

# REHABILITATION OF STRUCTURES VIA MEMBRANES

A. CATARIG, L. KOPENETZ and P. ALEXA

Technical University of Cluj-Napoca, Faculty of Civil Engineering, C. Daicoviciu 15, RO-3400 Cluj-Napoca, Romania

#### **ABSTRACT**

The paper presents a new procedure of refitting seismically affected structural massonry elements by using membrane type placating solutions. The possibility of obtaining the required level of ductility for the rehabilitated structure is emphasized and proved by computing and comparing numerical values of some parameters that govern the structural dynamic behaviour. Practical technology of ductilization of structural elements via membranes is presented for massonry columns and walls. An extensive part of the study deals with the structural analysis problem in the FEM technique emphasizing the discretization, the nonlinear formulation and the algorithm. The nonlinearity comes into the analysis mainly due to the taking into account of the deformed geometry and of the constitutive relations. The numerical results have been obtained by using SUM02 computer program worked out by the authors. Comparison of the obtained numerical values with values from the literature proves the correctness and the accuracy of the proposed procedure.

#### **KEYWORDS**

Earthquake; massonry structures; rehabilitation; membrane; ductility; structural analysis.

#### INTRODUCTION

The natural desasters, mainly the earthquakes, as well as those generated and conducted by man, mainly the wars, are imensly affecting the built heritage of the mankind. If a construction is more affected than another one from its vicinity, a fair conclussion is that the former has not been properly conceived and built. The financial reasons will usually determine the owner of a seismic affected structure to rehabilitate it, rather than built a new one. There are clear cut advantages of

structural rehabilitation versus reconstruction: a certain economy of material resources and energy and the preserving of the existing constructions. There is, of course, another class of structures that have to be rehabilitated and not rebuilt, as it is the architectural and cultural heritage. The new, rehabilitated structure has, on one hand, to reproduce - geometrically and functionally - the former, affected structure and, on the other hand, to improve its behaviour under a new seismic impact. Also, depending on the natural legislation, the rehabilitated structure has to observe the existing standards which are not always the same as those under which the old structure has been designed. The new building regulations refer mainly to the structural behaviour during the earthquake and, therefore, ductility requirements have to be fulfilled. The ductility requirements together with the need of reusing the material substance of the existing structure make the structural rehabilitation a highly demandind engineering process and open the door for innovation. The use of membranes and membrane-type elements in structural rehabilitation is a possible answer to the challanges the civil engineering community is confronted to.

### MATERIALS USED FOR MEMBRANES

The membranes are made up of several types of materials, from geosynthetics (Koerner, 1986), to concrete reinforced with sprayed steel fibres (Robins and Austin, 1986).

Two classes of materials for membranes are currently used: isotropic and unisotropic materials. Among the isotropic materials the following may be mentioned: the sheets made up of metal, polyesters, polyetilene, vinylpoliclorure (PVC) and vinylpolifluoride (PVF or TEDLAR). From the unisotropic class of materials, the most suitable ones for ductility activation of certain structural zones are: corrugated or sinuosidal shaped metallic sheets or even isotropic materials reinforced with mineral fibres (glass, carbon, graphitte) or synthetic fibres (DRALON, KEVLAR, PERLON, etc.). The reinforcing fibres may be used in one or several layers.

### STRUCTURAL ANALYSIS

A membrane introduced into, or augmented to a structure has to be carefully analysed in what regards its stress and strain states.

### Coordinate systems, strain and stress tensors

The initial underformed configuration °C and the current deformed configuration are expressed in a convective Lagrangean system of coordinates (Fig. 1).

The elongation of a line tangent to the surface may be computed as

$$(d^{1}s)^{2} - (d^{o}s)^{2} = 2^{1}e_{\alpha\beta} d^{o}\theta^{\alpha} d^{o}\theta^{\beta}$$

$$(1)$$

where

$${}^{1}e_{\alpha\beta} = \frac{1}{2} \Big( {}^{1}a_{\alpha\beta} - {}^{o}a_{\alpha\beta} \Big)$$

is the Green or Lagrangean strain tensor, while  ${}^{1}a_{\alpha\beta}$  and  ${}^{o}a_{\alpha\beta}$  are the metric covariant coefficients of the surface.

The Lagrangean technique implies the use of the Lagrangean stress tensor  $t^{ij}$  expressed in the initial configuration  ${}^{\circ}C$  and whose entries, generally, are non-symetric. To obtain symetry and facilitate the computation, the stress tensor may be transformed into a symetric stress tensor  $n^{\alpha\beta}$  called Piola-Kirchoff stress tensor of second type.

