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PRACTICAL DIGITAL-OPTIMIZING CONTROL ALGORITHM OF MULTI-LOCATED ACTIVE CONTROL SYSTEM FOR EARTHQUAKE EXCITATIONS

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ABSTRACT

A 'digital-optimizing control method' is proposed as a new practical algorithm for active response control systems of building structures. By introducing this new control method, a 'multi-located active control system' using active braces is investigated. In this paper, an emphasis is put on the structural response control for earthquake excitations. On the point that the aseismic active control algorithm should be simple and effective, the digital-optimizing control method is fairly advantageous and applicable for practical use of active control system. As the first step to introduce this control method, several trial control forces represented as a step function are assumed to be generated by each control device. For those set of control forces, the capacity of each control device is considered. A 'digital-index function' is installed for the optimization of control performances, among of all combinations of control force by each device, the most adequate set which makes the digital-index function minimum at each control time step is selected. A three-degrees-of-freedom system arranged three control devices is adopted for investigating this control algorithm from numerical simulations. As a result, it appeared that remarkable control effects were obtained by introducing the digital-optimizing control method onto the multi-located active control system.

KEYWORDS

Active control; structural response control system; control algorithm; optimization; instantaneous optimal control; digital-optimizing control method; control performance; active brace system.

I. INTRODUCTION

On the architectural engineering fields in Japan, remarkable developments of structural response control technology have been achieved. About ninety base-isolated buildings and about fifty vibration-controlled buildings had been constructed in the last one decade [Inoue et al., 1993]. In general, for relatively rigid buildings, base-isolating systems are adopted to prevent resonance of structural vibrations with earthquake excitations. While, for high-rise building structures having large aspect ratio, active or passive response control systems are used for reducing the wind-induced structural vibrations. As for the reduction of structural vibrations caused by earthquakes, vibration control systems have not been so popularized as much as the wind-resistant response control systems. Although there are a few practical buildings introduced active vibration control systems for earthquakes, those control devices are almost designed as to stop their operation under strong motions. In order to establish the aseismic control system against strong ground motions, several practical problems, such as capacities of control devices or optimality of control performances and so on, which should be solved have been still remained. In this paper, from the point of view that the aseismic active control algorithm should be simple and easy to apply for practical use, a new active control algorithm named as a digital-optimizing control method is proposed. Through some case studies with numerical simulations, the effectiveness of this active control method is investigated.

II. CONSIDERATIONS FOR THE INSTANTANEOUS OPTIMAL CONTROL

The instantaneous optimal control method [Yang et al., 1987] is very attractive as a control algorithm on the cases that earthquake motions are supposed as external excitations. According to the traditional optimal control theory [Friedland, 1986], it is required to assume that the external excitation is a white noise, or at least, it has simple statistical characteristics. While, in the instantaneous optimal control algorithm, such conditions are not required by introducing a time-dependant index function. In spite of such actual merits, the other problems for practical applications have been remained in this algorithm. In order to point out those clearly, the outline of the instantaneous optimal control will be shown again in this chapter.

When considering an n degrees-of-freedom structural system equipped with r control devices, let $\{X_s\}$ denote a state vector having $2\times n$ components at the time of $t_s = \Delta t \times s$ (where Δt means a discrete control time interval and s is an integer number), and let $\{u_s\}$ denote a control force vector having r components. The equation of motion of this system can be described as a following expression by introducing the state vector.

$$\{\dot{X}_s\} = [A]\{X_s\} + [B]\{u_s\} + [D]\{w_s\}, \tag{1}$$

in which $[A]$, $[B]$ and $[D]$ are matrices defined from mass, damping and stiffness of the system, and arrangements of control devices. When supposing the disturbance vector $\{w_s\}$ as earthquake motions, it can be expressed by using the ground motion $\ddot{x}_{0,\,s}$. Let the control force vector $\{u_s\}$ define as a linear feed-back system,

$$\{u_s\} = [G_s]\{X_s\}, \tag{2}$$

in which $[G_s]$ means the time-dependant feed-back gain matrix. Then, the instantaneous optimal control algorithm is simply reduced to the unconstrained minimization problem [Luenberger, 1973]. To find out the feed-back gain matrix $[G_s]$ and to decide the control force vector $\{u_s\}$, an index function,

