



A LOCALIZED IDENTIFICATION OF MDOF STRUCTURES BY EXTENDED KALMAN FILTER CONSIDERING NOISES IN INPUT MOTION RECORDS

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ABSTRACT

The objective of this study is to develop the method for identifying structural parameters of the local part of a structure (substructure), which can consider the noises included in the input motions at the boundaries of the substructure. The absolute velocities and displacements at the boundaries, that are the input motions to the substructure, are incorporated into the system and measurement equations of extended Kalman filter, in order to reduce the influence of noises in the input motions. Numerical simulation is carried out for identifying structural parameters in the local part of a shear type MDOF structural model and the effectiveness of the present method is investigated.

KEYWORDS

Parameter identification; localized identification; MDOF structure; extended Kalman filter; noises in input motions; system equation; measurement equation

INTRODUCTION

It is very important to estimate the structural parameters such as stiffness and damping coefficients of real structures by using earthquake observation records, for estimating the existing condition and the damage degree of structures. For this purpose, the recursive identification method for structural parameters in time domain has been developed in recent years. Hoshiya *et al.* (1984, 1987) presented an algorithm to identify structural parameters by extended Kalman filter and illustrated that their method was very useful for identification of various types of vibrational systems subjected to earthquake motions. However, when the structure has a large number of degrees of freedom (DOFs), it is not practical to identify the complete structural parameters by the method at a time, because the accuracy and convergency in identification are deteriorated and the computation time required for convergence increases. From such a point of view, the substructure approach or the localized identification method has been developed (Koh *et al.*, 1991, Oreta and Tanabe, 1993). These identification methods not only improved the accuracy and convergency of structural parameters to be identified but also reduced computation time considerably.

In the localized identification, the observation records at the boundaries of a substructure to be identified are generally treated as the input motions which are assumed to be noise free. However, it is practical that the input motions to the substructure are considered to be noise corrupted because those are obtained from observation and the noises included in the observation records deteriorate the accuracy and convergency of identified parameters.

Recently, Koh and See (1994) have developed the method which can consider not only the noise in input motion but also the modeling error by using the adaptive extended Kalman filter and applied their method to identification of complete structural parameters. The method is useful for identifying the uncertainties of parameters but is rather complicated in the process of computing system noise covariance by adaptive filter.

The objective of this study is to present the method to reduce the influence of noises included in the input motions in parameter identification of a substructure. The absolute velocities and displacements at the boundaries of the substructure, which are the input motions to the substructure and are calculated from numerical integrals of absolute acceleration records, are incorporated into the system and measurement equations of extended Kalman filter. In numerical examples, the effectiveness of this procedure is investigated by using a shear type 10-DOFs structural model.

LOCALIZED IDENTIFICATION OF STRUCTURAL PARAMETERS

A shear type MDOF structural model as shown in Fig.1 is considered as an example to illustrate the localized identification method. In this study, the responses of the structural model are assumed to be given in the form of absolute acceleration, velocity and displacement time histories since the observation records are actually obtained as absolute response forms.

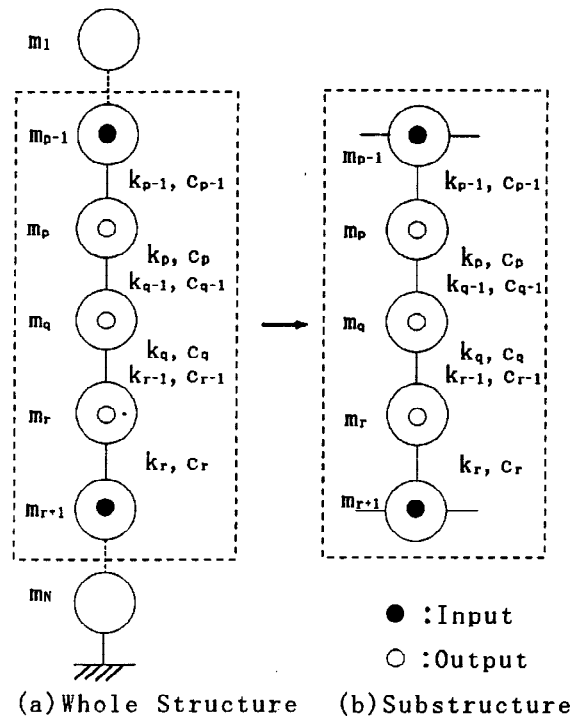


Fig.1 MDOF Structural Model

Equations of Motion of Substructure and Its Solution

The equations of motion for the substructure (local part of the structural model) in Fig.1 can be written as

$$M\ddot{z}(t) + C\dot{z}(t) + Kz(t) = f(t) \tag{1}$$

in which (·) denotes time derivative, M, C, K are the mass, damping and stiffness matrices, respectively, z(t) the absolute displacement vector and f(t) the excitation force vector which includes the input motion to the substructure.

