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EARTHQUAKE ANALYSIS OF MASONRY STRUCTURES

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ABSTRACT

Historical masonry structures, such as arches, vaults, and domes, depend solely on their funicular geometry to resist the forces imposed upon them. Many such structures have withstood earthquakes over the centuries, and some number have survived partial collapse conditions. We present an analytical method to model the behavior of such structures under earthquake motion, and to predict directly the earthquake motion which could cause collapse. The method is based upon the rigid limit state model of such geometric structures, in which excitation and collapse are characterized by the formulation of a kinematic mechanism. It has been noted that such structures move as one rigid body, with no mechanism formation, as long as the base acceleration remains below a threshold. That threshold acceleration can be determined by fundamental mechanics, and represents a useful measure of the earthquake resistance of such structures.

KEYWORDS

Masonry, arches, domes, rigid-body kinematics, limit state theory.

INTRODUCTION

In a recent article (Oppenheim, 1992) this author introduced the dynamic analysis of a masonry arch subjected to horizontal ground motion, modelling its response as a rigid body mechanism motion. That article presented the full derivation of the governing equation of motion, with solutions and sample results. In the same way that a rigid block (Housner, 1963) does not start to topple until a threshold value of ground acceleration is reached, the masonry arch is envisioned to remain intact at low values of ground acceleration, forming a mechanism only when such a comparable threshold is reached. This paper deals with the determination of that threshold value of horizontal ground acceleration using techniques that are well-known to practicing engineers.

Other researchers (Allen *et. al.*, 1986, Ulm *et. al.*, 1993, Facchini *et. al.*, 1994) have reported on similar analyses of masonry structures with rectangular portal geometry. There is also an extensive body of literature (see Augusti *et. al.*, 1992) on numerous theoretical questions of rocking block behavior, as well as literature addressing very real problems (see Sinopoli, 1989) in protecting historic masonry construction under seismic excitation. In particular, collapse mechanisms involving sliding have been noted, but this present paper ignores them in order to concentrate on the kinematic nature of masonry structure response to horizontal ground acceleration. Finally, there is a large body of work toward developing and applying finite element analysis to study the earthquake response of such historic masonry structures. The reader is advised to explore the extensive literature on these related topics.

ANALYSIS OF A MASONRY ARCH

Figure 1 depicts a bare part-circular arch with an embrace of 157.5°, radius a to its middle surface and thickness t equal to $0.15a$. It is assumed here that the masonry is rigid with infinite compressive strength and zero tensile strength, and the arch is subjected to a horizontal ground acceleration to the left. We assume that slip does not occur, and we seek to identify the mechanism motion by which the arch responds to the ground acceleration. As reported in previous work, the structure moves as one rigid body, with no mechanism formation, as long as the base acceleration does not exceed a threshold value αg.

Figure 1 depicts the example arch at the horizontal ground acceleration αg, creating a four-link mechanism $ABCDA$. We show the vertical self-weight force at the center of gravity in each link, W_{AB} and so on, as well as the effective horizontal force with respect to the ground in each link, αW_{AB} and so on. We intend to visualize a virtual displacement of the arch mechanism and to calculate the external virtual work performed by the six forces pictured in Figure 1, from which the threhold value of αg can be obtained. This is a direct counterpart to the use of virtual work in plastic analysis of steel frames, in which it is necessary to explore different possible hinge locations to find the governing collapse mechanism.

Figure 2 depicts the geometry of the arch as a conventional four-link mechanism, the motion of which is completely described by the rotations of each link: θ_{AB}, θ_{BC}, and θ_{CD}. The three rotations are not independent and the planar four-link mechanism possesses only one degree-of-freedom. The center of rotation for link BC is the instantaneous center I, obtained by a geometric construction commonly employed in the plastic analysis of steel gable frames. Note that this construction applies only to the instantaneous motion, but is perfectly adequate to determine the threshold ground acceleration αg. The kinematics can also be obtained analytically or by modern computer applications.

