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LOAD DEFLECTION CHARACTERISTICS OF PLASTIC HINGES IN DUCTILE CONCRETE BEAMS

Richard C. FENWICK, Leslie M MEGGET and Peter WUU

Department of Civil and Resource Engineering, The University of Auckland
Private Bag 92019, Auckland, New Zealand

ABSTRACT

Tests on reinforced concrete beams, which have been made to assess the seismic performance of ductile frame structures, show that plastic hinges when subjected to inelastic cyclic loading sustain both significant shear deformation and elongation (dilation). Typically at displacement ductility levels of six 40% of the beam deflection is due to shear while the elongation is of the order of 3% of the member depth. The mechanisms associated with both elongation and shear deformation are described and a method of predicting the shear deflection from the beam's loading history is outlined. Predictions obtained from this method are found to be in good agreement with the shear deflections measured on twelve beams.

KEYWORDS

Reinforced concrete; plastic hinges; shear deformation; elongation; dilation.

INTRODUCTION

With current practice multi-storey frame buildings are designed to form plastic hinges in a major earthquake in such a way that the structure behaves in a ductile manner. This is achieved by proportioning the beam and column elements so that a beam sway mode develops in preference to a column sway mode. With this arrangement the majority of the plastic hinges form in the beams, and it is the behaviour of these which largely determines the dynamic characteristics of the building once the structure has been deflected into the inelastic range. For this reason it is important that the behaviour of plastic hinges is understood so that their load deformation characteristics can be predicted. To achieve this a method of determining the shear deformation in plastic hinges is required.

It has been shown that two different forms of plastic hinge can occur in concrete structures; namely the reversing plastic hinges, which sustain both positive and negative inelastic rotations in the same region, and the uni-directional plastic hinges, where the negative and positive inelastic rotations are sustained in different regions of the beam (Megget and Fenwick (1989)). Reversing plastic hinges form in beams which carry only light gravity loading during an earthquake. In this case the critical positive and negative bending moments arise in the beam at the column faces. Where the member carries a higher level of gravity loading, however, the critical negative moments occur at the column faces but the critical positive moments can form in the span of the beam. In this situation as cyclic inelastic deformations are applied negative and positive inelastic rotations are sustained in different regions of the beam. These accumulate as the earthquake progresses with the vertical deflection of the beam increasing as each inelastic displacement is applied (Megget and Fenwick (1989), and Douglas *et al* (1996)).

DEFORMATION IN PLASTIC HINGE ZONES

The deformation in a plastic hinge in a beam can be described in terms of three components; namely, flexural rotation, shear deformation and elongation. The way in which the flexural rotation and elongation arises in the two forms of plastic hinge is illustrated in Fig.1. In this figure the results of displacement measurements, which have been made on the longitudinal reinforcement in the plastic hinges of two beams, are reproduced. In one of these a uni-directional plastic hinge was formed, while in the second a reversing plastic hinge developed. In the beam with the reversing plastic hinge two complete cycles to plus and minus 3/4, 2, 4, 6 and 8 displacement ductilities were applied to the beam. With the uni-directional plastic hinge a similar series of load cycles were applied, but in this case the upward load was limited so that the maximum bending moment did not exceed 3/4 of the theoretical ultimate flexural strength. Consequently the bottom reinforcement did not yield.

With the uni-directional plastic hinge it can be seen that the average strain in the reinforcement on the bottom side of the beam was small, and for practical purposes it may be assumed to be zero when this side of the beam is in compression. In this case the rotation arises essentially from the extension of the top reinforcement. The resultant elongation of the mid-depth of the beam is given by the expression

$$\text{elongation} = e + \theta\,(d-d')/2 \qquad \text{(Eq.1)}$$

where θ is the rotation sustained by the plastic hinge, $d-d'$ is the distance between the top and bottom reinforcement and e is the extension of the compression reinforcement in the beam. As noted above with the uni-directional plastic hinge the value of e is negligible when the reinforcement in the compression zone has not been yielded.

