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MODELLING OF REINFORCED CONCRETE PLASTIC HINGES

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ABSTRACT

A finite element sub-structure model, which represents uni-directional and reversing plastic hinges in reinforced concrete beams, has been developed and implemented in a dynamic structural analysis program. The model allows for the increase in plastic hinge length as strain hardening occurs, and it is capable of predicting flexural, shear and elongation deformations. Results from simulations of separate experiments are presented and found to be in good agreement with experimentally measured values.

KEYWORDS

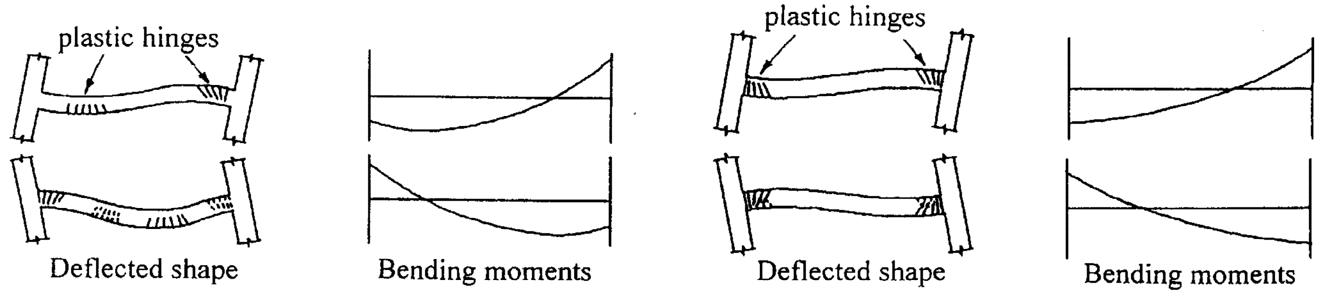
Reinforced concrete; plastic hinges; shear deformation; elongation; analytical simulation;

INTRODUCTION

To avoid structural collapse during major seismic events most multi-storey structures are designed using a combination of strength and ductility. To achieve the required ductility they are proportioned to ensure that in the event of a major earthquake a beam sway failure mode develops in preference to a column sway mode. Specified areas within the structure, which are referred to as potential plastic hinges, are designed to withstand inelastic deformations, while the remainder of the structure deforms elastically. Most of these potential plastic hinges are located in the beams. Once yielding has occurred it is these zones which predominantly determine the dynamic performance of the structure. Consequently it is important that they are modelled realistically.

REINFORCED CONCRETE BEAM PLASTIC HINGES

Two forms of plastic hinge may develop in beams subjected to seismic actions, with the type of plastic hinge depending upon the relative magnitudes of the seismic and gravity loads which act. Where the gravity loading dominates, as illustrated in Fig.1a, as the structure sways backwards and forwards negative moment plastic hinges develop in the beam at the column faces and positive moment plastic hinges in the span of the beam. With each inelastic displacement the vertical deflection of the beam increases and the inelastic



(a) Uni-directional plastic hinge (b) Reversing plastic hinge

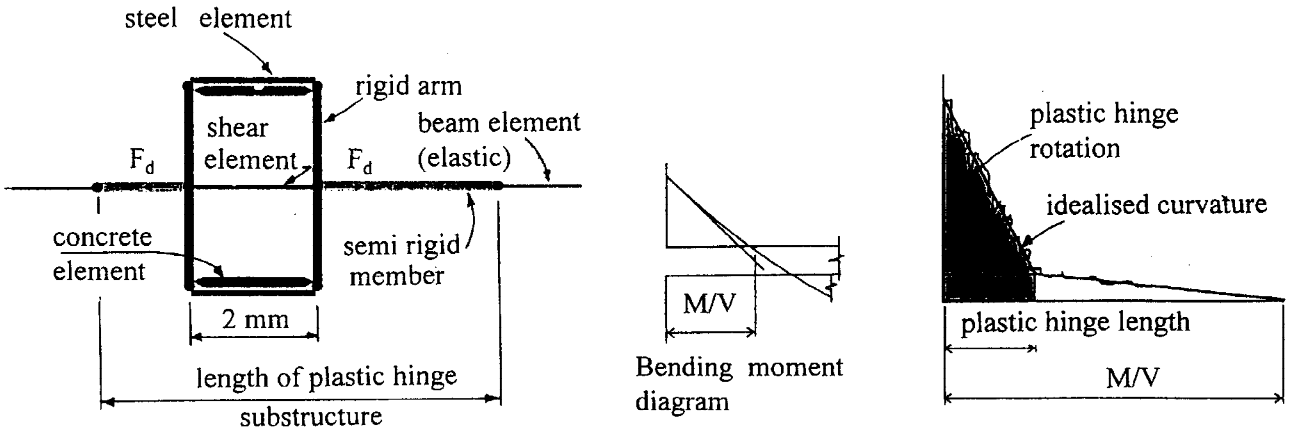
Fig. 1 Formation of uni-directional and reversing plastic hinges.

