



## SEISMIC ANALYSIS FOR SAFETY EVALUATION OF TAJ MAHAL MONUMENT

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### ABSTRACT

There is an increasing awareness to protect historical monuments, in future earthquakes. Taj Mahal is one such monument known for its cultural heritage worldwide and must be protected from damage. The strength evaluation of the structure taking into account future earthquake forces besides gravity and other loads is of concern for determining its safety. The seismic evaluation forms the essential step for retrofitting of such structures under occasional earthquake loads. In the present study, dynamic analysis of the Taj Mahal monument has been carried out on the basis of two simplified 3D mathematical models for fixed and flexible base condition using IS code and site dependent spectra. The free vibration characteristics and seismic response for bending moments, bending stresses, torsional moments, shear forces and axial forces have been studied.

### KEYWORDS

Taj Mahal; monument; 3D mathematical model, seismic analysis; response spectra; seismic response; safety evaluation

### INTRODUCTION

The Taj Mahal is one of the finest architectural monuments of the world and has been included in the cultural treasures of the world heritage. Because of the increasing awareness to protect the historical monuments and its importance, seismic safety evaluation of the marble mausoleum is of great value for its future safety against possible earthquakes.

### THE TAJ MAHAL MONUMENT

The monument lies in the seismic zone III of India on the bank of river Yamuna in the historical city of Agra. The structure has survived small to moderate earthquakes in its life. The Taj Mahal monument (Figure 1) consists of a central main dome resting on two storey thick brick masonry walls square in plan, four small domes each at corner with four 40m high independent minars at the four corners. The whole monument is resting on large platform of thick brick masonry raft which is approximately 18m above the

ground level. The center to center distance of columns is 32m and the storey height is measured as 11 m. The foundation soil consists of a deep alluvial deposit.

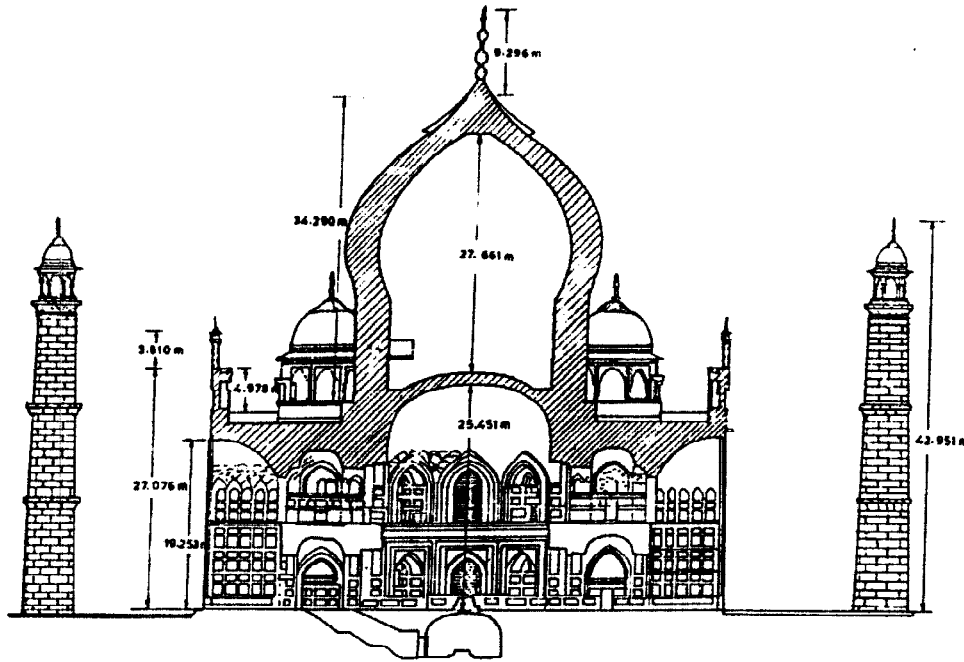


Fig. 1. Sectional elevation of Taj Mahal

## MODELING OF THE STRUCTURE

Dynamic analysis of the monument has been carried out for two simplified 3D mathematical models (i) fixed base and (ii) flexible base conditions. The first model is considered fixed at the center of raft while other is considered to be resting on soil springs. Figure 2 shows the 3D mathematical model of the complete structure, each portion of structure wall/ dome are represented by assemblage of beam elements with 6 degree of freedom at each node.

## MODELING OF THE FOUNDATION

### Type of Soils

There is sandy soil strata beneath the monument having the weight densities  $1.8 \text{ ton/m}^3$  and  $1.6 \text{ ton/m}^3$  at *masjid* site and *mehman-khana* site respectively. The Poisson's ratio is 0.25. The sandy soil is followed by clayey soil. Both the sandy and clayey soil strata beneath the monument are submerged. No material change in the mechanical properties of the various soil layers would therefore occur due to further impoundment of water in the river Yamuna.