$$J_s = \{X_s\}^{\mathrm{T}}\,[Q]\,\{X_s\} + \{u_s\}^{\mathrm{T}}\,[R]\,\{u_s\}, \tag{3}$$

should be minimize, where $[Q]$ and $[R]$ mean weight matrices. Since $\{u_s\}$ can be expressed by $\{X_s\}$ via the Exp. (2), J_s can be expressed as the function of the state vector $\{X_s\}$ (In general, the equation of motion with the Lagrangean multiplier is added to right hand side of the Exp. (3)). If the finite difference method is introduced, $\{X_s\}$ is determined from $\{X_{s-1}\}$ and $\ddot{x}_{0,\,s}$ without using the Duhamel integration [Hatada et al., 1989]. When using the instantaneous optimal control algorithm for the multi-located active control system, the cases that the required control forces (each control force is derived from the Exp. (2)) will exceed the capacities of control devices may occur. At such cases, overflowed values may be cut off and the control forces become no more optimal solutions in the sense of the mathematical meaning. So, it will be reasonable to change the Exp. (3) into the 'constrained minimum problems' as follows. Find out the set of control forces $\{u_s\}^*$ that makes the index function,

$$J_s = \{X_s\}^{\mathrm{T}} [Q] \{X_s\} + \sum_{j=1}^{r} R_j \cdot u_{s,j}{}^2 \qquad (u_{min,j} < u_{s,j} < u_{max,j}, \quad j = 1, 2, \cdots, r), \tag{4}$$

minimize, where $u_{max,j}$ and $u_{min,j}$ are the upper and lower bounds of control forces of the j-th device, respectively, $u_{s,j}$ is the control force of the j-th device and R_j is its weight coefficient. However, those type of calculations require much CPU time, so, the 'time delay' will be coming up as definitely significant problems in the active control system.

III. DIGITAL-OPTIMIZING CONTROL METHOD

In order to overcome those practical problems, a new control algorithm which may be named the digital-optimizing control method is proposed [Tachibana et al., 1994]. Let control force of each control device assume to limit into a few kinds of trial control forces $< \bar{u}_j >$ for the j-th device, where the notation $<\cdot>$ represents a 'set'. For instance, in the case that those control forces are limited into N kinds,

$$< \bar{u}_j > \; = \; < \bar{u}_{j,1}, \bar{u}_{j,2}, \cdots, \bar{u}_{j,N} > \qquad (u_{min,j} < \bar{u}_{j,a} < u_{max,j}, \quad a = 1, 2, \cdots, N). \tag{5}$$

The 'digital-index function' $\bar{J}_s$ is defined as,

$$\bar{J}_s = \{\bar{x}_{s+1}\}^{\mathrm{T}} [Q_d] \{\bar{x}_{s+1}\} + \{\dot{\bar{x}}_{s+1}\}^{\mathrm{T}} [Q_v] \{\dot{\bar{x}}_{s+1}\}, \tag{6}$$

where $\{\bar{x}_s\}$ and $\{\dot{\bar{x}}_s\}$ represent the inter-story displacement and velocity vector at the time instant t_s, respectively, and $[Q_d]$ and $[Q_v]$ mean those weight matrices. Since no spillover of control forces is in the digital-optimizing control method, the item of the control forces in the digital-index function is neglected in Exp. (6). When $\{\bar{x}_s\}$ and $\{\dot{\bar{x}}_s\}$ will be calculated from the following transition equation by selecting a set of $\{\bar{u}_s\}$,

$$\begin{Bmatrix} \dot{\bar{x}}_{s+1} \\ \bar{x}_{s+1} \end{Bmatrix} = [\bar{A}] \begin{Bmatrix} \dot{\bar{x}}_s \\ \bar{x}_s \end{Bmatrix} + [\bar{B}] \begin{Bmatrix} w_{s+1} \\ w_s \end{Bmatrix} + [\bar{D}] \begin{Bmatrix} \bar{u}_{s+1} \\ \bar{u}_s \end{Bmatrix}, \tag{7}$$

the values of $\bar{J}_s$ for all combinations of $< \bar{u}_j >$ can be computed. In which $[\bar{A}]$, $[\bar{B}]$ and $[\bar{D}]$ are matrices defined from mass, damping and stiffness of the system, and arrangements of control devices. The most adequate control forces are determined as real control forces by finding out the set that makes the digital-index function $\bar{J}_s$ minimum. Those digital-optimizing procedures can be written as following flows.