rotations sustained by the plastic hinges increase in magnitude. As each of these zones sustains inelastic rotations in one direction only, they are referred to as uni-directional plastic hinges. The load deflection characteristics of structures which form uni-directional plastic hinges show little stiffness degradation when subjected to inelastic cyclic deflections. The exception to this occurs just prior to failure when the compression reinforcement buckles (see Fig.7 in the last two load cycles). The situation where the seismic actions dominate is illustrated in Fig.1b. In this case, as the structure sways backwards and forwards, negative and positive moment plastic hinges form in the beam at the column faces, with the direction of the rotation in each of these reversing with the direction of motion. These are referred to as reversing plastic hinges.

The behaviour of the reversing plastic hinges is in sharp contrast to that of uni-directional plastic hinges. With reversed inelastic cyclic loading either the top or the bottom reinforcement yields in tension in one half cycle and it does not completely yield back in compression in the next half cycle. The flexural cracks remain open adding significantly to the elongation of the plastic hinge. The failure of the cracks to close in the compression zone, arises from the dislocation of the aggregate particles at the crack faces and from the truss like action associated with the shear resisting mechanism. This is described in greater detail in a companion paper (Fenwick et al 1996). With the reversal of loading two sets of diagonal cracks develop. These destroy the shear resistance provided by the so called "concrete ", that is the shear resisted by dowel and aggregate interlock actions and the flexural resistance of the concrete between the cracks. This results in all the shear being resisted by a truss like action with the stirrups resisting tensile forces and the concrete the diagonal compression forces. The elongation that develops in reversing plastic hinges before strength degradation is typically of the order of 2 to 4 per cent of the member depth. In addition significant shear deformation also occurs. This reduces the stiffness at low load levels, causing the characteristic pinched load deflection relationship, which can be seen in Fig.5a.

REINFORCED CONCRETE BEAM PLASTIC HINGE MODEL

The complete hinge sub-structure is shown in Fig. 2a. Two rigid arms, which are separated by 2 mm, are attached to the ends of two semi-rigid members. Four truss elements and a shear link couple the two rigid arms. All the inelastic deformation is confined to these five linking elements. Consequently to generate the correct deformations it is important that the lengths of the semi-rigid beams are arranged so that these five elements are positioned close to the centroid of the plastic hinge. The total length of the hinge zone is taken as a member depth for negative and reversing plastic hinges and twice this value for positive moment plastic hinges which form in the span. The properties of the semi-rigid beams are derived to give the potential plastic hinge its correct initial elastic stiffness.



(a) Plastic hinge model. (b) Bending moments (c) Curvatures in plastic hinge.

Fig. 2 Plastic hinge model.

