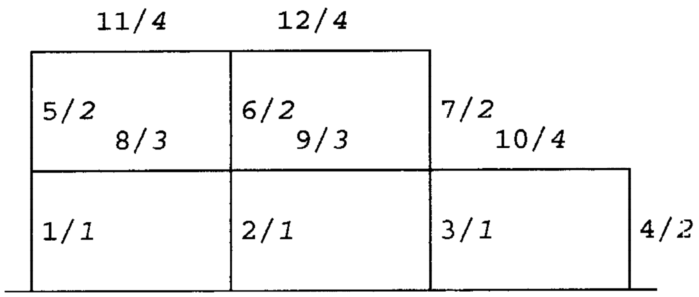

Fig.1 Frame structure with CCF classes
i/*k* : i = component label in *C*
k = CCF class

It is (probably) no need to explain the way the CCF classes were established; however, let us notice that the column elements 1, 2, 3 with one story above them are in class *1*, the other columns 4, 5, 6, 7 are in class *2*, the beams possibly subjected to live loads 8, 9 are in class *3* and the other beams 10, 11, 12 are in class *4*. Hence, for this example frame structure we have $\kappa = 4$. One of the many possible failure modes of such a structure is the "chain mechanism" consisting in the failure of all the columns of the first level. Such a mechanism would then be $M_1 = \{1, 2, 3, 4\}$ according to expression (8), respectively $M_1 = [3\ 1\ 0\ 0]$ according to expression (9).

Alpha Factor and MGL Methods for Structural Failure Modes

In view of the previous discussion on the CCF classes in a structural system, it follows that the total unavailability (or probability of failure) of a component cannot be the same for all of them in the system; instead, the similarity of the components in a CCF group or class allows to assign the same Q_t to all the members of such a group. More exactly,

$$Q_{t,i} = Prob[k \text{ fails}, \ k \in C_i] \ , \quad i = 1, 2, \ldots, \kappa. \tag{10}$$

This total failure probability, specific to each of the κ CCF classes, has to be estimated from statistical evidence and/or experimental evaluations for the type of structural system under analysis. Next, an αFM should be considered at the level of each CCF class C_i. According to Eq.(5) in the preceding section, the partial unavailability of ℓ components in the CCF class C_i will be given by an adapted version of this equation:

$$Q_{\ell,i} = \frac{\ell}{\binom{\kappa_i - 1}{\ell - 1}} \frac{\alpha_\ell}{\alpha_{t,i}} Q_{t,i}, \quad \ell = 1, 2, \ldots, \kappa_i. \tag{11}$$

The α factors $\alpha_1, \alpha_2, \ldots, \alpha_{\kappa_i}$ associated to every CCF class C_i have to be estimated using the Bayesian procedure given in Vol.2 of (NUREG/CR-4780, 1989). For a certain failure mode M_j of the form given in Eq.(9), we have to take - in Eq.(11) - $\ell \le k_{i,j}$. Now, the failure probability of the $k_{i,j}$ components of the subclass C_{ij} within the failure mode M_j will be given by an equation similar to Eq.(6).However, it has to be taken into account the structure of the F-mode as in Eq.(9) instead of the simple failure/success events of the form $[k/m]$ considered in the *homogeneous* (sub)systems of NPPs. Obviously, our concept of a failure mode essentially involves its *nonhomogeneity*. In fact, a failure mode in a structural system is a nonhomogeneous complex event as regards the CCF classes involved in it. In the terminology "*k* out of *m*" specific to risk studies in NPPs, an F-mode could be assimilated to a κ-tuple of CC failures:

$$M_j = [\,[k_{1,j}/\kappa_1] \ \ [k_{2,j}/\kappa_2] \ \ldots \ [k_{\kappa,j}/\kappa_\kappa]\,]\,. \tag{12}$$

Of course, certain subclass numbers $k_{i,j} = \kappa_{ij}$ can be $= 0$ (as in the previous example); the corresponding subclass events will be impossible, that is, $[k_{i,j}/\kappa_i] = \emptyset$ since $C_{ij} = \emptyset$. Now, a probability or frequency has to be evaluated for each failure mode M_j. The appropriate way to do it would consist in formulating an adapted version of the MGL method. The ground for such an approach lies in the nature of the MGL parameters which are factors on the *total component unavailability* Q_t. But we are going to apply them on the total unavailabilities associated with CCF classes, that is on $Q_{t,i}$'s as given in Eq.(10), taking into account the event structure of an F-mode given in (12). The unavailability associated to a failure mode M_j can be obtained as follows:

$$Q(M_j) = \mathbf{\Omega}_\kappa [\boldsymbol{\rho}_j * \mathbf{Q}_{t,j}] \tag{13}$$

where $\mathbf{Q}_{t,j}$ is a κ-tuple whose components are $Q_{t,i}$ given by Eq.(10), $\boldsymbol{\rho}_j$ is also a κ-tuple whose entries are the MGL coefficients given by