$$M = \begin{bmatrix} m_p & & 0 \\ & \ddots & \\ 0 & & m_r \end{bmatrix}_{n \times n} \tag{2}$$

$$C = \begin{bmatrix} c_{p-1}+c_p & -c_p & 0 \\ & \ddots & \\ 0 & -c_{r-1} & -c_{r-1}+c_r \end{bmatrix}_{n \times n} \quad (3)$$

$$K = \begin{bmatrix} k_{p-1}+k_p & -k_p & 0 \\ & \ddots & \\ 0 & -k_{r-1} & -k_{r-1}+k_r \end{bmatrix}_{n \times n} \quad (4)$$

$$z(t) = \begin{Bmatrix} z_p(t) \\ \vdots \\ z_r(t) \end{Bmatrix}_{n \times 1} \quad (5)$$

$$f(t) = \begin{Bmatrix} c_{p-1}\dot{z}_{p-1}(t) + k_{p-1}z_{p-1}(t) \\ 0 \\ c_r\dot{z}_{r+1}(t) + k_r z_{r+1}(t) \end{Bmatrix}_{n \times 1} \quad (6)$$

where subscripts 'p-1' and 'r+1' denote the mass numbers at the boundaries of the substructure as shown in Fig.1, and $n=r-p+1$ is the number of masses included in the substructure. Eq.1 can be solved in the discrete form by using Newmark's β method as follows.

$$\ddot{z}(k+1) = A^{-1} \{ f(k+1) - C \cdot a(k) - K \cdot b(k) \} \quad (7)$$

$$\dot{z}(k+1) = a(k) + \Delta t \cdot A^{-1} \{ f(k+1) - C \cdot a(k) - K \cdot b(k) \} / 2 \quad (8)$$

$$z(k+1) = b(k) + \beta \Delta t^2 \cdot A^{-1} \{ f(k+1) - C \cdot a(k) - K \cdot b(k) \} \quad (9)$$

in which matrix A, Vector a(k), b(k) and f(k+1) are as follows.

$$A = M + \Delta t \cdot C / 2 + \beta \Delta t^2 \cdot K \quad (10)$$

$$a(k) = \dot{z}(k) + \Delta t \cdot \ddot{z}(k) / 2 \quad (11)$$

$$b(k) = z(k) + \Delta t \cdot \dot{z}(k) + (0.5 - \beta) \Delta t^2 \cdot \ddot{z}(k) \quad (12)$$

$$f(k+1) = \begin{Bmatrix} c_{p-1}\dot{z}_{p-1}(k+1) + k_{p-1}z_{p-1}(k+1) \\ 0 \\ c_r\dot{z}_{r+1}(k+1) + k_r z_{r+1}(k+1) \end{Bmatrix}_{n \times 1} \quad (13)$$

where k denotes $k\Delta t$ and Δt is the time increment in response time history.

Consideration of Input Noise in Localized Identification

Now consider the excitation force vector $f(k+1)$ in right hand side of Eqs.7-9, which includes the input motions $(\dot{z}_{p-1}, z_{p-1}, \dot{z}_{r+1}, z_{r+1})$ at the boundaries and the parameters $(c_{p-1}, k_{p-1}, c_r, k_r)$ in the substructure as shown in Eq.13. In previous studies (Koh et al., 1991, Oreta and Tanabe, 1993), the identification has been carried out assuming that the input motions at the boundaries of the substructure are to be known. However, it is practical that the input motions are regarded as being unknown since those are obtained from observation records and generally include some noises which deteriorate the identification accuracy.

In this study, the input motions are incorporated into the system and measurement equations by the following procedure. Assuming that the absolute acceleration responses at the boundary masses of p-1 and r+1 are observed, the velocities and displacements can be calculated by using Newmark's β method as follows.

