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VIBRATION CONTROL OF STRUCTURES WITH MULTIPLE SUPPORT EXCITATION

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ABSTRACT

This paper analyzes the feasibility and efficiency of active and passive control systems for the vibration control of structures subjected to multiple support excitation. This type of external excitation represents the earthquake-induced spatially varying ground motion and it is extremely important for horizontally extended structures. The structural response is decomposed into a dynamic component and a quasi-static component. Both active (tendons) and passive (base isolation) vibration control systems have been analyzed. While the active tendon control system is quite effective in reducing the dynamic response, the quasi-static component of the structural response can be controlled by such a system only in particular structural conditions. The importance of the quasi-static component of the response is also confirmed in base isolated systems, which, however, seem to be more effective in reducing the shear and bending moments in the vertical supports.

Keywords: active control, passive control, active tendons, base isolation, spatial variation of the ground motion, quasi-static response, dynamic response

INTRODUCTION

The theory of vibration control for structures subjected to earthquake excitation is usually presented by assuming uniform ground support motion at the structure's foundations (e.g. Soong (1990)). Such an assumption is acceptable for structures whose base dimensions are "limited" so that the soil properties and the effects of the source magnitude and arrival time of seismic waves can be considered identical at all the supporting points. However, for horizontally extended structures, such as pipelines, long-span bridges, elevated highways and railways, or for extended structures with greatly varying soil conditions at the supports, the effects of the spatial variability of the earthquake induced ground motion on the structural response become extremely important, as proven by the vast structural damage produced in this type of structures during recent earthquakes (i.e. Northridge (1994), Hyogo-Ken Nanbu (1995)). The importance of multiple support excitation on the response of horizontally extended structures has been emphasized in previous studies (Mindlin and Goodman (1950)) and recent studies have confirmed the urgent need to include the effects associated with the spatial variability of the ground motion in regular engineering practice (Der Kiureghian and Neuenhofer (1993)). However, little has been done to incorporate the effects of the multiple support excitation in the vibration control theory of structures. Many research investigations had focused their attention on the analysis of control systems for building-type structures (e.g. Yang et al. (1988)) which had proven active tendon systems and active tuned mass dampers effective in reducing the response of such structures to strong external excitations. Recently, researchers have extended their analyses to include control mechanisms for horizontally extended structures such as bridges (Feng and Shinozuka (1990), Fujino et al. (1994), Xu et al. (1994), Betti and Panariello (1995)). In these studies, different control techniques (tuned mass dampers, aerodynamic appendages, active tendons) were used to reduce the vibrations of the bridge superstructure subjected to earthquake and wind excitation. However, only few of these studies (Xu et al. (1994), Betti and Panariello (1995)) have included the spatial variability of the ground motion in their control analysis.

In this paper, a comparison between different vibration control systems suitable for structures subjected to spatially varying ground motion is presented. Both active control and passive control strategies are examined using simple structural models with multiple support excitation. Advantages and disadvantages of both types of control strategies are outlined.

MULTIPLE SUPPORT EXCITATION

To highlight the effects of the multiple support excitation, let's consider a single-degree-of-freedom (SDOF) system with two supports (Fig.1). The mass of the system, m, is supported by two columns, with bending stiffness k_1 and k_2, respectively, while the damping of the system is provided by two dash-pots with damping constants c_1 and c_2. The external excitation is represented by the time histories of the ground displacements ($x_{g1}(t)$ and $x_{g2}(t)$) and of the ground velocities ($\dot{x}_{g1}(t)$ and $\dot{x}_{g2}(t)$) at the two supports. The equation of motion of the SDOF subjected to two different input ground motions can then be written as:

$$m\ddot{y} = -k_1(y-x_{g1}) - k_2(y-x_{g2}) - c_1(\dot{y}-\dot{x}_{g1}) - c_2(\dot{y}-\dot{x}_{g2}) \tag{1}$$

