



ANALYSIS ACCURACY IMPLIED BY EQUIVALENT LINEAR MODELS OF BRIDGE ISOLATION

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ABSTRACT

Four equivalent linear models applicable for the determination of the maximum seismic responses of base-isolated regular bridges are evaluated in this paper. The systematic evaluation is conducted to investigate the accuracy of the models in predicting the maximum inelastic seismic responses of bi-linear hysteretic bearings. A single degree of freedom (SDOF) system composed of a rigid block isolated by bi-linear hysteretic bearings is excited by 55 earthquake ground motions. The discrepancies of maximum seismic responses predicted by the equivalent linear analysis method and inelastic seismic analysis method are measured using root-mean-square (RMS) errors. The average of RMS errors over the 55 earthquake ground motions are used to evaluate the formulations of equivalent linear analysis methods.

INTRODUCTION

Two equivalent linear models of bi-linear hysteretic bearings such as lead-rubber bearings have been provided by the American Association of State Highway and Transportation Officials (AASHTO) (Guide 1991) and Japanese Public Works Research Institute (JPWRI) (Manual 1992, Sugita and Mahin 1994). Two other similar models have recently been proposed as the alternatives (Hwang et al. 1994, Hwang et al 1995). For the regular bridges defined by AASHTO (Standard 1991), the equivalent linear analysis is presumed to be appropriate in approximating the maximum inelastic responses of base isolated bridges subjected to major earthquakes. The use of these equivalent linear model is in compliance with the need for the simplicity of practical design. All the equivalent linear models are composed of the effective stiffness (or effective period) and equivalent damping ratio corresponding to the period shift and damping increase due to the implementation of base isolators into a bridge structure.

The equivalent linear analysis of bridge isolation has been developed and studied to some extent previously (Hwang, et al. 1994, Hwang et al. 1995). In this paper, the accuracy of these specified equivalent linear models and a refined model (Hwang and Chiou 1995, Hwang, et al. 1995) in predicting the inelastic maximum responses of bi-linear hysteretic bearings will be evaluated using an earthquake ensemble consisting of 55 ground motions. The evaluation is composed of two aspects: (1) the fidelity of the equivalent linear models in simulating a bi-linear hysteretic bearings; and (2) the accuracy of the equivalent linear models implemented with an iteration procedure in predicting maximum seismic responses of the bi-linear hysteretic bearings. The accuracy of prediction is measured based on the root-mean-square (RMS) errors between analysis results from the equivalent linear methods and an inelastic

inelastic seismic analysis.

EQUIVALENT LINEAR MODELS

In the follows, four equivalent linear models corresponding to a bi-linear model shown in Figure 1 are very briefly summarized:

(I) Effective stiffness:

$$K_{eff} = \frac{1 + \alpha (\mu - 1)}{\mu} K_u \quad (AASHTO) \quad (1)$$

$$K_{eff} = \frac{1 + \alpha (c_B \mu - 1)}{c_B \mu} K_u \quad (Japan) \quad (2)$$

$$K_{eff} = \frac{K_u}{\{1 + \ln [1 + 0.13(\mu - 1)^{1.137}]\}^2} \quad (Hwang, et al. 1994) \quad (3)$$

$$K_{eff} = \frac{1 + \alpha (\mu - 1)}{\mu} \left[\frac{1}{1 - 0.737 \frac{\mu - 1}{\mu^2}} \right]^2 K_u \quad (Hwang, et al. 1995) \quad (4)$$

(II) Equivalent viscous damping ratio:

$$\xi_e = \frac{2(1 - \alpha) \left(1 - \frac{1}{\mu}\right)}{\pi [1 + \alpha (\mu - 1)]} \quad (AASHTO) \quad (5)$$

$$\xi_B = \frac{2(1 - \alpha) \left(1 - \frac{1}{c_B \mu}\right)}{\pi [1 + \alpha (c_B \mu - 1)]} \quad (Japan) \quad (6)$$

$$\xi_e = 0.0587 (\mu - 1)^{0.371} \quad (Hwang, et al. 1994) \quad (7)$$

$$\xi_e = \left[\frac{2(1 - \alpha) \left(1 - \frac{1}{\mu}\right)}{\pi [1 + \alpha (\mu - 1)]} \right] \frac{\mu^{0.58}}{6 - 10 \alpha} \quad (Hwang, et al. 1995) \quad (8)$$

where μ = the shear displacement ductility ratio defined as the design displacement d_i divided by the yielding displacement d_y of the isolation bearing; α = the strain hardening ratio of the inelastic stiffness K_d to the elastic stiffness K_u ; $c_B = 0.7$ given by the Japanese design manual.