With the reversing plastic hinge the situation is different. In one half load cycle reinforcement on one side of the beam is yielded in tension. In the following half cycle, when it is subjected to compression, it does not fully yield back to close the cracks. This occurs for two reasons. Firstly, when the longitudinal steel yields in tension many cracks form in the concrete surrounding the reinforcement and these, together with

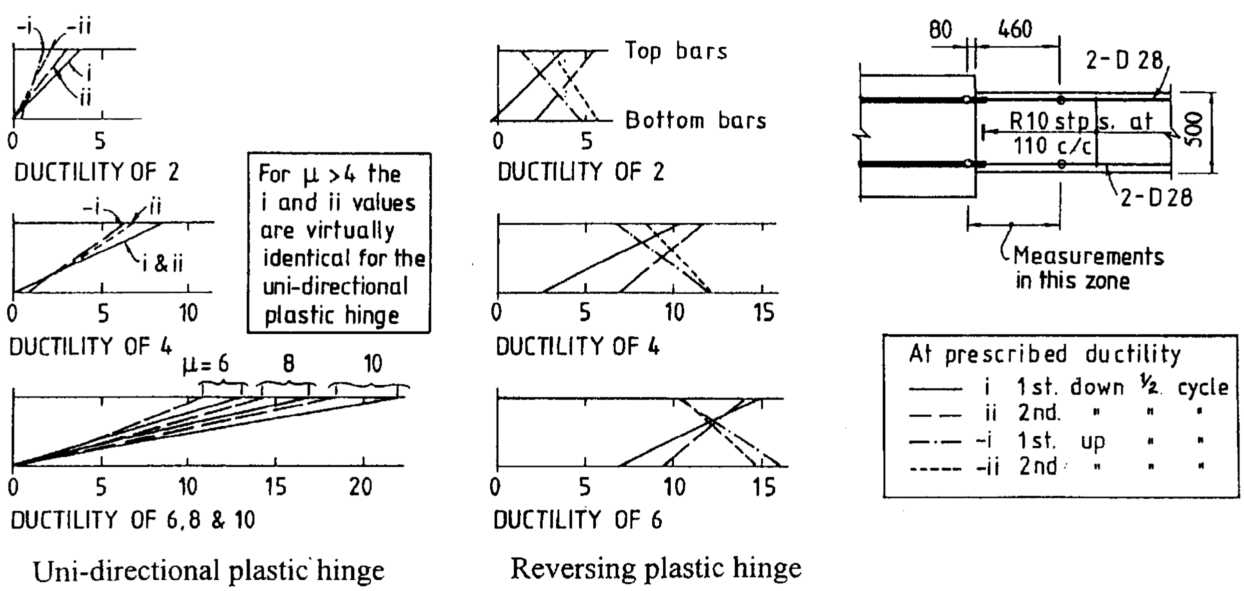


Fig. 1 Elongation in uni-directional and reversing plastic hinges

the dislocation of the aggregate particles at the crack interfaces tend to wedge the cracks open. Secondly, the shear in the beam, as illustrated in Fig.2(d), is resisted by a truss like action. The longitudinal component of the diagonal compression forces in the web results in the flexural compression force always being smaller than the flexural tension force. The difference in these two values means that the rotation tends to develop by yielding of the tension steel in preference to yielding of the compression reinforcement. With inelastic cyclic loading the member continues to elongate until the reinforcement buckles. Displacement measurements made on beams which sustain reversing plastic hinges show that typically two thirds of the elongation arises from the extension of the compression reinforcement, e, with the remaining one third arising from the rotation sustained by the plastic hinge.

MECHANISM OF SHEAR RESISTANCE IN REVERSING PLASTIC HINGES

As outlined in the following paragraphs a number of key observations, which have been made on reversing plastic hinges in beams, enables their mechanism of shear resistance to be defined.

(1) In reversing plastic hinges once the longitudinal reinforcement on one side of the beam has yielded in tension it does not fully yield back in compression (see Fig.1), leaving open cracks in the compression zone. This ensures that the flexural compression force is resisted principally by the compression reinforcement and hence the location of this force is determined. Consequently for a reversing plastic hinge in a uniform beam no shear can be resisted by the inclination of the flexural compression force. This condition does not hold if a compressive axial force acts, enabling the cracks in the compression zone to close.