where $y(t)$ represents the total displacement of the structural mass with respect to a fixed system of reference. By comparing eq. (1) with the classical equation of motion for an SDOF with uniform input excitation, the fundamental difference between the two formulations is that now the equation of motion is expressed in terms of the total displacements rather than in terms of relative displacements. This factor makes these multiple support systems unique and particular care has to be placed in order to effectively control their vibrations. A common procedure for solving eq. (1) is to decompose the displacements $y(t)$ into 1) a quasi-static component $y_{qs}(t)$, which includes in the analysis the different excitation at the supports and 2) a dynamic component $y_{dy}(t)$, which is associated with the inertial forces for the case of the fixed base conditions. This structural response decomposition will be assumed for both the active and passive formulations.

ACTIVE CONTROL STRATEGY

Among the various active control strategies, the one that presents the most effective results is the so-called active tendon control system with two independent actuators (Fig. 1b). In this case, the equation of motion for a SDOF with active tendon control forces becomes:

$$m\ddot{y} = -k_1(y-x_{g1}) - k_2(y-x_{g2}) - c_1(\dot{y}-\dot{x}_{g1}) - c_2(\dot{y}-\dot{x}_{g2}) + f_{u1} + f_{u2} \tag{2}$$

where $f_{u1}(t)$ and $f_{u2}(t)$ are the control forces associated with the first and second actuator, respectively. As a consequence of the structural response decomposition into a dynamic and a quasi-static component, also the control force of each actuator can be decomposed into two components:

$$f_{ui} = f_{ui}^{qs} + f_{ui}^{dy} \tag{3}$$

where $f^{qs}(t)$ is the control force necessary to control the quasi-static components of the structural response and $f^{dy}_{ui}(t)$ the control force associated with the dynamic component of the structural response. For the quasi-static component of the control force, we assume that it is proportional to the relative displacements of the mass with respect to the corresponding support. This yields:

$$\begin{aligned} f^{qs}_{u1} &= -2g_1^{qs}\cos\alpha(y_{qs}-x_{g1}) \\ f^{qs}_{u2} &= -2g_2^{qs}\cos\alpha(y_{qs}-x_{g2}) \end{aligned} \tag{4}$$

where α is the tendons inclination while g_1^{qs} and g_2^{qs} are two proportionality constants that have to be determined. By substituting these expressions for the control forces into the equation motion, the quasi-static component of the structural response can be obtained by static condensation of eq. (2):

$$y_{qs} = \frac{(k_1+2g_1^{qs}\cos\alpha)}{(k_1+k_2+2(g_1^{qs}+g_2^{qs})\cos\alpha)}x_{g1} + \frac{(k_2+2g_2^{qs}\cos\alpha)}{(k_1+k_2+2(g_1^{qs}+g_2^{qs})\cos\alpha)}x_{g2} \tag{5}$$

The constants g_1^{qs} and g_2^{qs} can be determined by looking at both the structural characteristics and the control strategies. For symmetric structures, the quasi-static response is independent by the values of g_1^{qs} and g_2^{qs} and eq. (5) becomes:

$$y_{qs} = \frac{x_{g1}+x_{g2}}{2} \tag{6}$$

while for non-symmetric structures the quasi-static response depends on the selected control strategy. In this study, two control techniques have been selected for the case of non-symmetric structures: 1) minimum strain energy associated with the quasi-static configuration and 2) equal relative displacements between the top and the bottom of the two columns.