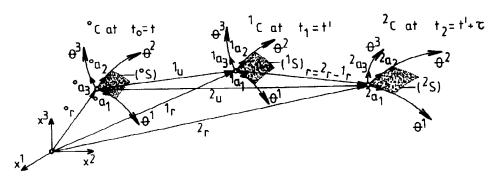


Fig. 1. Lagrangean system of coordinates

#### Constitutive relations

Assuming that during the deformation process temperature changes take place, the Piola-Kirchoff second type stress can be expressed in terms of Green strains (Kopenetz, 1989, Catarig et al., 1994) in the most general form

$$n^{\alpha\beta} = H^{\alpha\beta\gamma\lambda} \cdot {}^{1}e_{\rho\lambda} + H^{\alpha\beta\rho\lambda\delta\gamma} \cdot {}^{1}e_{\rho\lambda} \cdot {}^{1}e_{\delta\gamma}$$
 (2)

where  $H^{\alpha\beta\gamma\lambda}$  and  $H^{\alpha\beta\rho\lambda\delta\gamma}$  are the elastic tensors.

Taking into account that, in spite of large displacements, due to seismic actions the strains are, in their final stage, small, relation (2) may be liniarized by neglecting its second term, i.e.

$$n^{\alpha\beta} = H^{\alpha\beta\gamma\lambda}^{1} e_{\alpha\gamma} \tag{3}$$

The constitutive relation for the reinforcing fibres of the membrane can be obtained assuming that the fibres are equivalent to an homogenous layer in the proper direction. In this way, the constitutive relation expressed in the Cartezian system reads

$$\begin{cases} \sigma^{11} \\ \sigma^{22} \\ \sigma^{12} \end{cases} = \begin{bmatrix} E_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} + \varepsilon_{21} \end{bmatrix}$$

$$\sigma = \mathbf{E}_f \boldsymbol{\varepsilon}$$

In the case when damping is accounted for, the constitutive relations have to be altered.

For the usual computation, a linear visco-elastic constitutive law may be considered. In this way, relation (3) is substituted by

$$n^{\alpha\beta} = H^{\alpha\beta\rho\lambda} \cdot {}^{1}e_{\rho\lambda} + C^{\alpha\beta\rho\lambda} \cdot {}^{1}e_{\rho\lambda} \tag{4}$$

where

- $C^{\alpha\beta\rho\lambda}$  is the visco-elastic tensor,
- $-{}^{1}e_{\alpha\lambda}$  is the Lagrangean tensor of the strain rate.

### Incremental formulation of the nonlinear problem

In the case of nonlinearity, the Lagrange's dynamic equations make up a system of nonlinear second order differential equations. The linearization can be achieved using an incremental technique that leads to the substitution of the initial nonlinear problem with a succesive set of linear problems. The incremental formulation requires the introduction of an intermediate configuration  ${}^{1}C$  between the initial configuration  ${}^{\circ}C$  and the final configuration  ${}^{2}C$  (Fig. 1).

## Structural modelling

For structural modelling a FEM technique is employed using izoparametric Zienkiewicz-Irons type finite elements (Fig.2).

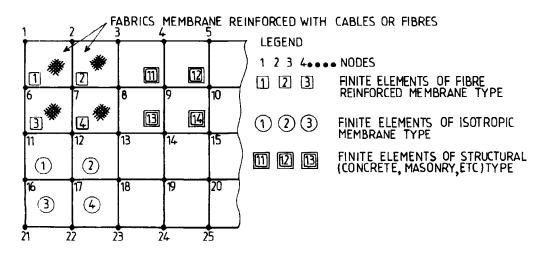


Fig.2. Wall modelling

Both the geometry and the displacement admissible fields are described by the same shape or interpolations functions. The structural geometry is referred to the global coordinates  $X^{N_i}$ .

### Finite element technique formulation via displacement method

The structure is considered in  ${}^2C$  configuration at the time  $t+\tau$ . The dynamic equilibrium condition at time  $t+\tau$  and in the  $j^{th}$  itteration in terms of nodal forces reads

$$f(\mathbf{u}^j, \dot{\mathbf{u}}^j, \ddot{\mathbf{u}}^j) = f(\hat{\mathbf{u}}^j) = 0$$
 (5)

where

$$\hat{\mathbf{u}}^j = \begin{bmatrix} \mathbf{u}^j & \dot{\mathbf{u}}^j & \ddot{\mathbf{u}}^j \end{bmatrix}^T$$

By expanding in Taylor's series and neglecting the second and higher order terms, the above relation yields into

$$\mathbf{M}.\Delta\ddot{\mathbf{u}}^{j} + {}^{t+\tau}\mathbf{C}.\Delta\dot{\mathbf{u}}^{j} + {}^{t+\tau}\mathbf{K}^{j-1}.\Delta\mathbf{u}^{j} =$$

$$= {}^{t+\tau}\mathbf{Q}^{ext} - {}^{t+\tau}\mathbf{Q}^{int}.\mathbf{u}^{j-1} - {}^{t+\tau}\mathbf{M}.\ddot{\mathbf{u}}^{j-1} - {}^{t+\tau}\mathbf{C}.\dot{\mathbf{u}}^{j-1}$$
(6)

where:

- $-^{t+\tau} \mathbf{K}^{j-1}$  is the tangent stiffness matrix,
- $-^{t+\tau}$  M is the mass matrix,
- $^{t+r}$ **C** is the damping matrix,
- $-^{t+\tau} \mathbf{Q}^{int}$  is the nodal stress resultant vector,
- $-^{t+r} \mathbf{O}^{ext}$  is the external forces vector.