The steel and concrete truss elements, which are pinned at each end to the rigid arms, are placed at the centroids of the top and bottom beam reinforcement. They represent the corresponding material within each half of the beam section and their combined response simulates the flexural and axial characteristics of the plastic hinge. Because of the linear nature of the solution scheme employed by the structural analysis package used for this research, the stress-strain relationships of these elements are described as a series of straight lines joining defined, but continuously changing reference points. The properties of the steel element are based on a model proposed by Menegotto and Pinto (1973) and later updated by Trokjodimuljo (1985). Both strain hardening and the Bauschinger effect are incorporated. To make some allowance for longitudinal reinforcing bar kinking, which occurs in plastic hinges, the user can define a magnitude corresponding to the percentage of softening occurring in the inelastic strain hardening region of the elements stress-strain response. The concrete element has a relatively simple stress-strain relationship which incorporates stiffness degradation at high compressive loads, tensile cracking and the wedging type action of the dislocated aggregate particles in the cracks, which is often referred to as the contact stress effect. The properties of both these elements are user defined and based on the properties of the region they are required to simulate.

The strains and associated stresses sustained by the hinge sub-structure elements are set to correspond to those occurring at the maximum moment end of the plastic hinge region and an idealised linear strain distribution is assumed to occur along its length. Consequently, once the length of the plastic hinge has been established, the curvatures occurring along this length for any given rotation can be determined, see Fig. 2c. The shear resistance of the hinge is provided by the shear link. It can only resist shear forces as it has no axial or flexural stiffness. Its hysteretic response is based on a theory developed by Fenwick and Thom (1982), which is explained in depth in Douglas (1996) and outlined in a companion paper by Fenwick *et al* (1996).

To avoid the possibility of unrealistic stiffnesses caused by the 2mm fixed length of the sub-structure, the truss elements are assigned a theoretical length, which is used to determine their load deflection characteristics. In the initial computations, for reversing and negative moment uni-directional plastic hinges this length is assumed to equal the longitudinal projection of the diagonal cracks that develop in the section of the associated plastic hinge in which the stirrups yield. This is defined as the distance "f" in a companion paper (Fenwick *et al* (1996). While for positive moment uni-directional plastic hinges it is set to twice this value.

Once the reinforcement in the plastic hinge region has begun yielding in tension the hinge length increases and the length of the corresponding steel element is updated at every subsequent change in stiffness along the tensile yielding curve. When subjected to tensile loads the concrete element cracks and is assumed to have a negligible stiffness. The changing plastic hinge length therefore has very little impact on the cracked concrete element response during the tensile loading stage. Consequently its length is only updated if the loading direction reverses.

The length of the plastic hinge region, *lp*, is defined as the length over which the longitudinal reinforcement yields [Fenwick and Thom (1982)]. It is derived by assuming that the moments sustained by a member vary linearly along its length and that the concrete sustains negligible tensile forces. It is given by:

$$lp = \chi\left(\left(T - T_y\right)\frac{(d-d')}{|V|} + f + \lambda\right) \qquad \text{(Eq.1)}$$

where f, the longitudinal projection of the diagonal cracks, is included to make allowances for the shear lag or tension shift resulting from diagonal cracking. In addition the factors χ and λ are included to give greater flexibility to the equation. For reversing and negative moment uni-directional plastic hinges, λ and or χ can be used to represent the amount of yield penetration into the beam-column joint zone adjacent to the plastic hinge. The uni-directional positive moment plastic hinge in the span of a beam extends in two directions from the point of maximum bending moment, thus effectively doubling its length. To allow for this the χ factor is set to 2. The shear at these locations is low and consequently there is no diagonal cracking or shear lag. Hence in this case the value of λ is set to -f.

To represent the action of the diagonal compression forces sustained by the concrete in the web of the beam, which arise from the truss action associated with shear, two self equilibrating horizontal forces F_d, are applied at the end nodes of the shear link, as illustrated in Fig. 2a. The lateral component of the sum of the diagonal compression forces at a section is equal to the shear force resisted by the action of the web reinforcement at the high moment end of the plastic hinge. Assuming the average angle at which these forces act in a reversing plastic hinge, α, is given by

$$\alpha = \frac{\left(\dfrac{f_p}{(d-d')} + \dfrac{f_n}{(d-d')}\right)}{2} \qquad \text{(Eq.2)}$$

where f_p and f_n are the longitudinal projections of the diagonal cracks in the plastic hinge corresponding to the positive and negative moment actions, and *d-d'* is the distance between the centroids of the top and bottom reinforcement, the longitudinal component of the diagonal forces, F_d, is given by