Once the control constants have been determined and the quasi-static response has been computed, it is possible to obtain an expression of the equation of motion (eq. (2)) in terms of the dynamic component of the structural response and of the control force. For either symmetric structures or non-symmetric structures which satisfy the control strategy of equal differential displacements at the supports, the equation of motion becomes:

$$m\ddot{y}_{dy}+c\dot{y}_{dy}+ky_{dy} = -\frac{1}{2}m(\ddot{x}_{g1}+\ddot{x}_{g2})+f^{dy}_{u1}+f^{dy}_{u2} \tag{7}$$

while for non-symmetric structures with the condition of minimal quasi-static strain energy, it becomes:

$$m\ddot{y}_{dy}+c\dot{y}_{dy}+ky_{dy} = -m\frac{k_1}{k_1+k_2}\ddot{x}_{g1}-m\frac{k_2}{k_1+k_2}\ddot{x}_{g2}+f^{dy}_{u1}+f^{dy}_{u2} \tag{8}$$

where $c=(c_1+c_2)$ and $k=(k_1+k_2)$ indicate the total structural damping and stiffness, respectively. Eq. (7) and eq. (8) are similar to the classical equation of motion of an SDOf subjected to uniform ground excitation. The only difference is that here the ground excitation is represented by a "weighted average" of the ground accelerations at the two supports. The control forces in eq. (7) and (8) are provided by the two sets of pretensioned tendons whose constants can be obtained using any available control strategy. In this study, an instantaneous optimal closed-open loop control has been used (Yang *et al.* 1987).

BASE ISOLATION STRATEGY

To seismically isolate a single-degree-of-freedom system with two supporting columns, it is necessary to introduce a base mass between each column and its respective isolator. The base mass provides a suitable surface to which the isolator is attached and acts as the column foundation. The SDOF system is thus transformed into a three-degree-of-freedom system.

Consider the previously analyzed SDOF system as shown in Figure 1c. Each column is attached to a base mass, m_b, which is fixed atop the isolator. The isolator is represented by a Kelvin body, composed of a shear spring, with lateral stiffness k_b , and a dashpot with damping constant c_b, in parallel. The equations of motion for this system can be written as:

$$\begin{aligned} &m\ddot{y}+c_1(\dot{y}-\dot{y}_{b1})+k_1(y-y_{b1})+c_2(\dot{y}-\dot{y}_{b2})+k_1(y-y_{b2})=0 \\ &m_b\ddot{y}_{b1}+c_b(\dot{y}_{b1}-\dot{x}_{g1})+k_b(y_{b1}-x_{g1})-c_1(\dot{y}-\dot{y}_{b1})-k_1(y-y_{b1})=0 \\ &m_b\ddot{y}_{b2}+c_b(\dot{y}_{b2}-\dot{x}_{g2})+k_b(y_{b2}-x_{g2})-c_2(\dot{y}-\dot{y}_{b2})-k_2(y-y_{b2})=0 \end{aligned} \tag{9}$$

where y(t), $y_{b1}(t)$, $y_{b2}(t)$ represent the displacement of the structural mass, the first base mass, and the second base mass respectively, measured with respect to the original, undeformed configuration. As before, each total displacement mentioned above can be decomposed into a dynamic component, e.g. $y^{dy}(t)$, and a quasi-static component, e.g. $y^{qs}(t)$, whose expressions can be obtained from static condensation of eq. (9):

$$\begin{aligned} y^{qs}(t)&=\frac{k_1(k_b+k_2)x_{g1}+k_2(k_b+k_1)x_{g2}}{k_bk_1+k_bk_2+2k_1k_2}, \\ y_{b1}^{qs}(t)&=\frac{(k_bk_1+k_bk_2+k_1k_2)x_{g1}+k_1k_2x_{g2}}{k_bk_1+k_bk_2+2k_1k_2}, \\ y_{b2}^{qs}(t)&=\frac{k_1k_2x_{g1}+(k_bk_1+k_bk_2+k_1k_2)x_{g2}}{k_bk_1+k_bk_2+2k_1k_2}, \end{aligned} \tag{10}$$

It can be seen that when the isolator stiffness is small, the displacement of the mass is the average of the ground displacements. When the isolator stiffness is large, the base mass's displacement equals the ground displacement below it.