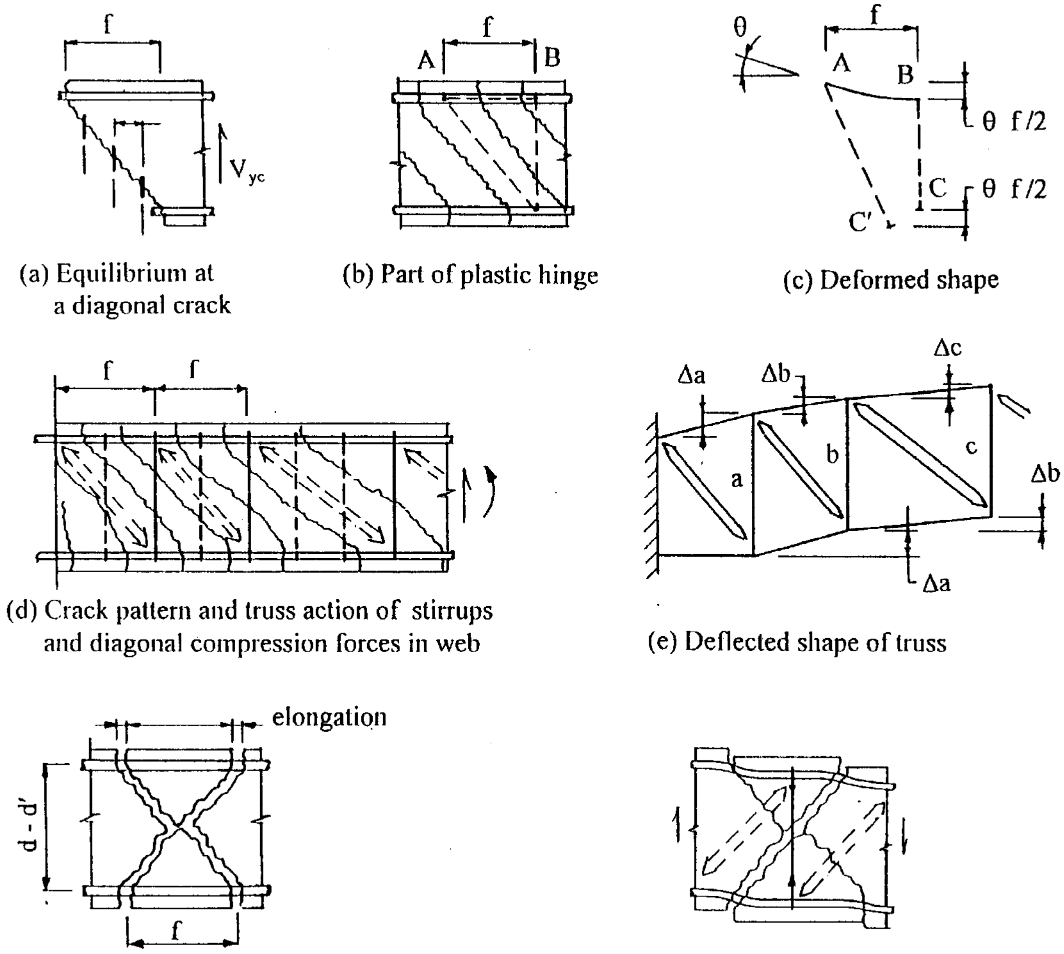
(2) Due to the reversal in direction of the shear force with cyclic loading two sets of diagonal cracks form. These divide the web of the beam up into diamond shaped blocks, see Fig.2(f). With the yielding of the longitudinal reinforcement wide cracks form in the web in the web, and these destroy the shear resistance provided by aggregate interlock and dowel actions and the flexural resistance provided by the concrete between the cracks (Park and Paulay (1975). Thus the shear resisted by the concrete becomes negligible and the only viable shear resisting mechanism is a truss like action where the concrete in the web sustains diagonal compression forces and the stirrups resist the tension forces.

(3) Photographs and measurements made on test beams show that plastic hinge zones increase in depth with cyclic loading. This indicates that the stirrups yield in tension even when these stirrups have been proportioned by conventional theory to carry all the shear. Furthermore the photographs indicate that the inclination of the cracks can be more steeply inclined at the high moment end of the plastic hinge than the traditional assumption of 45^0.