$$F_d = |V|\frac{(f_p + f_n)}{2(d-d')} fp_{const} \qquad \text{(Eq.3)}$$

where fp_{const}, is a factor which is defined by the user. For reversing plastic hinges this value is taken as unity. However, for uni-directional plastic hinges, where the concrete can sustain an appreciable amount of shear by a combination of shear transfer in the compression zone, aggregate interlock and dowel action, the truss like shear resistance mechanism does not develop to the same extent. Consequently, the factor fp_{const} can be adjusted to reflect the reduction in the diagonal compressive forces.

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

The analytical model was used to predict the response of a number of test units which had been subjected to inelastic cyclic loading. In this section the results of two such analyses are outlined. Further details on these and additional analyses may be found in Douglas (1995).

A series of cantilever beams were tested by Fenwick, Tankat and Thom (1981). Each test unit consisted of two cantilever beams springing from a central block, which was stressed down to the strong floor. The two beams were tested independently. To limit the yield penetration of the flexural reinforcement into the springing, additional 10mm bars were welded to each longitudinal reinforcing bar as shown in Fig.3. The reinforcement details complied with the current NZ code of practice for concrete (NZS3101-1995). However, in this beam the stirrups provided a surplus shear resistance. Using the code equation ($V_s = A_v f_{vy} d / s$) the shear resistance provided by the stirrups was in excess of 1.75 times the measured maximum shear that acted in the plastic hinge zone during the test.

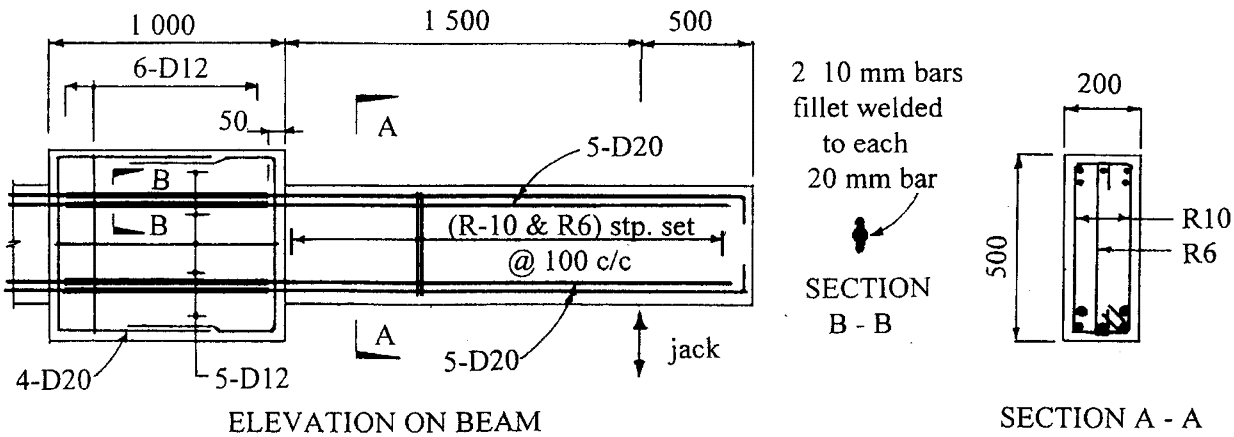


Fig.3 Details of beam which formed a reversing plastic hinge.

The beam was instrumented with 24 displacement transducers which enabled the rotation, shear deformation and elongation to be measured through out the test. The measurements made from these gauges were also used to predict the deflection of the jack. These values can then be compared with the deflections measured directly at this position. The two sets of values were generally found to agree to within a few percent.