The calculation of external virtual work for the mechanism in Figures 1 and 2 involves six product terms. The three horizontal forces are all paired with rightwards virtual displacements, and thus all yield positive virtual work. The vertical force W_{AB} is paired with a downwards virtual displacement, thus yielding positive virtual work, but W_{BC} and W_{CD} are both paired with upwards virtual displacements, yielding negative virtual work. With zero horizontal ground acceleration the total external virtual work calculated in this process is negative; this demonstrates that if the arch is slightly displaced into this geometry it will restore itself to its original configuration. Setting the total external virtual work to zero yields the threshold value of α at which mechanism motion forms.

If a scale model is constructed from any material of uniform density and placed on a tilt table, the static forces it experiences under gravity in the normal and tangential (downhill) directions are exactly proportional to those depicted in Figure 1 in the vertical and horizontal directions, respectively. A static tilt table test on a scale model will produce the governing geometric mechanism, which will develop at an angle with tangent equal to α. We have found such tilt table tests of scale models to be useful and instructive.

A MULTIPLE SPAN SYSTEM; A SUPPORT PIER

The preceding section treated only a single arch supported by rigid abutments. In the absence of sliding between blocks, the only mechanism motion which would occur under horizontal acceleration is the four-link mechanism as outlined. However, arches are often constructed in series, and the support conditions at interior springing points may admit numerous additional kinematic mechanisms. For example, Figure 3 depicts two arches of the geometry treated in the preceding example, for which αg, the horizontal ground acceleration causing mechanism motion individually, would be $0.37g$. However, if the central support moves horizontally, as shown in Figure 3, an alternate mechanism geometry develops with a spreading motion in the left-hand span and a hogging motion in the right-hand span. (In actuality, the hinges may not all form at the bottom springing points but perhaps instead at the first or second block joints, a detail which would complicate this discussion, so this example is demonstrated for the simpler geometry as pictured.) Figure 3 also shows the instantaneous centers of rotation used to construct the rotations and displacements used in the virtual work calculation. If the central support is a frictionless bearing condition, then αg, the horizontal ground acceleration causing mechanism motion in Figure 3, is only $0.14g$. If a friction coefficient μ characterizes the reaction against the horizontal plane at the central support, then αg increases to $(0.14 + \mu)g$. (This last result appears gratuitous, but is not. If one poses a unit horizontal displacement at the central support position and establishes the displacement vectors at the center of mass for each arch segment, for the part-circular arches of uniform thickness the virtual work done by the effective forces in the horizontal direction equals one-half the total weight of the system multiplied by α. Correspondingly, the negative virtual work done by friction under that unit displacement equals μ multiplied by the vertical reaction, which in this particular geometry is also precisely one-half the total weight of the system. This somewhat coincidental condition accounts for the appearance of μ as a simple addend to the frictionless result.)

Figure 4 depicts the two arches supported by a central pier. In this example the pier has a width $0.30a$, a height H, a depth into the figure equal to that of the arch, and a corresponding self-weight. The candidate mechanism geometry involves a rotation of the pier about its toe, producing a spreading motion in the left-hand span, a hogging motion in the right-hand span, and a slight increase in elevation of the central support. The horizontal ground acceleration causing that mechanism motion is shown in Plot 1 as a function of the pier height. For a pier height less than approximately $0.9a$ the governing mechanism motion remains that of Figure 2, for which αg is $0.37g$, while for taller piers the horizontal ground acceleration causing mechanism motion decreases.

Another potential mechanism, not shown, corresponds to a tall pier in which the arches maintain their geometry and thereby prevent the top of the pier from moving horizontally, causing the pier to fail like a beam at its mid-height while raising the arches at their central support. The horizontal ground acceleration causing that mechanism motion, αg, is also shown in Plot 1 as a function of the pier height. In this particular case that mechanism does not govern the behavior, but under other geometries and other pairings of arch weight and pier weight it could govern.