### Type of Foundation

The raft foundation is solid square platform measuring approximately  $100\text{m} \times 100\text{m}$  in plan and 35m thick. The portion of the raft above the ground level is about 18m. The raft is built in brick masonry consisting of thick fire burnt clay bricks and mortar joints of varying thickness. The platform rests on thick sandy layer followed by clayey layer underneath. In the two mathematical models the bottom most vertical members are assumed to be rigid and half the weight of the raft is lumped at the four bottom nodes

## Soil Springs

Based on the meager data published in the literature, soil spring constants for an embedded massless rigid rectangular foundation which forms a prism have been used. The increase in stiffness for embedment leads to a factor with which the value for surface foundation is multiplied and which is for surface foundation equal to one. For three different values of shear wave velocities, different soil spring constants are calculated and one fourth of each spring constant is used at all the four bottom most nodes as shown in Fig. 2.

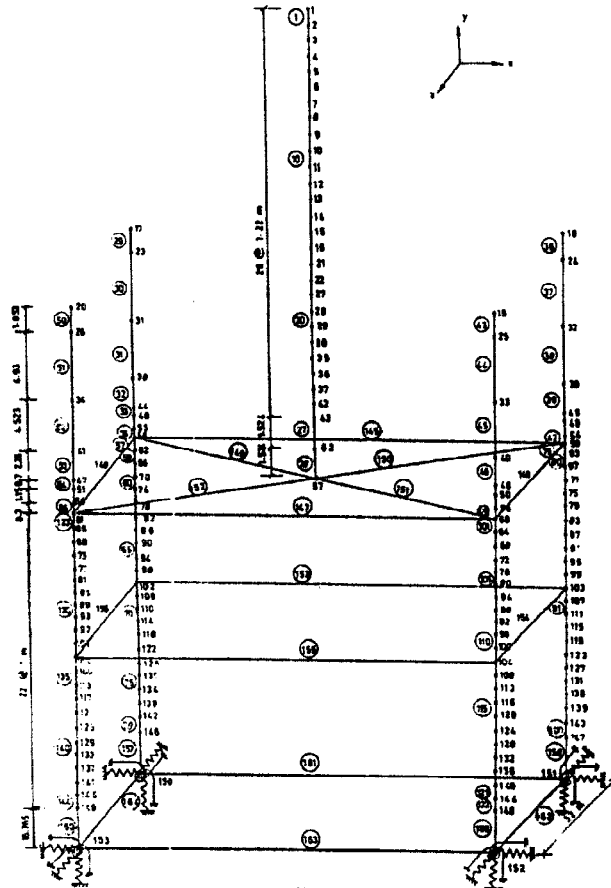


Fig. 2. Mathematical model of the monument with springs at the bottom nodes

## MATERIAL PROPERTIES

Materials used in the Taj Mahal are sand stone, brick work in lime mortar and marble for cladding. Modulus of elasticity, Poisson's ratio and unit weight densities of all the materials are given in Table 1. The unit weights of soils at *masjid* site is  $1.8 \text{ ton/m}^3$  and at *mehman-khana* site is  $1.6 \text{ ton/m}^3$ . The unit weight of soil for analysis purposes is taken as  $1.7 \text{ ton/m}^3$ , the average of weight densities at the above two sites. The Poisson's ratio for the soil is taken as 0.25. The analysis has been carried out for three different shear wave velocities, 158 m/sec, 300 m/sec and 600 m/sec and shear modulii have been calculated accordingly.

Table 1. Material properties of the Taj monument

Material	Modulus of Elasticity ( $\text{ton/m}^2$ )	Poisson's ratio	Weight density ( $\text{ton/m}^3$ )
Sand stone brick work	$2.5 \times 10^6$	0.2	2.40
Brick work mortar	$1.0 \times 10^5$	0.2	1.60
Marble cladding	$2.5 \times 10^6$	0.2	2.65

## RESPONSE SPECTRA

The site dependent spectra has been worked out (EQ 92-10) at the site of Taj Mahal monument, taking into consideration local soil conditions, geology and seismic history. The zero period acceleration for this spectra is 0.2g while it is 0.1g for code response spectra. The response spectrum analysis has been carried out for both fixed base and flexible base models using IS code spectra and site dependent spectra. For fixed base condition damping of the structure has been taken to be 7% of critical and for flexible base condition 10%. As the city of Agra falls in seismic zone III, zone factor is taken as 0.2, importance factor as 1.5 and soil foundation factor as 1.2 for the analysis using code spectra. The digitized values of spectral acceleration (Sa/g) for IS code as well as site dependent spectra are given in Table 2 both for 7% and 10% damping.