$$\ddot{z}_{p-1}(k+1) = \ddot{z}_{p-1}(k) + w_1(k) \quad (14)$$

$$\dot{z}_{p-1}(k+1) = \dot{z}_{p-1}(k) + \Delta t \cdot \ddot{z}_{p-1}(k) + \Delta t \cdot w_1(k) / 2 \quad (15)$$

$$z_{p-1}(k+1) = z_{p-1}(k) + \Delta t \cdot \dot{z}_{p-1}(k) + \Delta t^2 \cdot \ddot{z}_{p-1}(k) / 2 + \beta \Delta t^2 \cdot w_1(k) \quad (16)$$

$$\ddot{z}_{r+1}(k+1) = \ddot{z}_{r+1}(k) + w_2(k) \quad (17)$$

$$\dot{z}_{r+1}(k+1) = \dot{z}_{r+1}(k) + \Delta t \cdot \ddot{z}_{r+1}(k) + \Delta t \cdot w_2(k)/2 \quad (18)$$

$$z_{r+1}(k+1) = z_{r+1}(k) + \Delta t \cdot \dot{z}_{r+1}(k) + \Delta t^2 \cdot \ddot{z}_{r+1}(k)/2 + \beta \Delta t^2 \cdot w_2(k) \quad (19)$$

where $w(k)$ is the difference of the acceleration time history as follows.

$$w_1(k) = \ddot{z}_{p-1}(k+1) - \ddot{z}_{p-1}(k) \quad (20)$$

$$w_2(k) = \ddot{z}_{r+1}(k+1) - \ddot{z}_{r+1}(k) \quad (21)$$

Representing Eqs.14-19 in matrix form

$$d(k+1) = B \cdot d(k) + D \cdot w(k) \quad (22)$$

in which d , w , B and D are represented by

$$d = \{ \ddot{z}_{p-1}, \dot{z}_{p-1}, z_{p-1}, \ddot{z}_{r+1}, \dot{z}_{r+1}, z_{r+1} \}^T \quad (23)$$

$$w = \{ w_1, w_2 \}^T \quad (24)$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \Delta t & 1 & 0 & 0 & 0 & 0 \\ \Delta t^2/2 & \Delta t & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \Delta t & 1 & 0 \\ 0 & 0 & 0 & \Delta t^2/2 & \Delta t & 1 \end{bmatrix}_{6 \times 6} \quad (25)$$

$$D = \begin{bmatrix} 1 & 0 \\ \Delta t/2 & 0 \\ \beta \Delta t^2 & 0 \\ 0 & 1 \\ 0 & \Delta t/2 \\ 0 & \beta \Delta t^2 \end{bmatrix}_{6 \times 2} \quad (26)$$

Substituting Eq.22 into Eq.13

$$f(k+1) = E \cdot d(k+1) = E \cdot B \cdot d(k) + E \cdot D \cdot w(k) \quad (27)$$

in which matrix E is given as

$$E = \begin{bmatrix} 0 & c_{p-1} & k_{p-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_r & k_r \end{bmatrix}_{n \times 6} \quad (28)$$

Next, substituting Eq.27 into Eqs.7-9, the following state equations for response of each mass are obtained.

$$\ddot{z}(k+1) = A^{-1} \{ E \cdot B \cdot d(k) - C \cdot a(k) - K \cdot b(k) \} + A^{-1} E \cdot D \cdot w(k) \quad (29)$$

$$\dot{z}(k+1) = a(k) + \Delta t \cdot A^{-1} \{ E \cdot B \cdot d(k) - C \cdot a(k) - K \cdot b(k) \} / 2 + \Delta t \cdot A^{-1} E \cdot D \cdot w(k) / 2 \quad (30)$$

$$z(k+1) = b(k) + \beta \Delta t^2 \cdot A^{-1} \{ E \cdot B \cdot d(k) - C \cdot a(k) - K \cdot b(k) \} + \beta \Delta t^2 \cdot A^{-1} E \cdot D \cdot w(k) \quad (31)$$

The system equation of extended Kalman filter is then composed of Eqs.22 and 29-31 together with the next equations representing the transition of identified parameters.

$$k(k+1) = k(k) \quad (32)$$

$$c(k+1) = c(k) \quad (33)$$

in which c and k are as follows.

$$k = \{ k_{p-1}, \dots, k_r \}^T \quad (34)$$

$$c = \{ c_{p-1}, \dots, c_r \}^T \quad (35)$$

In the Eqs. 22 and 29-31, the vector $w(k)$ is regarded as system noise with the 2×2 covariance matrix $Q(k)$ whose elements $Q_{ij}(k)$ are calculated by the next equation.

$$Q_{ij}(k) = \frac{1}{T_a} \int_{k\Delta t - T_a}^{k\Delta t} w_i(t) \cdot w_j(t) dt \quad (36)$$

where T_a is the averaging time in calculation of $Q(k)$. In this study, $T_a = 4\text{sec}$ is used in numerical analysis.