These equivalent linear models are applied to the analysis of base-isolated bridges in conjunction with

an iteration procedure illustrated in Figure 2.

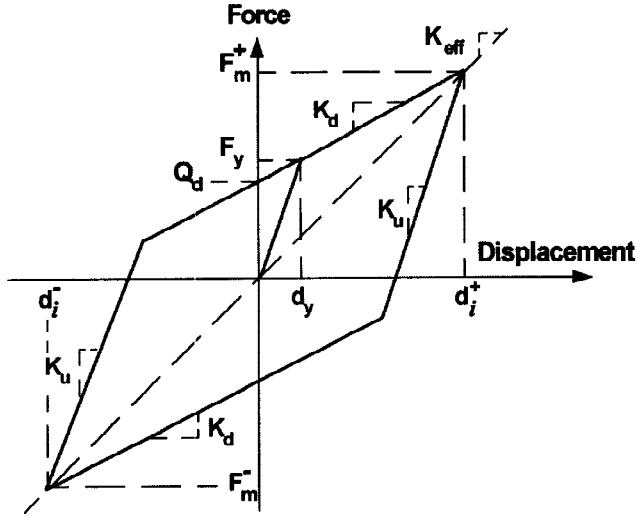


Figure 1 Bi-linear hysteresis model

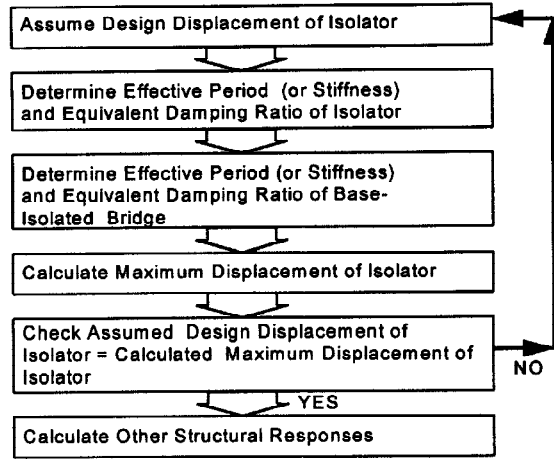


Figure 2 Equivalent Linear Analysis Procedure

EVALUATION OF EQUIVALENT LINEAR ANALYSIS METHODS

(1) Fidelity of Equivalent Linear Models : An isolated rigid block is modeled as a bi-linear hysteretic SDOF system subjected to 55 strong earthquake ground motions given in Table 1. The inelastic displacement response spectra of constant ductility ratios are calculated and used as the exact solutions for the evaluation. These inelastic spectral displacements are then employed as the design displacement (step 1 of Figure 2) instead of using an arbitrarily assumed displacement. Because the inelastic spectral displacement rather than an arbitrarily presumed displacement is used as the design displacement, it is not necessary to perform the iteration process as described in Figure 1. In addition, the calculated displacements will be exactly the same as the assumed displacements if the equivalent linear models are perfectly accurate. However, the difference between the inelastic spectral displacement and the calculated maximum displacement is expressed in terms of root-mean-square (RMS) errors to be used as a measure of the fidelity of various equivalent linear models in representing the bi-linear hysteretic bearings.

The RMS errors of maximum displacements for each earthquake ground motion are expressed as a function of natural periods T_i and strain hardening ratios α_k

$$RMS [S_d(T_i, \alpha_k)] = \sqrt{\frac{1}{N_\mu} \sum_{j=1}^{N_\mu} \left[\frac{S_{\mu d}(T_i, \mu_j, \alpha_k) - S_d(T_i, \mu_j, \alpha_k)}{S_{\mu d}(T_i, \mu_j, \alpha_k)} \right]^2} \quad (9)$$

where $S_{\mu d}$ = the maximum inelastic displacement response determined from the inelastic analysis; S_d = the maximum elastic displacement response determined from the equivalent linear analysis; T_i = the natural period of the SDOF system ranging from 0.2 to 1.6 seconds with a period increment of 0.1 second; N_μ = the total number of constant ductility ratios μ_j , for which 15 constant ductility ratios equal to 2, 4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 36, 40, 45 and 50 are used; and four strain hardening ratio $\alpha_k = 0.05, 0.10, 0.15$ and 0.20 are assumed.