The purpose of these examples is to illustrate the kinematic possibilities for mechanism formation. Each possible mechanism can be analyzed, and the governing mechanism is the one which corresponds to the lowest threshold of horizontal ground acceleration. This is comparable to the exploration of governing failure mechanisms in plastic analysis of steel frames.

A DOME

Previous work on the kinematics of domes (shells of revolution) has been limited to visualizing mechanisms associated with static collapse under self-weight or other radially symmetric conditions. Before discussing the earthquake response of domes envisioned here, it is first appropriate to distinguish the dome and its three-dimensional behavior from that of the arch. If such a dome is sufficiently thick, then diametrically opposed lunes can function as an arch. However, domes can be built, and typically are found, with lesser thickness than would be needed for such lune-arch behavior. Rather, the masonry in thinner domes forms a rigid top cap, and diametrically opposed slices do not abut one another but instead abut this top cap.

The existence and geometry of such a top cap can be visualized by subjecting the dome to a small radially outward displacement at its base. Figure 5 is a sketch depicting the subsequent development of meridional cracks, with the observation that the cracks do not extend to the crown. Rather, they end at the height at which the rigid cap forms, which can be calculated from a combination of kinematics and statics. For a hemispherical dome of radius a with thickness $0.05a$, the rigid cap develops approximately 30^o away from the vertical axis of the dome. The portion of any lune below that rigid cap, bounded by two meridional cracks, is simply termed a slice. (In Figure 6 the thrust line which defines the rigid cap under gravity load alone is sketched on the left half of the section.)

The mechanism motion envisioned here is simply the toppling of the slice that is oriented such that the effective horizontal force acts outward normally; in a tilt table test, it is the slice occupying the downhill position. Figure 5 also depicts the outcome of such a mechanism failure, and Figure 6 depicts that vulnerable slice, which topples by simple overturning about its toe. For the hemispherical dome discussed above, the horizontal acceleration at which that mechanism motion would occur, αg, is determined to be $0.39g$. (In theory, at horizontal ground accelerations exceeding αg a growing zone of such slices would topple outward.) Note that the geometry of this mechanism, while reasonable, is only approximate, because possible changes in the geometry of the rigid cap have not been fully explored.

This mechanism was envisioned only after conducting model experiments on a thicker dome with a non-circular meridian. That dome was constructed with a total of 32 blocks, 8 in the circumferential direction and 4 in the meridional direction, and was laid as a segmental (octagonal) dome. Tilt table testing clearly demonstrated the mechanism to be the outward toppling as one rigid body of the downslope segment, comprising one whole octant of the dome, with the other seven-eighths of the dome remaining intact. These experimental results were unambiguous. They also demonstrated the funicular behavior of the remaining partial (seven-eighths) dome at that tilt angle. The model dome was relatively thick, such that it could originally display lune-arch behavior, and therefore the generalization of those observations to thinner domes required the additional envisioning of the rigid top cap within the dome geometry.

The theoretical threshold resistance of domes to horizontal ground acceleration appears to be quite high. In actuality, the behavior of a dome will be largely influenced by motion of its supporting piers, including elastic movements and including incoherent motion effects. Nonetheless, this mechanism is interesting to contemplate, and it was instructive to observe the stability of the remaining partial dome. We note that the main dome in Hagia Sophia has twice demonstrated its integrity when reduced to a partial (approximately three-quarter) dome.

CONCLUSIONS

The geometry of the masonry structure establishes its capacity to resist static and dynamic loads. Proper visualization of the mechanism kinematics allows direct calculation of the theoretical threshold of horizontal ground acceleration, below which the structure should behave as one rigid body. Excitation below such a threshold value can therefore be broadly interpreted as (theoretically) survivable, making this calculation of fundamental interest to the engineer. The calculation can be performed using principles of mechanism kinematics and virtual work calculations, familiar to engineers working with plastic analysis of steel frames. Examples have been presented, for the first time, for multiple-arch series and for domes.