Application of extended Kalman Filter

System Equation In order to apply extended Kalman filter to parameter identification, the state variables and parameters are transformed as follows.

$$\begin{cases} X_1 = \{ X_1, \dots, X_n \}^T = \{ \ddot{z}_p, \dots, \ddot{z}_r \}^T = \ddot{z} & (37) \\ X_2 = \{ X_{n+1}, \dots, X_{2n} \}^T = \{ \dot{z}_p, \dots, \dot{z}_r \}^T = \dot{z} & (38) \\ X_3 = \{ X_{2n+1}, \dots, X_{3n} \}^T = \{ z_p, \dots, z_r \}^T = z & (39) \\ X_4 = \{ X_{3n+1}, \dots, X_{4n+1} \}^T = \{ k_{p-1}, \dots, k_r \}^T = k & (40) \\ X_5 = \{ X_{4n+2}, \dots, X_{5n+2} \}^T = \{ c_{p-1}, \dots, c_r \}^T = c & (41) \\ X_6 = \{ X_{5n+3}, \dots, X_{5n+8} \}^T = \{ \ddot{z}_{p-1}, \dot{z}_{p-1}, z_{p-1}, \ddot{z}_{r+1}, \dot{z}_{r+1}, z_{r+1} \}^T & (42) \end{cases}$$

Then, the system equation of extended Kalman filter, which is constructed by Eqs. 29-33 and 22, are transformed as follows.

$$\begin{cases} x_1(k+1) = g_1\{x(k)\} + A^{-1}E \cdot D \cdot w(k) & (43) \\ x_2(k+1) = g_2\{x(k)\} + \Delta t \cdot A^{-1}E \cdot D \cdot w(k)/2 & (44) \\ x_3(k+1) = g_3\{x(k)\} + \beta \Delta t^2 \cdot A^{-1}E \cdot D \cdot w(k) & (45) \\ x_4(k+1) = x_4(k) & (46) \\ x_5(k+1) = x_5(k) & (47) \\ x_6(k+1) = B \cdot x_6(k) + D \cdot w(k) & (48) \end{cases}$$

in which $w(k)$ is system noise and x, g_1, g_2, g_3, A, a and b are as follows.

$$x = \{ x_1^T, x_2^T, x_3^T, x_4^T, x_5^T, x_6^T \}^T \quad (49)$$

$$\begin{cases} g_1\{x(k)\} = A^{-1}\{ E \cdot B \cdot x_6(k) - C \cdot a(k) - K \cdot b(k) \} & (50) \\ g_2\{x(k)\} = a(k) + \Delta t \cdot A^{-1}\{ E \cdot B \cdot x_6(k) - C \cdot a(k) - K \cdot b(k) \} / 2 & (51) \\ g_3\{x(k)\} = b(k) + \beta \Delta t^2 \cdot A^{-1}\{ E \cdot B \cdot x_6(k) - C \cdot a(k) - K \cdot b(k) \} & (52) \end{cases}$$

$$\begin{cases} A = M + \Delta t \cdot C / 2 + \beta \Delta t^2 \cdot K & (53) \\ a(k) = x_2(k) + \Delta t \cdot x_1(k) / 2 & (54) \\ b(k) = x_3(k) + \Delta t \cdot x_2(k) + (0.5 - \beta) \Delta t^2 \cdot x_1(k) & (55) \end{cases}$$

Consequently, Eqs. 43-48 are represented in the following.

$$x(k+1) = g\{x(k)\} + \Gamma(k) \cdot w(k) \quad (56)$$

in which vector g and matrix Γ are given as follows.

$$g = \{ g_1^T, g_2^T, g_3^T, x_4^T, x_5^T, (B \cdot x_6)^T \}^T \quad (57)$$

$$\Gamma(k) = \begin{bmatrix} A^{-1}E \cdot D \\ \Delta t \cdot A^{-1}E \cdot D / 2 \\ \beta \Delta t^2 \cdot A^{-1}E \cdot D \\ 0 \\ 0 \\ D \end{bmatrix}_{(5n+8) \times 2} \quad (58)$$

The response of the complete structure is analyzed by using Newmark's β method, in which the 1940 El Centro acceleration record (Fig.3) with a time interval of 0.02 sec and duration of 20.48 sec is used as input ground motion at the base. The observation records are then generated by adding band limited white noise with a frequency band width of 20 Hz to the acceleration responses of the masses 9 and 10 as well as the input ground motion, in which the intensity of noise is set by the rate (per cent) to rms intensity of each response. The identified substructure includes mass 10 as shown in Fig.2, while the identified parameters consist of the stiffness and damping coefficients of masses 9 and 10, i.e., k_9 , k_{10} , c_9 , and c_{10} . In the identification, time dependent state variables are initially set at 0. The initial values of the error covariance matrix are set at 1 for all state variables. The noise variance for the observation response records is given as 10^{-5} . Assuming the initial values of 2 times of true values for all unknown parameters, the localized identification is carried out by the method in this study as well as in previous study (Koh et al.,1991).