Similar to Eq. (9), the RMS errors of seismic coefficients for each earthquake excitation are written as

Year	Earthquake Station	PGA (g)	Magnitude
--	AASHTO Isolation Design Earthquake	0.40	--
--	CALTRANS ARS Curve S.7GA51	0.70	--
--	CALTRANS ARS Curve S.6GB51	0.60	--
--	JPWRI Isolation Design Earthquake S_{20}	0.36	--
1940	Imperial Valley, El Centro, Irrigation District, S00E	0.34	$M_L = 6.3$
1952	Kern County, Taft, Lincoln School Tunnel, S69E	0.17	$M_S = 7.7$
1966	Parkfield, Cholame, Shandon, N69E	0.49	$M_S = 5.6$
1971	San Fernando, Pacoima Dam, S16E	1.17	$M_L = 6.5$
1979	Imperial Valley, El Centro, Meloland Overpass, Channel 15	0.32	$M_L = 6.6$
1983	Coalinga, Cantua Creek School, 360 degree	0.29	$M_S = 6.5$
1986	San Salvador, CIG, 90 degree	0.71	$M_S = 5.4$
1986	San Salvador, CIG, 180 degree	0.42	$M_S = 5.4$
1986	San Salvador, IGN, 180 degree	0.40	$M_S = 5.4$
1986	San Salvador, IGN, 270 degree	0.54	$M_S = 5.4$
1986	San Salvador, IVU, 90 degree	0.37	$M_S = 5.4$
1986	San Salvador, IVU, 180 degree	0.72	$M_S = 5.4$
1987	Whittier Narrows, Alhambra, Fremont School, 270 degree	0.40	$M_L = 6.1$
1987	Whittier Narrows, Altadena, Eaton Canyon Park, 360 degree	0.32	$M_L = 6.1$
1987	Whittier Narrows, Los Angeles, Obregon Park, 270 degree	0.44	$M_L = 6.1$
1989	Loma Prieta, Corralitos, Eureka Canyon Road, 90 degree	0.48	$M_L = 7.0$
1989	Loma Prieta, Corralitos, Eureka Canyon Road, 90 degree	0.63	$M_L = 7.0$
1989	Loma Prieta, Capitola, Fire Satation, 90 degree	0.47	$M_L = 7.0$
1989	Loma Prieta, Capitola, Fire Station, 360 degree	0.54	$M_L = 7.0$
1989	Loma Prieta, Gilroy #1, Gavilan College, 90 degree	0.44	$M_L = 7.0$
1989	Loma Prieta, Gilroy #1, Gavilan College, 360 degree	0.43	$M_L = 7.0$
1989	Loma Prieta, Gilroy #3, Gilroy Sewage Plant, 90 degree	0.37	$M_L = 7.0$
1989	Loma Prieta, Gilroy #3, Gilroy Sewage Plant, 360 degree	0.55	$M_L = 7.0$
1989	Loma Prieta, Oakland, 2 Story Office Building, 290 degree	0.26	$M_L = 7.0$
1989	Loma Prieta, San Francisco International Airport, 90 degree	0.33	$M_L = 7.0$
1989	Loma Prieta, San Francisco International Airport, 360 degree	0.24	$M_L = 7.0$