**Table 2. Digitized values of spectral acceleration coefficients (Sa/g) for IS code and site dependent spectra**

Time period (sec.)	Spectral acceleration coefficients for site dependent spectra		Time period (sec.)	Average spectral acceleration coefficients for IS code spectra	
	7% damping	10% damping		7% damping	10% damping
0.00	0.200	0.200	0.00	0.100	0.100
0.08	0.340	0.320	0.10	0.182	0.155
0.16	0.460	0.400	0.20	0.184	0.160
0.24	0.520	0.453	0.30	0.184	0.160
0.32	0.550	0.473	0.40	0.168	0.150
0.40	0.550	0.470	0.50	0.151	0.130
0.48	0.537	0.447	0.60	0.138	0.120
0.56	0.500	0.410	0.70	0.125	0.110
0.64	0.457	0.377	0.80	0.115	0.100
0.72	0.417	0.347	0.90	0.105	0.090
0.80	0.387	0.320	1.00	0.096	0.082
0.88	0.360	0.303	1.10	0.091	0.078
0.96	0.333	0.280	1.20	0.085	0.070
1.00	0.320	0.273	1.30	0.080	0.064
1.20	0.279	0.230	1.40	0.071	0.058
1.40	0.240	0.200	1.50	0.067	0.054
1.60	0.211	0.173	1.70	0.058	0.048
1.80	0.193	0.160	1.90	0.053	0.042
2.00	0.180	0.150	2.10	0.049	0.040
2.20	0.170	0.143	2.20	0.046	0.040
2.40	0.160	0.140	2.40	0.045	0.038

## RESULTS OF ANALYSIS

Based on the dynamic analysis carried out for the Taj Mahal monument, the free vibration characteristics and seismic response for bending moments, bending stresses, torsional moments, shear forces and axial forces have been studied and are presented herein. A comparative study of structure with fixed base and spring base has also been made.

### Time periods

Table 3 shows the time periods of the structure in fixed base as well as in the three different cases of the

flexible base condition. The fundamental period for the model with fixed base works out to be 0.487 sec which is very close to the period of the model with spring base and shear wave velocity ( $V_s$ ) 600 m/sec.

Table 3. Time periods of the complete structure for fixed and flexible base conditions

Mode	Time period (second)			
	Model with fixed base	Model with flexible base		
		$V_s = 158\text{m/sec}$	$V_s = 300\text{m/sec}$	$V_s = 600\text{m/sec}$
1	0.487	0.716	0.547	0.501
2	0.485	0.711	0.543	0.496
3	0.443	0.709	0.445	0.443
4	0.423	0.448	0.423	0.423
5	0.422	0.423	0.411	0.422

### Dynamic displacements

The dynamic displacements of the structure at critical points namely top and bottom of main dome and at the top of small dome for fixed and spring base conditions are given in Table 4 and 5 using code spectra and site dependent spectra respectively. The maximum displacement is at the top of small dome and is equal to 0.87 cm and 8.44 cm for code and site dependent spectra respectively. While at the top of main dome the displacement is 0.29 cm and 2.77 cm for code and site dependent spectra respectively. The displacement at top of small dome is more than the top of main dome which is due to the fact that the main dome is more rigid than small dome.

Table 4. Displacements at critical locations in X-direction using code spectra

Location	Displacements (cm)			
	Model with fixed base	Model with flexible base		
		$V_s = 158\text{m/sec}$	$V_s = 300\text{m/sec}$	$V_s = 600\text{m/sec}$
Top of main dome	0.29	0.46	0.30	0.30
Top of small Dome	0.87	0.60	0.61	0.78
Bottom of main dome	0.12	0.27	0.15	0.13

Table 5. Displacements at critical locations in X-direction using site dependent spectra

Location	Displacements (cm)			
	Model with fixed base	Model with flexible base		
		$V_s = 158\text{m/sec}$	$V_s = 300\text{m/sec}$	$V_s = 600\text{m/sec}$
Top of main dome	2.77	4.10	2.82	2.90
Top of small dome	8.45	5.36	5.63	7.38
Bottom of main dome	1.14	2.43	1.37	1.20

## Bending stresses

The bending stresses at the bottom of main and small domes for fixed and spring base conditions are given in Table 6 and 7 using code spectra and site dependent spectra respectively. It is observed that stresses developed are more in fixed base model as compared to the three cases of the spring base model.

Table 6. Bending stresses at critical sections using code spectra

Location	Bending stresses (ton/m <sup>2</sup> )			
	Model with fixed base	Model with flexible base		
		V <sub>s</sub> = 158m/sec	V <sub>s</sub> = 300m/sec	V <sub>s</sub> = 600m/sec
At the bottom of main dome	3.06	2.39	2.63	3.04
At the bottom of small dome	3.86	1.53	2.33	3.35

Table 7. Bending stresses at critical sections using site dependent spectra

Location	Bending stresses (ton/m <sup>2</sup> )			
	Model with fixed base	Model with flexible base		
		V <sub>s</sub> = 158m/sec	V <sub>s</sub> = 300m/sec	V <sub>s</sub> = 600m/sec
At the bottom of main dome	29.71	25.86	24.34	28.54
At the bottom of small dome	37.45	16.61	21.61	31.54

## CONCLUSIONS

The paper describes various aspects of 3D seismic analysis and safety evaluation of Taj Mahal monument with the aim of retrofitting of structure for future earthquake. The influence of variation of soil properties on seismic response has been particularly studied. It is found that bending stresses in the domes are much smaller for soft soil conditions as compared to fixed base situation.

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