From these observations it is concluded that at peak load levels all the shear at the high moment end of a plastic hinge is resisted by stirrups acting at yield. To maintain equilibrium, as illustrated in Fig.2(a), the longitudinal projection of the diagonal crack, f, must be such that sufficient stirrups are intersected to resist the total shear. This gives the expression

$$V_{yc} = A_v f_y f / s \qquad \text{(Eq.2)}$$

where V_{yc} is the shear force which corresponds to the theoretical flexural strength of the beam, $A_v f_y$ is the stirrup force at yield and s is the spacing of the stirrups. The longitudinal reinforcement carries both the flexural compression and tension forces and consequently the theoretical flexural strength is given by the expression $A_s f_y$ (d - d'), where d-d' is the distance between the top and bottom reinforcement and $A_s f_y$ is the tension force in the flexural tension reinforcement when the steel first yields. With cyclic loading high strains are imposed on both the longitudinal and shear reinforcements. This leads to strain hardening with a corresponding increase in the load carried by the beam. It is assumed that the stirrups and longitudinal



(a) Equilibrium at a diagonal crack

(b) Part of plastic hinge

(c) Deformed shape

(d) Crack pattern and truss action of stirrups and diagonal compression forces in web

(e) Deflected shape of truss

(f) Elongation in plastic hinge

(g) Shear deflection due to elongation

Fig. 2 Shear deformations in a plastic hinge

reinforcement strain harden at the same rate so that the value of "f" does not change. At the low moment end of the plastic hinge the stirrups are not at yield and the inclination of the cracks approaches 45°.

COMPATIBILITY IN PLASTIC HINGE

Fig. 2(b) shows a portion of a plastic hinge in which the diagonal cracks have a longitudinal projection of f, while part (c) shows the deformed shape of this zone. In the compression zone a rotation of θ occurs between the points A and B, which are located a distance of "f" apart. This causes the point A to rise a distance of "θ f/2" above the tangent drawn from point B, and it increases the depth of the beam at B by "θ f/2". If this distance exceeds the elastic extension of the stirrup a permanent yield extension is induced. From this computability requirement the curvature of the plastic hinge which induces yielding of the stirrups and ensures that the longitudinal projection of the crack reaches its critical value of "f" can be established.

Shear deformation in plastic hinge zones arises from two distinct actions; firstly the extension of the stirrups and secondly the elongation in the plastic hinge zone. To find the shear deformation associated with the first of these the deflection of one of the trusses spanning this zone, as illustrated in Fig.2(d), is found. The strains associated with the diagonal compression forces in the concrete are small and for practical purposes can be neglected. On this basis, as illustrated in Fig. 2(e), the shear deflection equals the sum of the stirrup extensions making up the truss. In determining a stirrup extension it should be noted that at any particular load stage it is equal to the sum of all the inelastic extensions sustained in previous load cycles plus the additional extension (elastic plus inelastic) arising since the load last reversal. At each inelastic load stage the additional strain imposed on an individual stirrup can be found from the longitudinal projection on the diagonal crack and the change in curvature sustained in the compression zone since the last shear reversal.

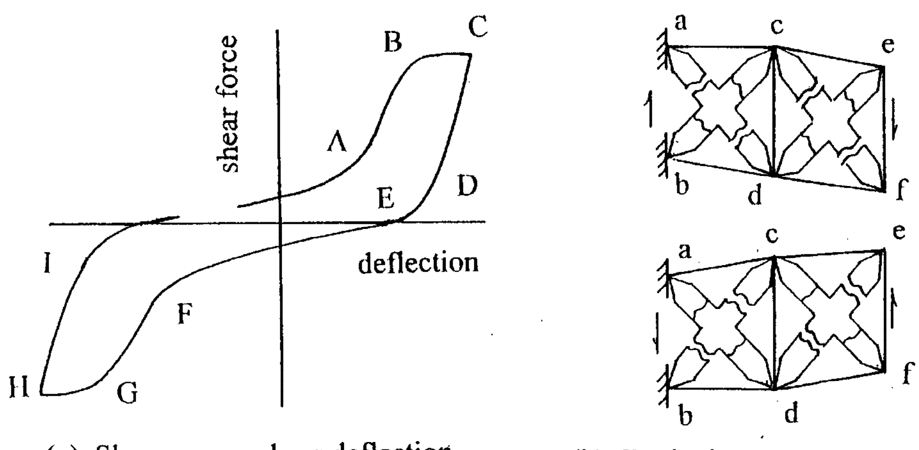
The shear deformation associated with elongation is illustrated in Fig.2(f) and (g), where an elongated section of a plastic hinge is shown. When shear is applied one set of the diagonal cracks must close to allow the diagonal compression forces to be sustained. Assuming that the major diagonal cracks are located in the region where the longitudinal projection is f, this component of shear displacement, s_e, is given by