Nonetheless, the actual response of such structures will be weaker than predicted herein, largely because of elastic support motion, incoherent support motion, local sliding, and so on. It is recommended that such additional influences be investigated, consistent with the framework of mechanism motion outlined herein.

ACKNOWLEDGEMENTS

The author is grateful to undergraduate students who conducted model experiments and analysis as part of their 1995 senior project activities in the Department of Civil and Environmental Engineering. Mr. Kevin Johns performed the experiments on a part-spherical dome, revealing the mechanism whereby one segment, or one slice, would topple. Mr. James Tyler performed the experiments on a mutliple-arch series. In all cases we are indebted to our faculty colleague, Lawrence G. Cartwright, for his direction of those project activities.

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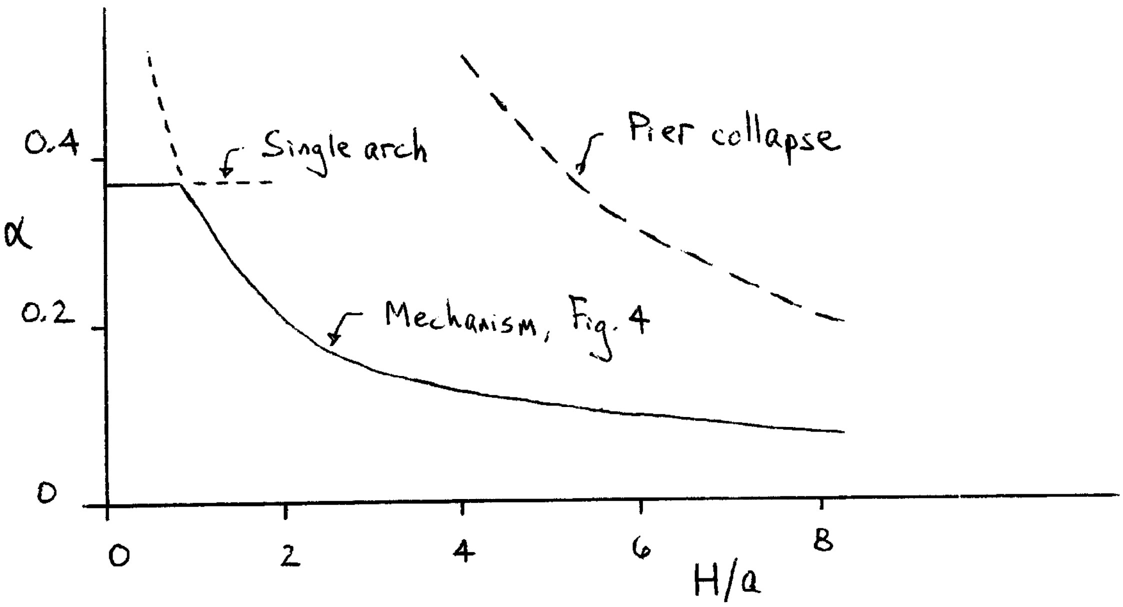