The analytical model is shown in Fig.4. A yield penetration length of 100mm was assumed with the χ factor taken as unity. The dead load of the beam was applied and the force representing the jack was incremented in small steps to generate the deflection history of the jack. The analytical and experimental results are shown in Fig.5 for the load versus deflection, moment rotation, shear versus shear displacement and load versus elongation relationships. It can be seen from these that the curves are generally in close agreement. There is some discrepancy in the shear versus shear deflection, particularly in the shape of the unloading and loading curves at the low shear levels. The structural response in this range depends principally on the contact stress effects, that is the wedging action of the aggregate particles in the cracks during closing (see the companion paper Fenwick *et al* (1996)). The discrepancy in the results reflects the difficulty of modelling this action. The elongation which occurs is the most difficult factor to predict. It is believed that this is due to the problems involved in allowing for the kinking of the longitudinal reinforcement and the buckling of the bars that generally occurs in the last few load cycles of the test.

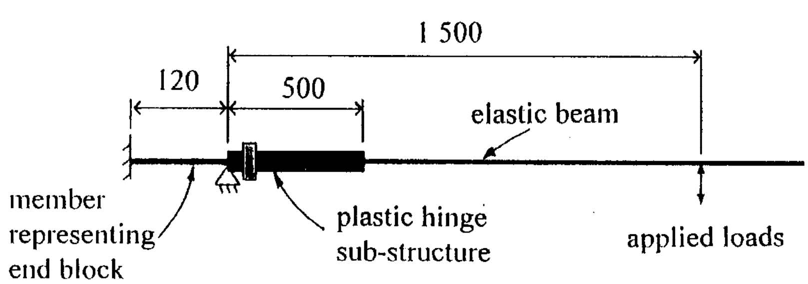
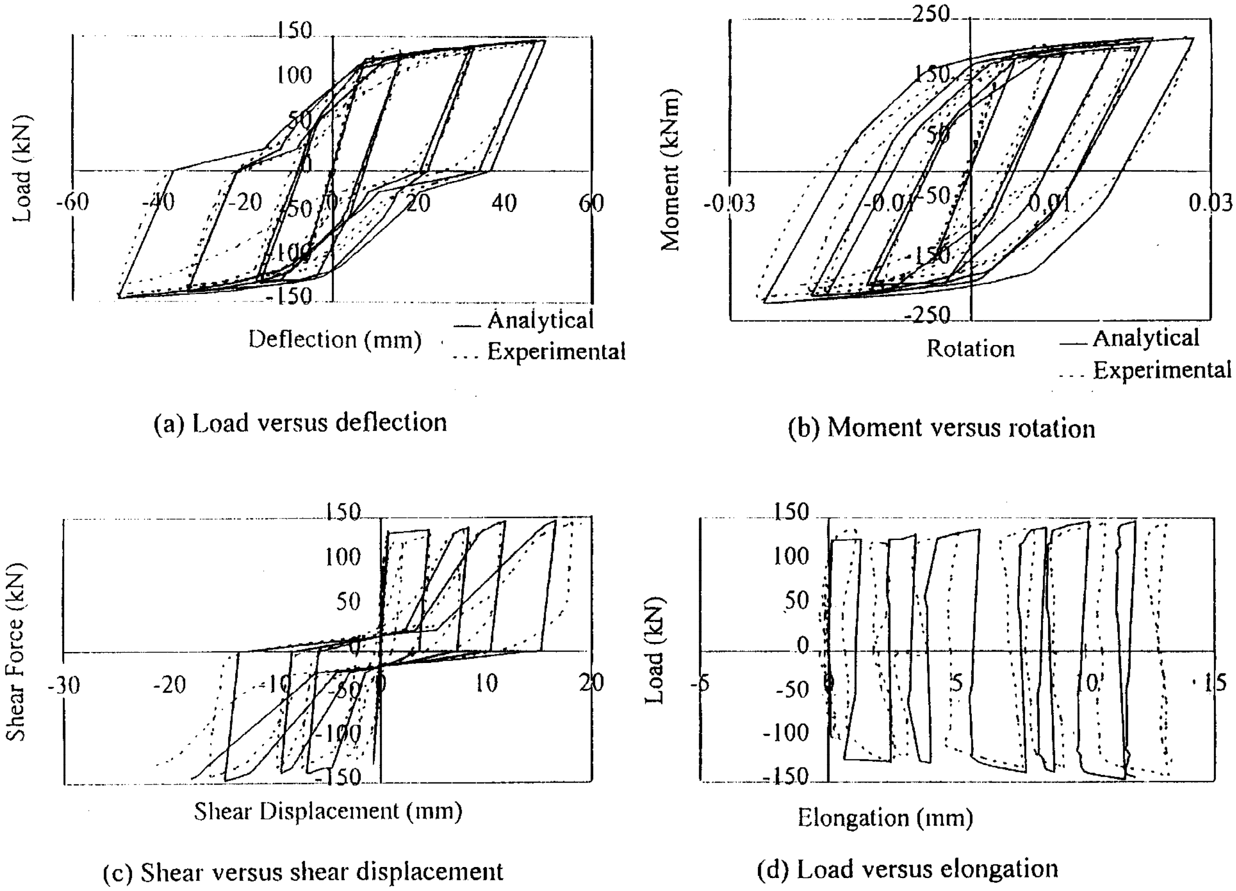