Substituting eqs. (9) into eqs. (10), the equations of motion can now be rewritten in terms of the dynamic components of the structural response:

$$m\ddot{y}^{dy}+(c_1+c_2)\dot{y}^{dy}-c_1\dot{y}_{b1}^{dy}-c_2\dot{y}_{b2}^{dy}+(k_1+k_2)y^{dy}-k_1y_{b1}^{dy}-k_2y_{b2}^{dy}=-m\frac{k_1(k_b+k_2)\ddot{x}_{g1}+k_2(k_b+k_1)\ddot{x}_{g2}}{k_bk_1+k_bk_2+2k_1k_2}, \tag{11}$$

$$m_b\ddot{y}_{b1}^{dy}+(c_1+c_b)\dot{y}_{b1}^{dy}-c_1\dot{y}^{dy}+(k_1+k_b)y_{b1}^{dy}-k_1y^{dy}=-m_b\frac{(k_bk_1+k_bk_2+k_1k_2)\ddot{x}_{g1}+k_1k_2\ddot{x}_{g2}}{k_bk_1+k_bk_2+2k_1k_2}, \tag{12}$$

$$m_b \ddot{y}_{b2}^{dy} + (c_2 + c_b)\dot{y}_{b2}^{dy} - c_2 \dot{y}^{dy} + (k_2 + k_b) y_{b2}^{dy} - k_2 y^{dy} = -m_b \frac{k_1(k_b + k_2)\ddot{x}_{g1} + k_2(k_b + k_1)\ddot{x}_{g2}}{k_b k_1 + k_b k_2 + 2k_1 k_2} \tag{13}$$

which represent three coupled equations in $y^{dy}(t)$, $y_{b1}^{dy}(t)$ and $y_{b2}^{dy}(t)$. To obtain the solution of eqs. (11), (12) and (13), it is convenient to use modal analysis, considering the generalized eigenproblem associated with the free undamped structural vibrations. The solution to the characteristic polynomial are real, distinct eigenvalues which may be obtained analytically. If the stiffnesses, k_1 and k_2, of the supporting columns have the same value, then the eigenvalues can be readily calculated, with the common column stiffeness denoted by $k = k_1 = k_2$, as:

$$\omega_1^2 = \frac{2km_b + km + k_b - \sqrt{(2km_b + km + k_b m)^2 - 8kk_b mm_b}}{2mm_b}$$

$$\omega_2^2 = \frac{k + k_b}{m}$$

$$\omega_3^2 = \frac{2km_b + km + k_b + \sqrt{(2km_b + km + k_b m)^2 - 8kk_b mm_b}}{2mm_b}.$$

and the correspondent eigenvectors are expressed as:

$$\Phi = \begin{bmatrix} \alpha & 0 & \beta \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Though the constants α and β can be determined analytically, it is simpler to approximate them by first utilizing the binomial theorem and then expanding them as a Taylor series in terms of the stiffness ratio k_b/k, where only those of first order are retained (Kelly (1993)). This yields:

$$\alpha = 1 + (\frac{m_b}{m})^2 + (1 - \frac{m_b}{m})\frac{k_b}{k},$$

$$\beta = -\frac{m_b}{m}[(\frac{m_b}{m} + 2) - \frac{k_b}{k}].$$

Once the parameters α and β have been obtained, classical modal superposition analysis will allow us to obtain the generalized coordinates associated with the determination of the dynamic component of the structural response.

NUMERICAL EXAMPLES

To validate the assumptions considered in this study, different SDOF systems have been analyzed. The structural properties are: m = mass = 800 tons, total elastic stiffness = $(k_1+k_2) = 5 \times 10^4$ KN/m, damping coefficient $\xi = 0.02$ (2% damping) and an undamped natural frequency of 1.258 Hz. For the active tendon system, two sets of cables with stiffness $k_{te} = 10^3$ KN/m and inclined of an angle α equal to 36° have been used. For the elastomeric isolators, the damping coefficient $\xi_b = 0.15$ (15 % damping) has been selected, as the range of critical damping factor for elastomeric isolators is between 10 and 20 % (Kelly (1993)). The mass ratio m_b/m used in this study has been chosen equal to 0.25 while the isolator horizontal stiffness has been selected equal to 2870 KN/m, based on a compact isolator design chart (Kelly (1993)). The earthquake excitation is represented by the acceleration and displacement records 4S50W and 7S50W from the 1979 El Centro earthquake.