Plot 1. α **v. pier height**

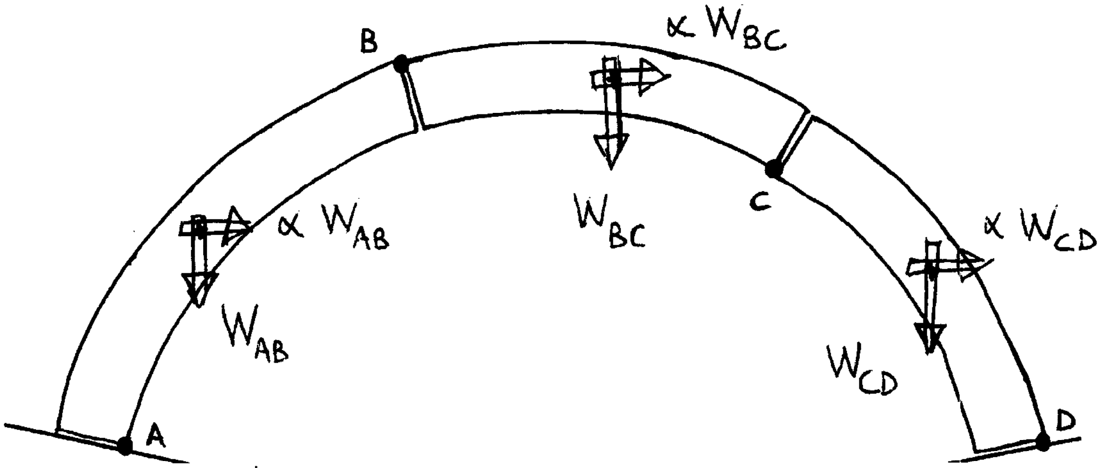

Figure 1. Part-Circular Arch showing Mechanism ABCDA

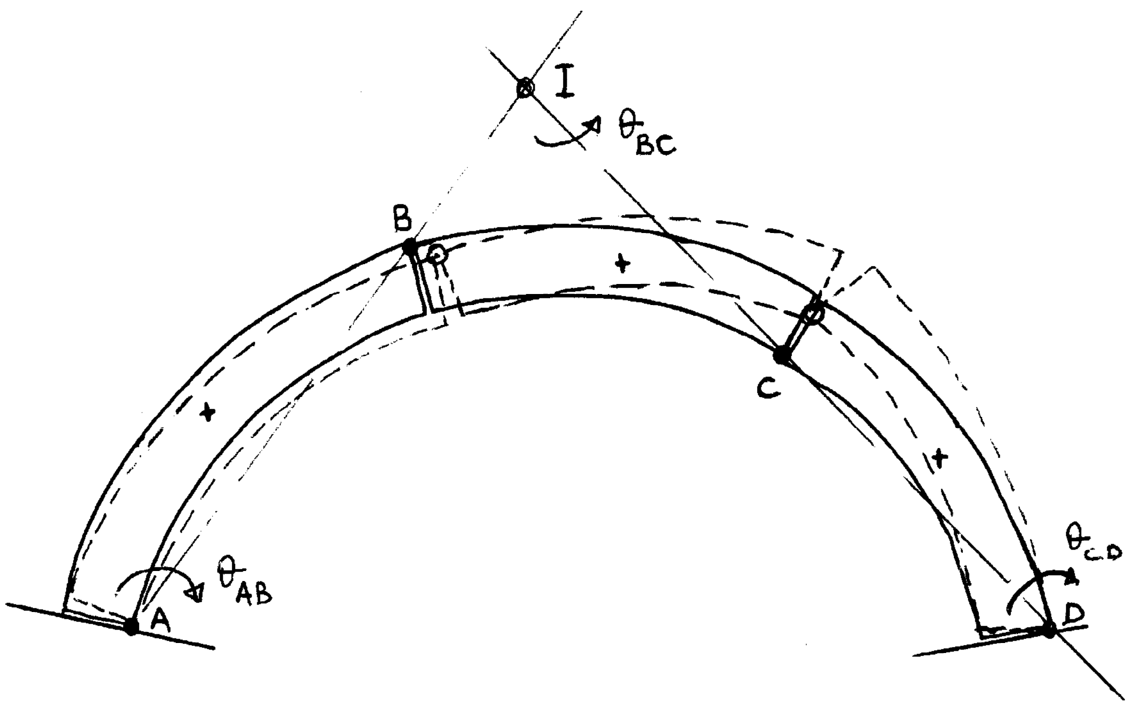