The entries of the above matrices can be computed at the finite element level using the work principle.

#### Computer program SUM02

Based on the above presented theory, a computer program called SUM02 (Kopenetz, 1989) has been worked out. The equilibrium equations are solved via Newton-Raphson type itterations independent of the type of the used finite element. The integration of the dynamic equations follows the Newmark and Wilson procedures. Three general case studies, also presented elsewhere, have been worked out aiming at checking the SUM02 computer program. In what follows only the numerical results obtained in one case (Robinson, 1973) are presented. As the aim was the testing of the computer program, the same structural discretization in the linear domain as in (Robinson, 1973) has been employed (Fig.3).

In Fig. 3 Mansfield's results taken from (Robinson, 1973) are also shown. The numerical results have been computed in two cases: (1) for  $\Phi = 30^{\circ}$  and (2) for  $\Phi = 60^{\circ}$ .

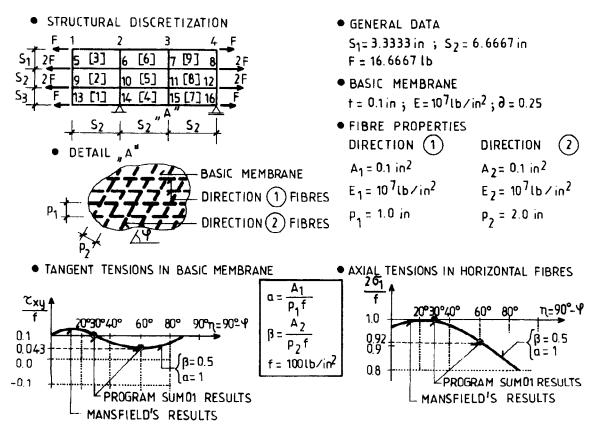


Fig.3. Structural discretization and results

In Fig.4 the  $P-\Delta$  effect on the first four natural frequencies of a massonry structure (Fig.5) placated with an izotropic (Fig.5,b) and an unizotropic (Fig.5,c) membrane are presented.

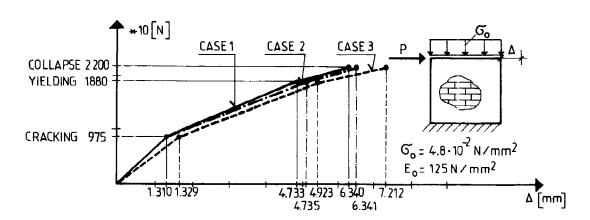


Fig. 4.  $P - \Delta$  effect on natural frequencies

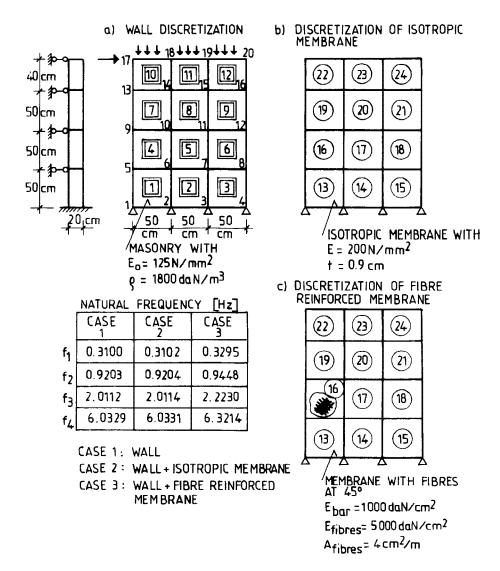


Fig. 5. Massonry structure placated with membranes

As it can be seen, the placating with membrane dramatically changes the natural frequencies which, in its turn, proves important changes in the structural ductility.

#### PRACTICAL METHODS

In the following several practical procedures to improve the ductility of structural elements by using membranes during the rehabilitation proces. In Fig.6 the practical technology of placating circular and rectangular columns with membrane-type solution is presented. The interior and exterior walls seismically affected may be rehabilitated and ensured with the required ductility by tensioned or non-tensioned membranes fixed to the walls (Fig.7).

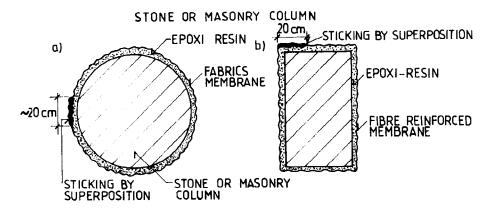


Fig. 6. Massonry column placated with membranes

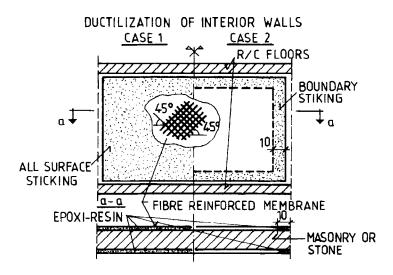


Fig. 7. Ductilization of walls

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