Table 1 and 2 show the errors of the identified values of parameters in various noise level, by previous study and this study. In these tables, errors ϵ (%) are distinguished by the following symbols.

- \odot : $\epsilon \leq 5\%$, \circ : $5\% \leq \epsilon \leq 10\%$, \square : $10\% \leq \epsilon \leq 20\%$
 \triangle : $20\% \leq \epsilon \leq 50\%$ \times : $50\% \leq \epsilon$

It is found from the tables that the stiffness and damping parameters of the masses 9 and 10 in the substructure can be reasonably estimated by the method in this study (see Table 3), while those parameters are not estimated accurately by previous study in the case of noise level larger than 0.5% (see Table 2).

Table 2 Errors of Identified Parameters by Previous Study

Noise Level (%)	Stiffness		Damping	
	k_9	k_{10}	c_9	c_{10}
0.1	\odot	\odot	\odot	\odot
0.2	\odot	\odot	\circ	\odot
0.3	\square	\square	\triangle	\square
0.4	\triangle	\triangle	\odot	\triangle
0.5	\times	\times	\times	\times
0.6	\times	\times	\times	\times
0.8	\times	\times	\times	\times
1.0	\times	\times	\times	\times

- \odot : $\epsilon \leq 5\%$, \circ : $5\% \leq \epsilon \leq 10\%$, \square : $10\% \leq \epsilon \leq 20\%$
 \triangle : $20\% \leq \epsilon \leq 50\%$ \times : $50\% \leq \epsilon$

Table 3 Errors of Identified Parameters by This Study

Noise Level (%)	Stiffness		Damping	
	k_9	k_{10}	c_9	c_{10}
1.0	\odot	\odot	\odot	\odot
2.0	\odot	\odot	\odot	\odot
3.0	\odot	\odot	\odot	\odot
4.0	\odot	\odot	\odot	\odot
5.0	\odot	\odot	\odot	\odot
6.0	\odot	\odot	\odot	\odot
8.0	\odot	\odot	\odot	\odot
10.0	\odot	\odot	\circ	\odot

- \odot : $\epsilon \leq 5\%$, \circ : $5\% \leq \epsilon \leq 10\%$, \square : $10\% \leq \epsilon \leq 20\%$
 \triangle : $20\% \leq \epsilon \leq 50\%$ \times : $50\% \leq \epsilon$

Fig.4 shows the convergence process of the identified parameters k_9 , k_{10} , c_9 and c_{10} by present method in the case of noise level of 5%. In the figure, horizontal axis is time in sec and vertical one the estimated/true value ratio of parameters. It is found from the figure that all parameters converge to true values about 4~5 sec (200~250 cycles), although the accuracy of the damping coefficient is not so good (2~10%).

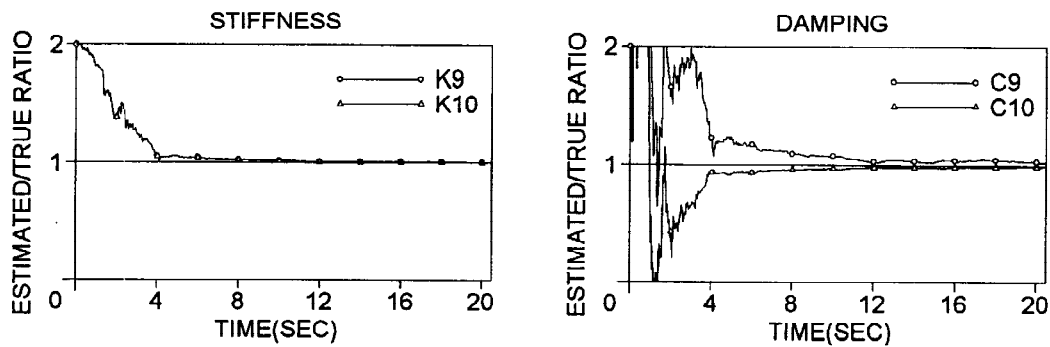


Fig. 4 Convergence Process of Stiffness and Damping Coefficients by Present Study
(Noise Level 5%)

CONCLUSION

The localized identification method for structural parameters using extended Kalman filter has been presented, in which the input motions at the boundaries of a substructure are treated as being noise corrupted. In the present procedure, the input motions at the boundaries of the substructure are incorporated into system equation and measurement equation of extended Kalman filter, in order to reduce the noise influence on identification of parameters. It has been shown from numerical examples that the present method works well even if the noise level of input motions is about 10%.

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