Table 1 Earthquake ground motions used for evaluation

Year	Earthquake Station	PGA (g)	Magnitude
1989	Loma Prieta, Santa Cruz, 90 degree	0.44	$M_L = 7.0$
1989	Loma Prieta, Santa Cruz, 360 degree	0.47	$M_L = 7.0$
1989	Loma Prieta, Saratoga, Aloha Ave., 90 degree	0.34	$M_L = 7.0$
1989	Loma Prieta, Saratoga, Aloha Ave., 360 degree	0.53	$M_L = 7.0$
1992	Petrolia, Cape Mendocino, 90 degree	1.20	$M_L = 6.4$
1992	Petrolia, Petrolia, 90 degree	0.69	$M_L = 6.4$
1992	Petrolia, Petrolia, 360 degree	0.62	$M_L = 6.4$
1992	Petrolia, Rio Dell, 272 degree	0.39	$M_L = 6.4$
1992	Petrolia, Rio Dell, 2 degree	0.55	$M_L = 6.4$
1992	Big Bear, Big Bear Lake, 270 degree	0.55	$M_L = 6.5$
1992	Big Bear, Big Bear Lake, 360 degree	0.57	$M_L = 6.5$
1994	Northridge, Castaic, Old Ridge Route, 360 degree	0.54	$M_L = 6.5$
1994	Northridge, Los Angeles, Hollywood Storage Building, 90 degree	0.24	$M_L = 6.4$
1994	Northridge, Los Angeles, Hollywood Storage Building, 360 degree	0.41	$M_S = 6.4$
1994	Northridge, New Hall, 90 degree	0.63	$M_S = 6.4$
1994	Northridge, New Hall, 360 degree	0.61	$M_S = 6.4$
1994	Northridge, Pacoima Dam Downstream 265 degree	0.44	$M_L = 6.4$
1994	Northridge, Pacoima Dam Downstream 175 degree	0.42	$M_L = 6.4$
1994	Northridge, Pacoima, Kagel Canyon 90 degree	0.30	$M_S = 6.4$
1994	Northridge, Pacoima, Kagel Canyon 360 degree	0.44	$M_L = 6.4$
1994	Northridge, Sylmar, County Hospital Parking Lot, 360 degree	0.84	$M_L = 6.4$
1994	Northridge, Sylmar, County Hospital Parking Lot, 90 degree	0.61	$M_L = 6.4$
1994	Northridge, Los Angeles, USC Hospital, Grounds, 5 degree	0.49	$M_L = 6.4$
1994	Northridge, Van Nuys, 7 Story Hotel, 270 degree	0.47	$M_L = 6.4$
1994	Northridge, Van Nuys, 7 Story Hotel, 360 degree	0.41	$M_L = 6.4$

Table 1 (continued) Earthquake ground motions used for evaluation

$$RMS [C_s (T_i , \alpha_k)] = \sqrt{ \frac{1}{N_\mu} \sum_{j=1}^{N_\mu} \left[\frac{C_{\mu s} (T_i , \mu_j , \alpha_k) - C_s (T_i , \mu_j , \alpha_k)}{C_{\mu s} (T_i , \mu_j , \alpha_k)} \right]^2 } \quad (10)$$

where $C_{\mu s}$ = the maximum inelastic seismic coefficient determined corresponding to the force determined from $S_{\mu d}$; and C_s = the maximum seismic coefficient calculated with respect to the force obtained from S_d .

In this study, both time history analysis and response spectrum analysis are conducted for the equivalent linear analysis. When the response spectrum analysis is used, the 5% damped elastic response spectrum is calculated first. The equivalent linear solutions corresponding to the effective stiffness and equivalent damping ratio are determined using the damping coefficient (Guide 1991, Manual 1992, Newmark and Hall 1982) to scaled the shifted 5% damped elastic spectrum. The average RMS errors of maximum displacement and seismic coefficient over the 55 earthquake ground motions are then calculated and shown in Figures 3 and 4. As can be seen from Figure 3, the average RMS errors predicted by the equivalent linear time history analyses are fairly constant and independent of the natural period of the SDOF system. All the equivalent linear model predict the maximum inelastic displacements and seismic coefficients with average RMS errors approximately below 20%. Based on Figure 4, the average RMS errors obtained from the response spectrum analysis are all larger than those of the time history analysis. Since the only difference between these two analyses is the use of damping coefficients, the damping coefficient is responsible for the loss of prediction accuracy. In general, the model proposed by Hwang et al (1995) predicts more accurately than the other three models.