$$s_e = \Delta l \ (d - d')/ f \qquad \text{(Eq.3)}$$

where Δl is the elongation. In practice this varies over the depth of the beam. However, analyses of a series of beams indicates that the Δl value should be taken as the smaller of the values measured at the mid-depth of the beam and the level of the compression reinforcement.

LOAD DEFLECTION CHARACTERISTICS

Fig.3(a) shows a typical shear versus shear deflection relationship for an inelastic cycle of loading in a reversing plastic hinge. In part (b) of this figure the effective trusses which act with each shear direction are illustrated. Between points A and B on the shear versus shear deflection curve the stirrups are stressed in the elastic range, while between B and C they yield in tension. This allows the shear deflection to increase with only a small increase in shear force. The near linear response which occurs on unloading between C to D is associated with the elastic recovery of the stirrups. Between D and the zero shear position at E, the recovery is non-linear. In this region the crack surfaces slide back past each other and this motion reduces the wedging action with the dislocated aggregate particles at the cracks allowing some closure to occur. With reference to Fig.3(b), with the reversal of the shear force the wide diagonal cracks in the members a - d and c - f, which opened in the previous half cycle of loading, have to close to enable the diagonal compression forces to be sustained in the opposite direction. The increase in shear between E and F is necessary to overcome the contact stress effects in these members and close these cracks. As described above it is the opening and closing of the diagonal cracks in the plastic hinge which gives the characteristic pinched shape of the shear deflection versus shear force relationship.



(a) Shear versus shear deflection

(b) Equivalent trusses associated with each direction of loading

Fig.3 Shear versus shear deflection

PRACTICAL APPLICATION OF THEORY

To apply the theory described in the previous sections both the length of the plastic hinge and the distribution of curvature over this length have to be defined. Measurements obtained on a series of beam tests (Fenwick and Thom(1982)), in which detailed strain measurements were made on the reinforcement, indicates that the length, l_p, over which the flexural tension reinforcement yields, can be predicted by the expression-

$$l_p = (M_{max} - M_{yc})\, M/V + f \qquad \text{(Eq.4)}$$

where M_{max} is the maximum bending moment which has acted in the loading direction being considered and M/V is associated moment to shear ratio. The tests also indicate that the strain distribution varies linearly over the length of the plastic hinge, and consequently the distribution of curvature along this length can be defined from the rotation sustained by the plastic hinge.

In calculating the increase in the extension of a stirrup for a particular load application it should be noted that the rotation θ , see Fig.2(c), is the value which has been sustained since the last shear reversal. Fig.2(c) does not show any existing shear deformations which may be present at the start of the half cycle of loading due to prior yielding of the stirrups. Including this deformation does not change the mechanics or the equations.

In practice the spacing of the stirrups in the beams does not coincide with the theoretical longitudinal projection length of the diagonal cracks and some averaging occurs. To simplify the calculations equivalent trusses are envisaged for the purposes of calculating the deflection. In these equivalent stirrups are located at bay lengths defined by the "f" values. In general the flexural strengths and moment to shear ratios differ for the two loading directions and this gives different f values; namely f_d and f_u for the downward and upward directions respectively. Consequently, as illustrated in Fig.4, it is necessary to have an equivalent truss for each shear direction.

In any inelastic loading excursion the additional extension of the equivalent stirrups can be found from the equivalent truss for the appropriate shear direction. However, it is necessary to determine the corresponding values for the equivalent stirrups in the truss for the opposite direction of loading. This second set of values may be found by interpolating in terms of distance between the additional extensions determined for the current loading direction. For example, with reference to Fig.4, for shear in the downward direction the additional extensions of the equivalent stirrups d_1 , d_2, and d_3 may be found by the theory as outlined. The corresponding additional extensions for the equivalent stirrups in the upward loading truss, u_1, u_2 and u_3 are found as follows. The value u_1 is found by interpolating between zero at the column face and the additional extension of the stirrup d_1 a distance of f_d out, while the u_2 and u_3 values are found by direct interpolation between the corresponding values for d_1, d_2 and d_3.