Figure 2. Kinematics of Mechanism Motion; Instantaneous Center of Rotation

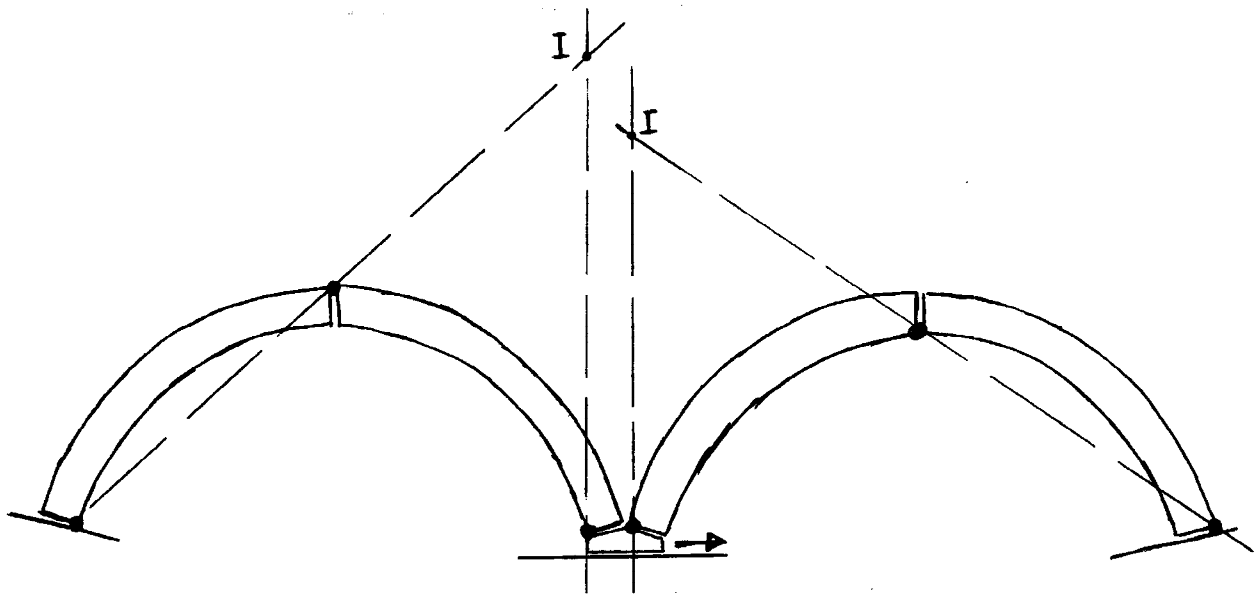

Figure 3. Series of Two Arches, Translation at Center Support