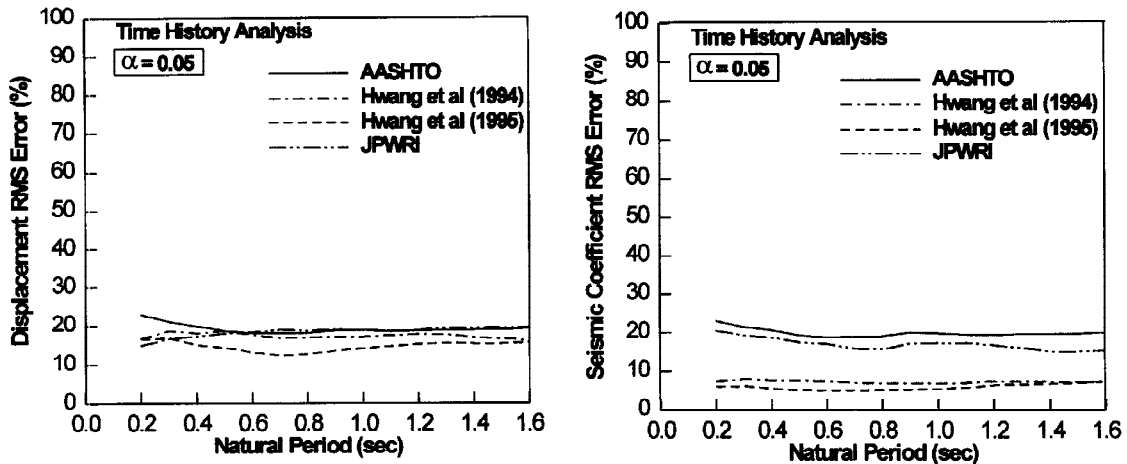


Figure 3 Comparison of average RMS errors predicted using time history analysis without iteration

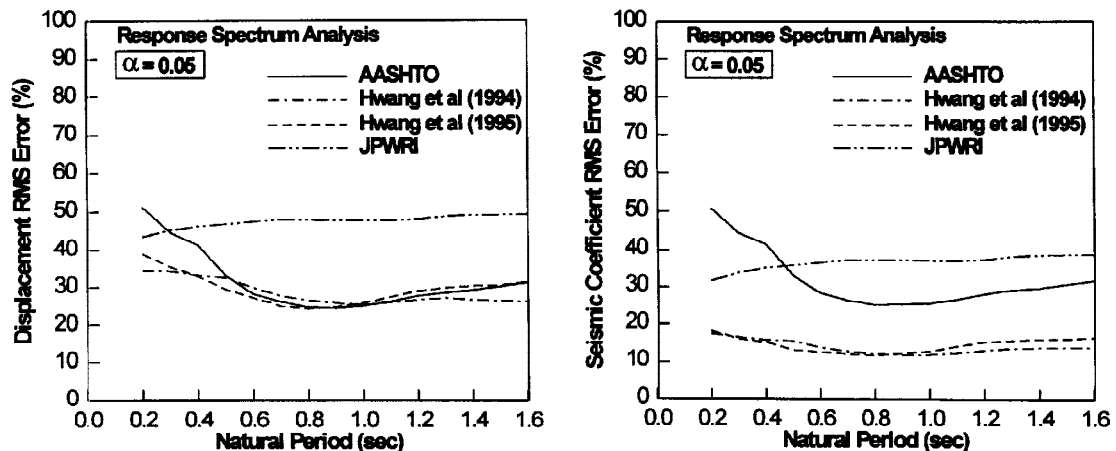


Figure 4 Comparison of average RMS errors predicted using response spectrum analysis without iteration

(2) Prediction Accuracy of Models Incorporated with Iteration Procedure : In addition to the evaluation on the fidelity of the equivalent linear models in representing the bi-linear hysteretic bearings, the accuracy of prediction using these equivalent linear models implemented with the iteration procedure will also be investigated. The design (or maximum) displacement of isolator is now arbitrarily assumed (step 1 of Figure 2) and the maximum displacement responses of the isolator are predicted through the iteration process. The average RMS errors of the prediction using time history analysis are summarized in Figure 5 for the maximum displacements and seismic coefficients of Comparing Figure 3 with Figure 5, it is observed that to use an arbitrarily assumed maximum displacement associated with the iteration procedure may increase the average RMS errors of maximum displacements and seismic coefficients for the range of short natural periods. For longer natural periods, e.g. 0.6-1.6 seconds, the prediction accuracy is not much affected. However, the prediction accuracy are much more amplified when using the response spectrum analysis in which both the iteration procedure and damping coefficient are employed for the calculation.