Fig. 4 Analytical model of beam.



(a) Load versus deflection

(b) Moment versus rotation

(c) Shear versus shear displacement

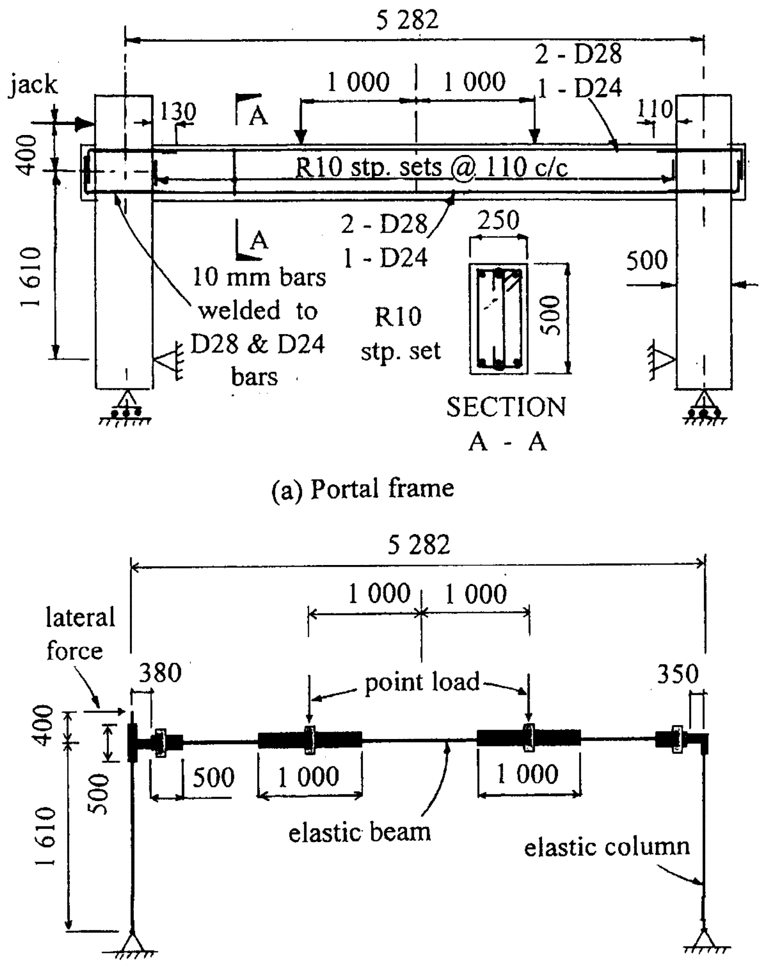
(d) Load versus elongation

Fig. 5 Experimental and analytical results for test beam

The second analysis to be illustrated was of a two pinned reinforced concrete portal frame, which was tested by Megget and Fenwick (1989). It was proportioned and loaded so that under lateral cyclic loading it would form a beam sway mechanism with uni-directional plastic hinges forming in the beam. Roller bearings were used to resist the vertical reactions on the columns while the horizontal reactions were taken out separately through pin ended members which incorporated load cells. Some of the details of the frame are shown in Fig.6a. The reinforcement details complied with the current NZ code of practice (NZS3101-1995). Under the combined action of two constant vertical loads and a cyclic lateral force, negative moment predominantly uni-directional plastic hinges formed in the beam close to the column faces, while positive moment uni-directional hinges developed near the vertical load points in the span. The columns and beam column joint zones were proportioned so that they would remain elastic through out the test. To prevent yield penetration into beam column joint zone additional 10mm bars were welded to the longitudinal reinforcement. This steel extended 130 and 110 mm into the beam on the left and right hand sides respectively.