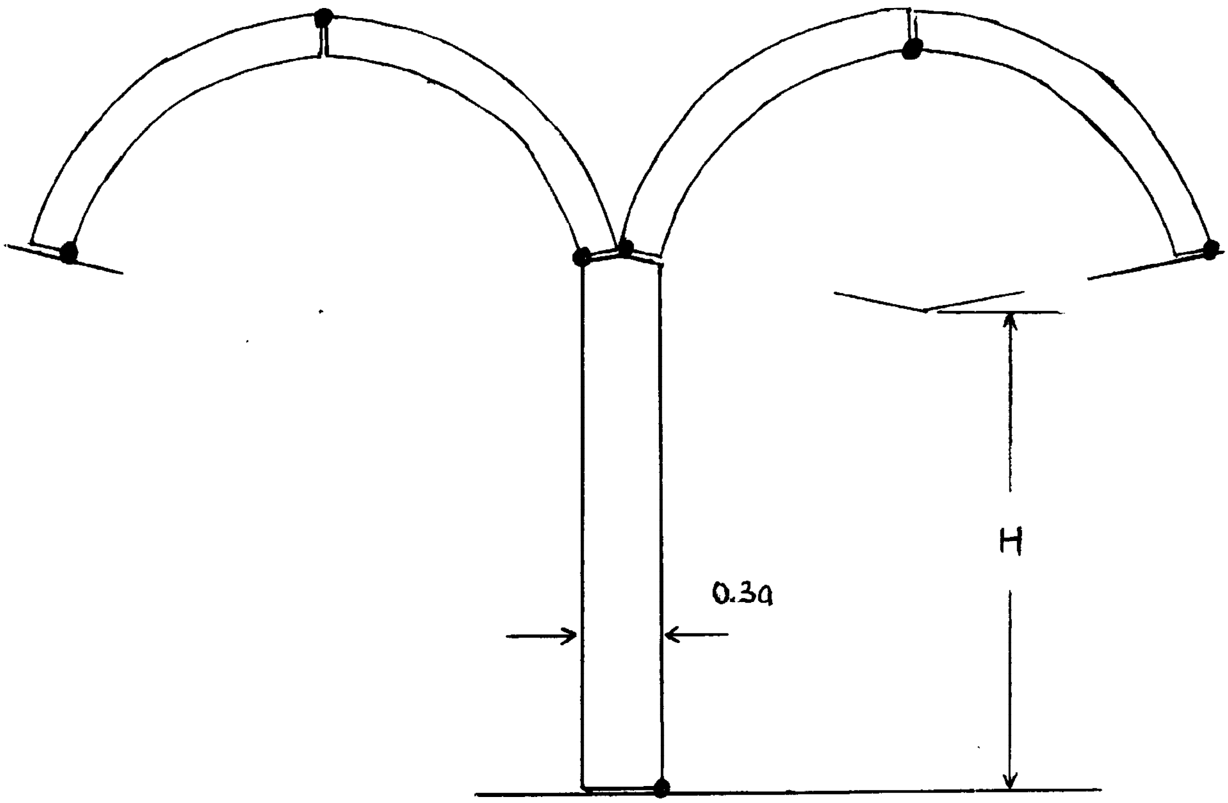

Figure 4. Series of Two Arches, Rotation of Central Pier

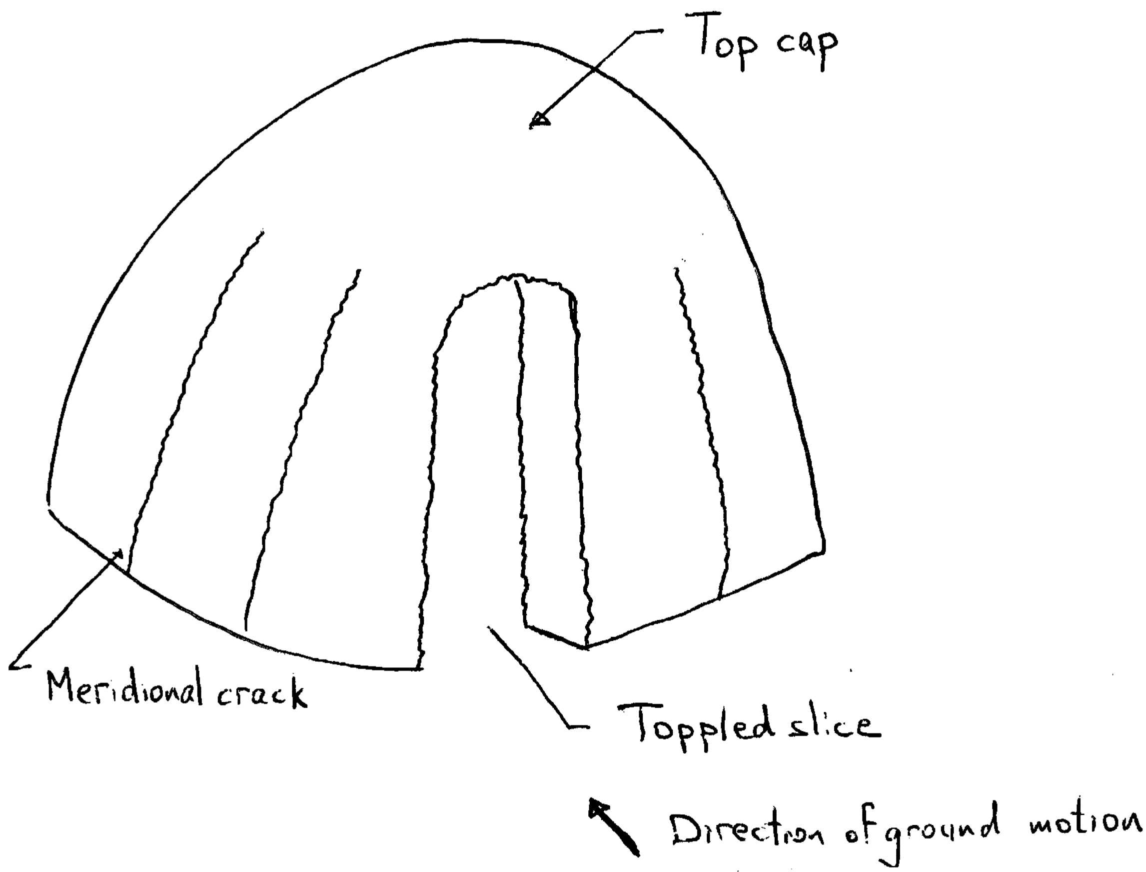