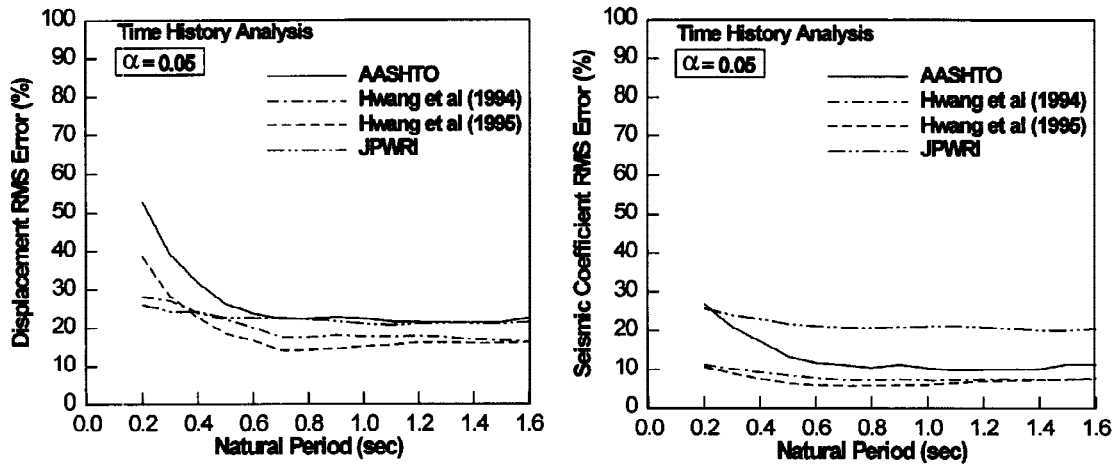


Figure 5 Comparison of average RMS errors predicted using time history analysis with iteration

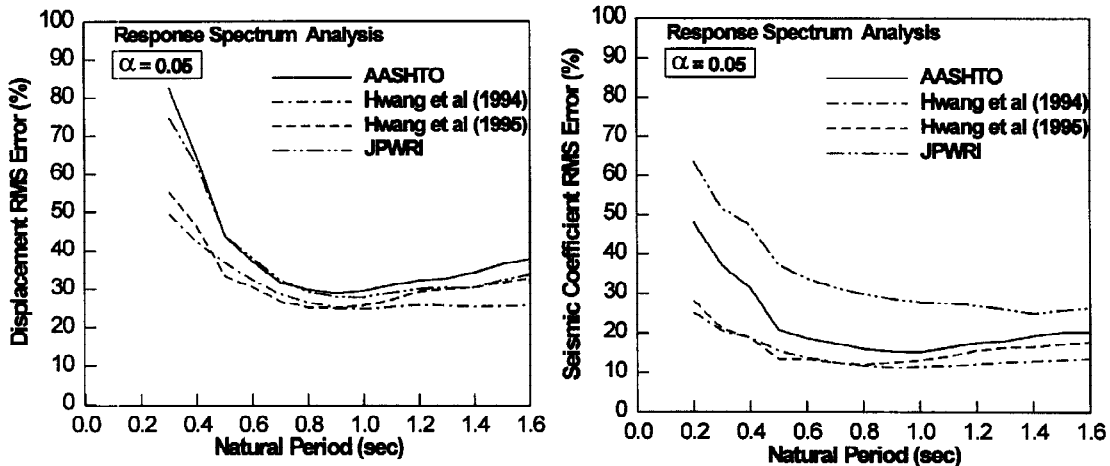


Figure 6 Comparison of average RMS errors predicted using response spectrum analysis with iteration

CONCLUSIONS

The total shear force transmitted by bi-linear hysteretic bearings is better determined using the models proposed by Hwang et al (1994, 1995) in which the force is calculated based on the hysteretic characteristics of the bearings when the maximum or design displacement has been obtained. Another method specified by the AASHTO isolation guide specifications and JPWRI manual is to determine the

total shear force directly from the multiplication of the calculated maximum displacement and effective stiffness. This method implies that the maximum inelastic acceleration response can be determined from the calculated maximum displacement using the simple relationship between the elastic displacement response spectrum and pseudo-acceleration response spectrum.

Based on the evaluation of prediction accuracy, it is concluded that, for practical analysis purpose, the equivalent linear time history analysis with the iteration procedure can predict the maximum inelastic response more accurately than the response spectrum analysis. The use of iteration procedure in general only slightly increase the discrepancy between the equivalent linear solution and inelastic solution. The increase of RMS errors resulting from the response spectrum analysis with iteration are due to the combined effects of the iteration procedure and damping coefficients with primary attribution to the damping coefficients.

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