The analytical model is shown in Fig.6b. In the analysis, as in the test, the vertical loads were applied to the beam and then the lateral force was applied to the portal to reproduce the same lateral deflection history as occurred in the test.

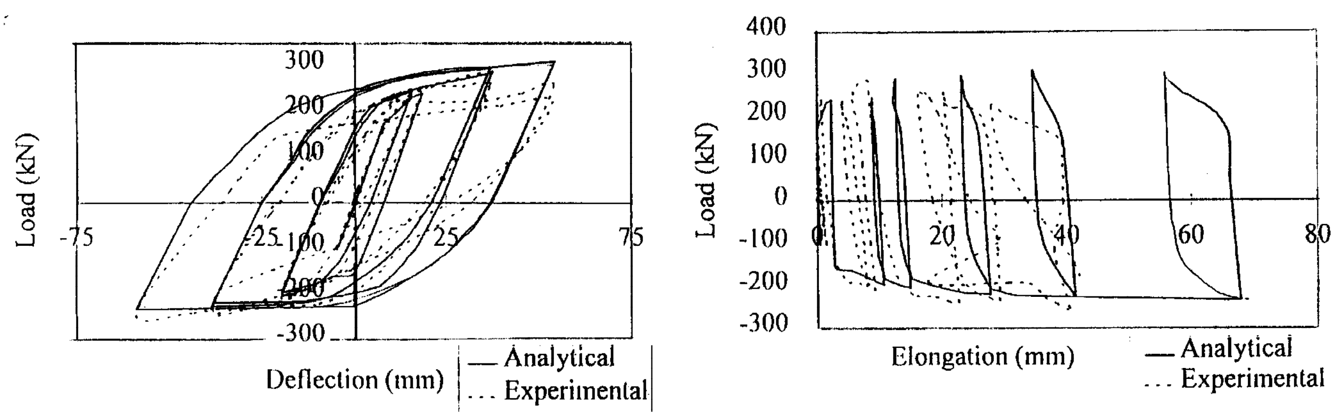
The analytical and experimental results for the portal frame are shown in Fig. 7. The lateral force versus deflection curves are in reasonable agreement up to the end of the displacement ductility 4 load cycles. For the ductility 6 cycles the lateral force is over predicted by the analysis and the amount of stiffness degradation is underestimated. It is believed that this discrepancy was due to the failure of the analytical model to allow for buckling of the reinforcement. It is believed that this also accounts for the discrepancy in the elongation versus lateral force curves shown in Fig. 7b.



(a) Portal frame

(b) Analytical model

Fig. 6 Portal frame and analytical model



(a) Lateral load versus deflection

(b) Lateral load versus elongation

Fig. 7 Experimental and analytical results for the portal frame.

DISCUSSION AND CONCLUSIONS

(1) The experimental results reproduced in this paper are typical of many others. They show that very significant shear deformation develops in reversing plastic hinges even when the stirrups have been proportioned to resist a considerably greater shear than the maximum value which acts at any stage of

the test.. In addition it can be seen that significant elongation occurs both in the uni-directional and the reversing plastic hinges; an effect which has important structural implications (Megget and Fenwick (1993)).

(2) A plastic hinge model has been developed which reproduces the main modes of deformation that occur in uni-directional and reversing plastic hinges in terms of the moment rotation, shear deflection and elongation characteristics. The agreement between experimental and analytical results is reasonable. To improve the accuracy further work is required on modelling the contact stress effects and bar buckling.

ACKNOWLEDGMENTS

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