$$\rho_j^i = \begin{cases} 0 & \text{if } k_{i,j} = 0 \\ \dfrac{k_{i,j}}{\binom{\kappa_i - 1}{k_{i,j}-1}} \left(\prod_{\ell=1}^{k_{i,j}} \rho_\ell \right) (1 - \rho_{k_{i,j}+1}) & \text{if } k_{i,j} \neq 0 \end{cases}, \tag{14}$$

the operation $*$ is defined by

$$\mathbf{a} = [a_1\ a_2\ \ldots\ a_\kappa] \wedge \mathbf{b} = [b_1\ b_2\ \ldots\ b_\kappa] \Rightarrow \mathbf{a} * \mathbf{b} = [a_1 b_1\ a_2 b_2\ \ldots\ a_\kappa b_\kappa], \tag{15}$$

and $\mathbf{\Omega}_\kappa$ is a scalar-valued operator (or functional) that gives an approximation to $Q(M_j)$. In fact, $\mathbf{\Omega}_\kappa$ turns the κ-tuple $\boldsymbol{\rho}_j * \mathbf{Q}_{\kappa,j} = \mathbf{Q}_j = [Q_{k_{1,j}}\ Q_{k_{2,j}}\ \ldots\ Q_{k_{\kappa,j}}]$ to a scalar as follows:

$$\mathbf{\Omega}_\kappa [\mathbf{Q}_j] = \left[\prod_{i=1}^{\kappa} \sum_{\ell=1}^{k_{i,j}} \binom{k_{i,j}}{\ell} Q_{k_{i,j}}^{k_{i,j}-\ell} \right]_\nu \tag{16}$$

where $[\ldots]_\nu$ denotes the truncation up to order ν of the (polynomial) expression between the brackets, by sorting out the terms of order $> \nu$ in $\rho_2 = \beta,\ \rho_3 = \gamma,\ \ldots \rho_{\max \kappa_i - \nu + 2}$ and $Q_{t,i}$. Let us remark that the vector $\mathbf{Q}_j$ is just the vector with probability components associated to M_j, that is, $\mathbf{Q}_j = \mathrm{Prob}(M_j)$ with M_j as given in Eq.(12). Equations (11) and (13) thru (16) are obviously extensions of the corresponding ones given in Chapter 3 of NUREG/CR-4780 (1988), where there are not considered *distinct* CCF classes within a (sub)system in an NPP. It still remains to check up to what degree the above presented approach to defining CCF classes in a general structural system is realistic.

Remark. Equation (13) and (16) - together with the other ones that are involved - approximates the probability of a failure mode under the hypothesis that failures in different CCF classes *are independent*;

besides that, *at most* $k_{i,j} = \kappa_{ij}$ failures are taken into account in each class. The exact probability of a failure mode of the form (12) - under the same assumption of independence between the failure in distinct classes - will be given by

$$Q(M_j) = \prod_{i=1}^{\kappa} \gamma_i \quad \text{with} \quad \gamma_i = \begin{cases} 1 & \text{if } k_{i,j} = 0 \\ \sum_{\ell=k_{i,j}}^{\kappa_i} \binom{\kappa_i}{\ell}^* Q_{\ell,i} & \text{if } k_{i,j} \neq 0 \end{cases}, \tag{17}$$

where $Q_{\ell,i}$ are given by Eq.(11) and $\binom{\kappa_i}{\ell}^*$ denotes the number of combinations reduced under the condition $\ell_{k,j} \in C_{ij}$.

A Numerical Example

We are going to apply some of the just presented formulas to the failure mode M_1 of the structure presented in Fig.1. The already mentioned chain failure mechanism for such a structure is $M_1 = [3\ 1\ 0\ 0]$. The corresponding event structure of this mode - see Eq.(12) - is $M_1 = [[3/3]\ [1/4]\ [0/2]\ [0/3]]$. According to the values for Q_t, ß and γ suggested by Fleming *et al.* (1983) as giving rather accurate approximations under operator $[\ldots]_v$ we take $\rho_2 = \beta = 0.1$, $\rho_3 = \gamma = 0.9$ and $Q_{t,1} = 0.06$, $Q_{t,2} = 0.04$, $Q_{t,3} = 0.05$, $Q_{t,4} = 0.03$. . The selected values for the α factors assigned to the four CCF classes are given in

Table 1. αFM parameters for CCF classes 1, 2, 3, 4

CCF class i	Size κ_i	α_1	α_2	α_3	α_4	$\alpha_{t,i}$
$i = 1$	3	0.3	0.4	0.3	-	2.0
$i = 2$	4	0.3	0.4	0.2	0.1	2.1
$i = 3$	2	0.3	0.7	-	-	1.7
$i = 4$	3	0.2	0.6	0.2	-	2.0