Figure 5. Dome, Meridional Cracking under Spreading, Location of Toppled Slice

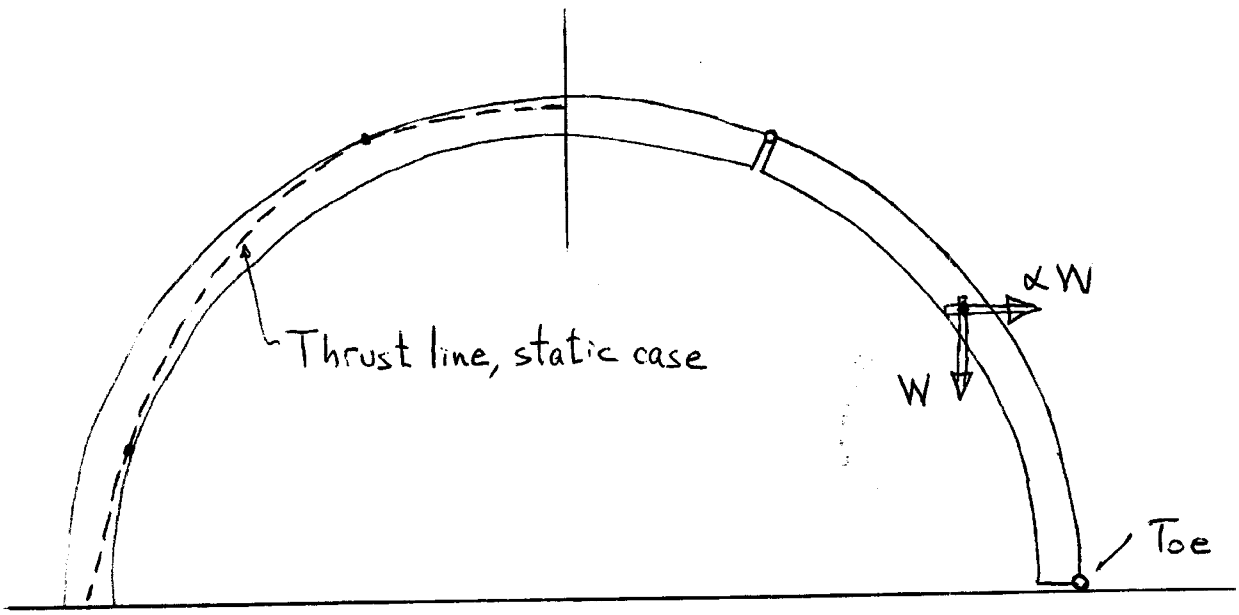

Figure 6. Forces on Slice, Rotating about Toe