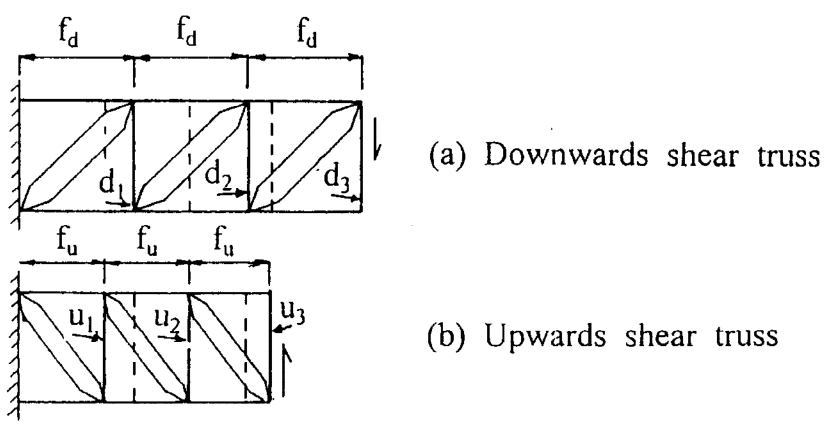
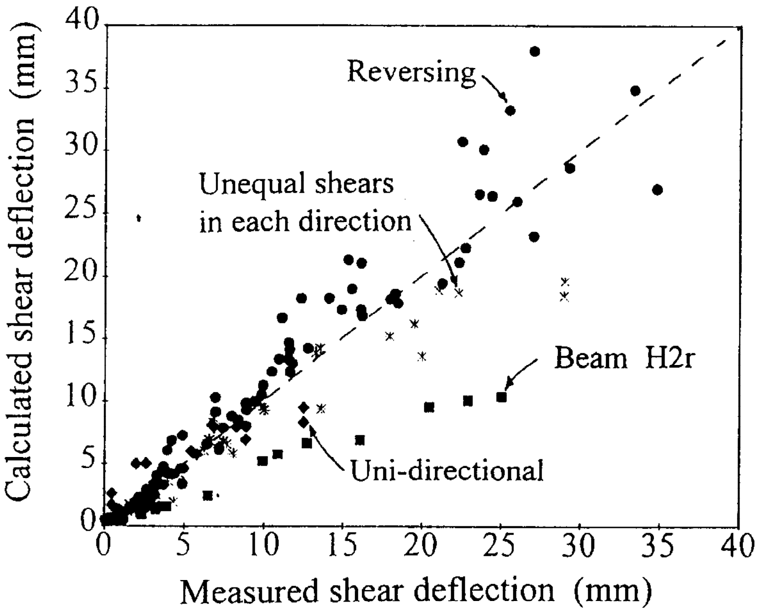


Fig. 4 Equivalent trusses for upward and downward shears.

COMPARISON THEORETICAL AND EXPERIMENTAL SHEAR DEFLECTIONS

In beam tests where the rotation and the elongation have been measured in the plastic hinges the shear deflection can be calculated by the theory outlined in the previous sections. Unfortunately there are few tests where all the necessary measurements have been made. However, over the last 15 years twelve beams have been tested at Auckland with the required instrumentation. The principal details of these are given in Table 1 and the typical instrumentation is illustrated in Fig.5. For these beams the shear deflection has been calculated at all the deflection peaks applied during the tests. These theoretical values are compared in Fig.5 with the shear deflection which was calculated from the array of diagonal gauges on the test beams. There is some experimental error in the tests, though generally the beam deflection calculated from the displacement transducer measurements was within 10% of the deflection measured directly at the loading point. Given the level of experimental error the agreement between the theoretical and experimental values is satisfactory in all cases but for beam H2r. This member was unusual in that it contained a large surplus of shear reinforcement giving a theoretical inclination of the diagonal compression forces in the plastic hinge of 68°. This implies that these forces on average crossed the diagonal tension cracks associated with the