On the base of these numerical data, we calculated the matrix

$$[Q_{\ell,i}]_j = \begin{bmatrix} .009 & .0057 & 0 & 0 \\ .012 & .0051 & 0 & 0 \\ .027 & .0038 & 0 & 0 \\ .000 & .0076 & 0 & 0 \end{bmatrix}, \quad j = 1. \tag{18}$$

The last two columns in this matrix are completed with zero entries since the CC classes *3* and *4* do not enter in M_1. The reduced binomial coefficients that occur in Eq.(17) are the entries of two vectors corresponding to classes *1* and *2*, namely $\mathbf{b}_1 = [0\ 0\ 3\ 0]^T$, $\mathbf{b}_2 = [1\ 3\ 3\ 1]^T$ whose dot product with the non-zero columns in matrix (18) give $\gamma_1 = 0.027$ and $\gamma_2 = 0.0438$, respectively. The resulting unavailability associated to failure mode M_1 - according to Eq.(17) - is $Q(M_1) = 0.0011826$.

Sesmic Risk Assessment by αFM and MGL Methods. The adapted versions of αFM and MGL methods we have just presented render so-called *unavailabilities* associated with certain failure modes. This terminology is "borrowed" from the NPP risk analysis, but it is not directly connected with the seismic risk evaluation. The concept of *seismic fragility* (and the models/methods based upon it) has to be combined with these methods, resulting in specific procedures for evaluating the seismic risk of structural systems. Although it was introduced about 15 years ago in probabilistic seismic safety studies for NPPs (Kennedy *et al.*, 1980), even in this area a limited amount of observed/experimental fragility data is available. Attempts to integrate these adapted extensions of the ß factor model (ßFM) with the seismic fragility concept and techniques will be an object of our future research work. It will be also necessary to take into account the correlation between components in different CCF classes but entering in the same failure mode; in other words, a correlation matrix should accompany a failure mode as given in Eq.(12).

REFERENCES

Apostolakis, G.E. (1989). The concept of dependence in PSA. In: *Trans. 10th SMiRT* (Anaheim -CA), Vol.P, pp.119-124.

Ballard, G.M. (1989). The use of engineering judgement in dependent failure analysis. In: *Trans. 10th SMiRT* (Anaheim - CA), Vol.P, pp.37-46.

Carauşu, A. and Vulpe, A. (1991). Structural reliability analysis by stochastic dependence models and matriceal techniques. In: *Reliability and Optimization of Structural Systems - Proc. 4th IFIP WG 7.5 Conf.* (Munich, Sept.11-13, 1991), *L.N. in Engineering*, 76, 141-152, Springer Verlag, Berlin.

Fleming, K.N. (1975). A reliability model for common mode failure in redundant safety systems. *Proc. Sixth Annual Pittsburgh Conf. on Modeling and Simulations.*

Fleming, K.N, and Kalinowski, A. (1983). An extension of the Beta Factor Method to systems with high level of redundancy. *PLG-*0289: Irvine (CA) and Washington (DC), Pickard, Lowe & Garrick.

Kaplan, S., H.F. Perla and D.C. Bley (1981). A methodology for seismic risk analysis of nuclear power plants. *Risk Analysis*, 3, 169-180.

Kaplan, S. (1985). A method for handling dependencies and partial dependencies of fragility curves in seismic risk analysis. *Trans. 8th SMiRT* (Brussels), Vol. M, 595-600.

Kennedy, R.P., A.C. Cornell, R.D. Campbell, S. Kaplan and H.F. Perla (1980). Probabilistic seismic safety study of an existing nuclear power plant. *Nuclear Engineering and Design*, 59, 315-338.

NUREG/CR-4780 (1988, 1989). *Procedures for Treating Common Cause Failures in Safety and Reliability Studies*, Vol.1 (Jan. 1988), Vol.2 (Jan. 1989). PLG - Irvine (CA) and EPRI - Palo Alto (CA).

Vulpe, A. and A. Carauşu (1991). Probabilistic safety assessment of structures sequentially failing. In: *Trans. 11th SMiRT* (Tokyo-Kyoto), Vol. M+SDO, pp.175-180.

Vulpe, A. and A. Carauşu (1993). On some types of dependence in PSA of nuclear power plants and their quantification. In: *Trans. 12th SMiRT* (Stuttgart), Vol. M, pp.23-28.

Vulpe, A. and A. Carauşu (1995). On modeling and quantification of dependent fragilities in NPP structures and equipment. In: *Trans. 13th SMiRT* (Porto Alegre), Vol. IV, pp.737-742.

Yamaguchi, A. (1991). Seismic fragility analysis of heat transport system of LFMBR considering partial correlation of multiple failure modes. In: *Trans. 11th SMiRT* (Tokyo-Kyoto), Vol. M+SDO, pp.91-96.