For the case of active tendon systems, figs (2) and (3) show the capability of this type of control systems in reducing the dynamic component of the structural response (from 13.93 cm for the uncontrolled case to 1.33 cm for the controlled case). In addition, figs (2) and (3) clearly indicate the importance of the non-uniform excitation in determining the relative displacements between the end-sections of the vertical columns and the inefficiency of such control systems in controlling the quasi-static component of the response. In fact, while the dynamic component of the response is almost reduced to zero by the action of the control forces, the quasi-static component of the structural response can be altered only in the case of non-symmetric structures where the criterion of equal quasi-static displacements has been considered. Other active control systems, such as Active Tuned Mass Dampers, present identical limitations in the vibration control of such extended structures: they are quite effective with the dynamic component of the response but they fail in reducing the quasi-static contribution to the structural response.

Base isolated systems seem to be much more effective in reducing the relative displacements between the end-sections of the vertical supports. In fact, while the inclusion of the two base isolators does not affect the absolute displacements of the structural mass, the relative displacements between the mass and the isolators are drastically reduced (fig 4). This follows by the fact that a rigid body displacement occurs which limits the amplitude of the relative displacements. This result is confirmed with the predominant mode of the dynamic response being the fundamental mode, whose frequency is 0.34 Hz. Figure (4) shows the total relative displacements between the two end-section of a vertical support and clearly indicates how the presence of base isolators reduces the amplitude of these differential displacements.

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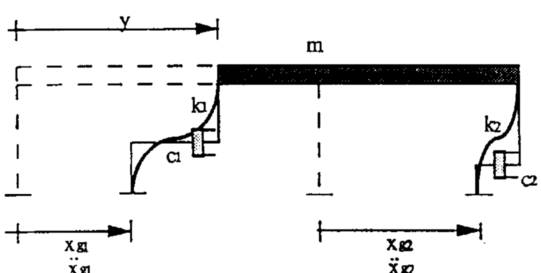