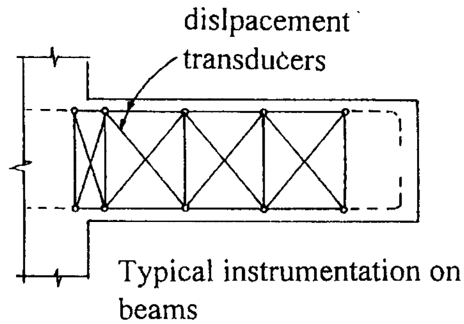


Fig. 5 Theoretical versus experimental shear deflections

previous direction of loading at an angle of close to 45°. The high shear to normal force ratio at these cracks may have resulted in additional shear deflections occurring here, with some rolling of the dislocated aggregate particles. Such an action would tend to increase the elongation which developed. There is some support for this possible mode of deformation as the measured elongation in beam H2r was 50% or more in excess of the values measured in the companion beam, H1r. In this beam (H1r) there was considerably less stirrup reinforcement and the theoretical angle between the diagonal compression force and the diagonal crack interface is 83°.

The theory described in this paper has been developed and incorporated into a model of a plastic hinge, which has been embedded in a structural dynamic analysis program. With this model the complete shear versus shear deflection and elongation behaviour of plastic hinges in beams can be predicted provided no significant axial loads act. This development is outlined in a companion paper (Douglas *et al* (1996)).

DISCUSSION AND CONCLUSIONS

(1) Two different forms of plastic hinge may develop in the beams of seismic resistant frames; namely uni-directional and reversing plastic hinges. Elongation develops in both of these though the mechanism is different in each case.

(2) Significant shear deformation develops in plastic hinges. This arises due to the yielding of the stirrups, which can be related to the curvature of the compression zone and to the elongation of the plastic hinge.

(3) The shear versus shear deflection relationship for a plastic hinge develops a characteristic pinched shape. The low stiffness at low load levels is associated with the closure of the diagonal cracks which opened up when the shear was acting in the opposite direction.

(4) Based on a mechanism for shear resistance a theoretical method of predicting the shear deflection from the previous loading history has been developed. The predictions from this theory are in good agreement with measured shear deflections in the plastic hinges of 12 beams.

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Table 1. Details test beams

Beam	Size DxB mm^2	Longitudinal Reinforcement	f_y MPa	Stirrup	f_{vy} MPa	f_c MPa	M/V ratio	d mm	d mm
T1Ar[1]	500 x 200	5 - 20, (t&b)[2]	311	2-10+1-6 @100 c/c[3]	307	33.2	1.50	444	56
T1Br	500 x 200	5 - 20, (t&b)	311	2-10+1-6 @100 c/c	307	42.1	1.50	455	56
T2Ar	500 x 200	5 - 20, (t&b)	306	2-10+1-6 @100 c/c	307	37.6	1.50	455	56
T2Br	500 x 200	3 - 20(t), & 5 - 20(b)	306	2-10+1-6 @100 c/c	307	37.6	1.50	455	40
T3Ar	500 x 200	5 - 20, (t&b)	306	2-10+1-6 @100 c/c	307	35.6	1.50 u 2.18 d	455	56
T3Br	500 x 200	5 - 20, (t&b)	306	2-10+1-6 @100 c/c	307	35.6	1.50 u 2.30 d	455	56
T4Ar	500 x 200	5 - 20, (t&b)	312	2-10+1-6 @100 c/c	307	34.3	1.25	455	56
M1Ar	500 x 200	2 - 28, (t&b)	317	2-10 @110	307	43.0	1.24	455	45
M1Bu	500 x 200	2 - 28, (t&b)	317	2-10 @110	307	43.0	1.24	455	45
N1r	500 x 250	2 - 28 + 1 - 24 (t&b)	278	2-10+2-6 @100 c/c	275	36.2	1.86	455	45
H1r	500 x 200	2 - 28, (t&b)	325	2-8 @ 110	380	36.5	1.30	455	45
H2r	500 x 200	2 - 28, (t&b)	325	2 - 12 @ 110	365	36.5	1.30	455	45

(1) r = reversing; u = uni-directional plastic hinge
(2) t = top; b = bottom
(3) 2-10 m legs + 1 - 6 mm leg @ 110 c/c
(4) Tee beam slab reinforced with 10 longitudinal 10 mm bars.