Figure 1

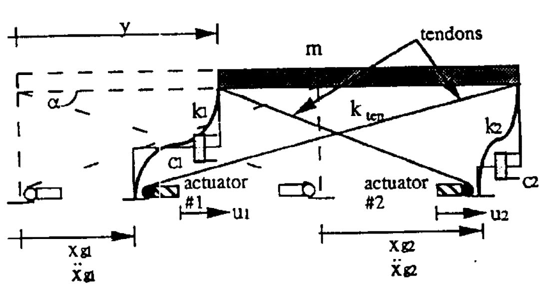

Figure 1b

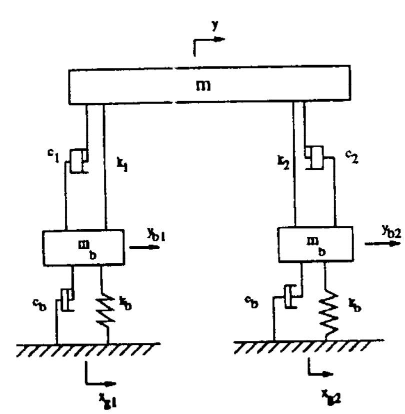

Figure 1c

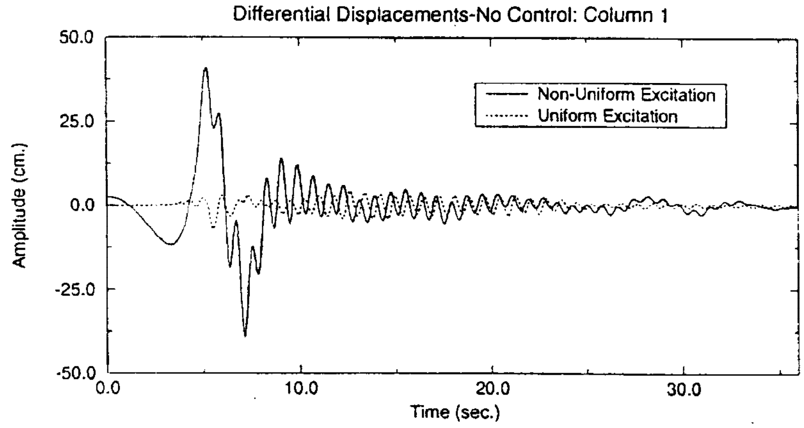

Figure 2

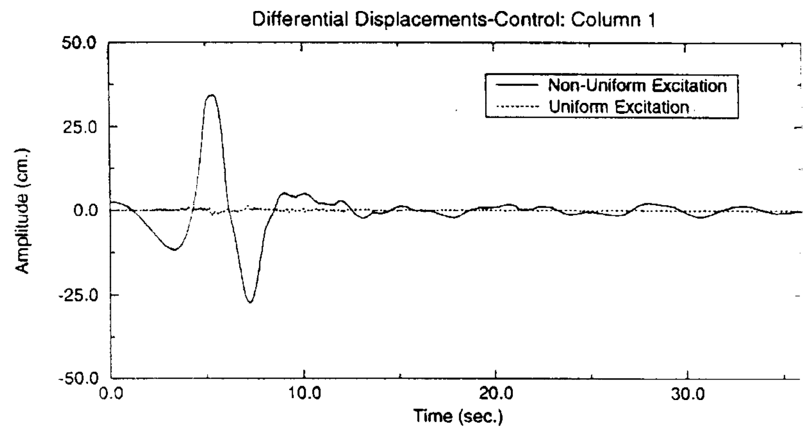

Figure 3

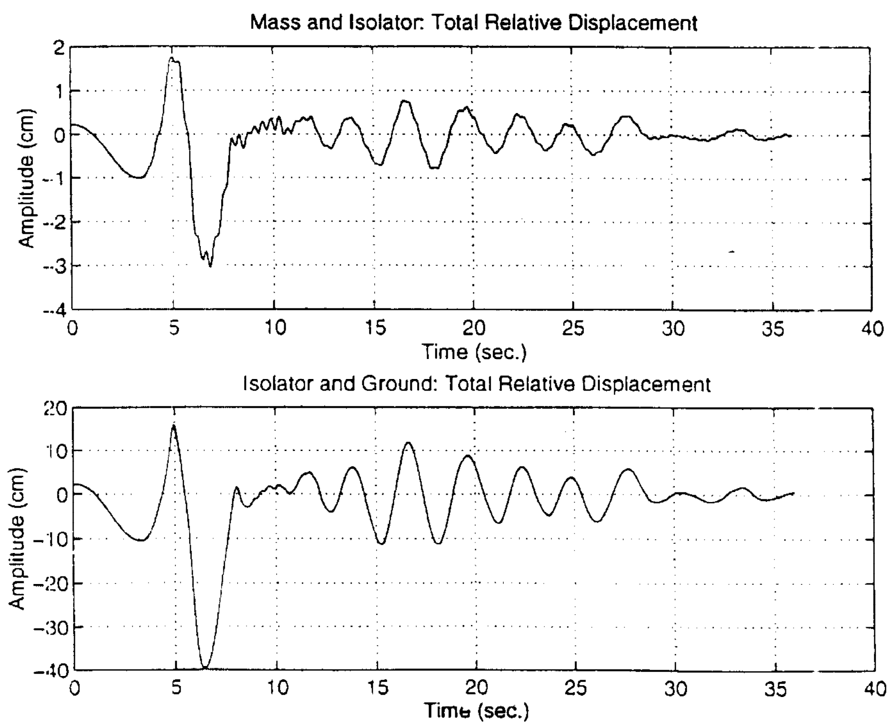

Figure 4