Digital-optimizing control algorithm-I

1) Set the basic coefficients of structure (Mass matrix [M], Damping matrix [C] and Stiffness matrix [K]).
2) Set the trial control forces $< \bar{u}_j >$, provided that the all component of $< \bar{u}_j >$ should be selected as to be no lager than $u_{max,\ j}$ and no less than $u_{min,\ j}$ in Exp. (5), and weight matrices [Q_d] and [Q_v].
3) Get $\{ \bar{x}_s \}$ and $\{ \dot{\bar{x}}_s \}$, and ground acceleration $\ddot{x}_{0,\ s}$ from sensers at the time $t_s = \Delta t \times s$.
4) Predict $\{ \bar{x}_{s+1} \}$ and $\{ \dot{\bar{x}}_{s+1} \}$ for all combination of $< \bar{u}_j >$ from Exp. (7).
5) Calculate the all $\bar{J}_s$ for all $\{ \bar{x}_{s+1} \}$ and $\{ \dot{\bar{x}}_{s+1} \}$.
6) Determine the set of control forces $\{ \bar{u}^*_s \}$ as the quasi-optimal control force vector which makes $\bar{J}_s$ minimum.

When using the digital-optimizing control algorithm, no complicated mathematical techniques are needed, and only knowledge about numerical response analysis is required. However, by increasing the number of control devices r or the number of trial control forces N, the total number of combinations of control forces will be growing. For instance, if each control device can generate N kinds of discrete forces, the total combinations of control forces are N^r (Universal searching method). In order to reduce the number of those combinations and to get quasi-optimal control forces quickly, following approximate searching method can be considered.

Let each control device make the order of each control device corresponding to its priority for searching sequence. The control device numbers ($j = 1, 2, \cdots, r$) according to its order of the searching sequence are denoted as a series of integer $\{gp \mid p = 1, 2, \cdots, r\}$, where $1 \le gp \le r$ and $gp \ne gi$ (gp means the device number which is examine on the p-th searching layer). This searching method can be described as following flows (by replacing the step 4), 5) and 6) in the *algorithm-I* mentioned above).

Digital-optimizing control algorithm-II

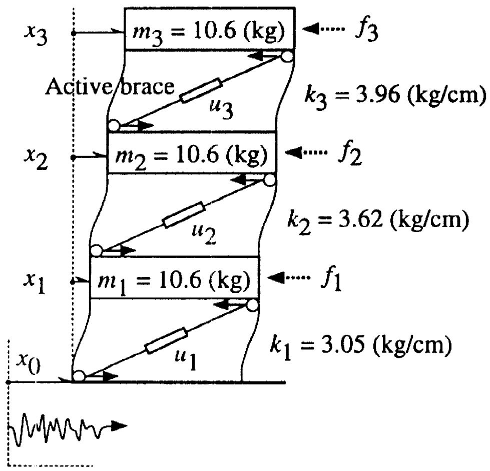
1) Same as the step 1), 2) and 3) for the *algorithm-I.*
2) Set $p = 1$.
3) Assume N kinds of trial control forces of gp-th device $< \bar{u}_{gp} >$ and $\{ < \bar{u}_{gi} > = < 0 >, i > p \}$.
4) Predict increments of $\{ \bar{x}_{s+1} \}$ and $\{ \dot{\bar{x}}_{s+1} \}$ for all combination of $< \bar{u}_{gp} >$ and $\{ \bar{u}_{gi}\ , i \ne p \}$ from Exp. (7).
5) Calculate the all $\bar{J}_{s,\ gp}$ for all $\{ \bar{x}_{s+1} \}$ and $\{ \dot{\bar{x}}_{s+1} \}$.
6) Determine gp-th control force $\bar{u}^*_{gp}$ which makes $\bar{J}_{s,\ gp}$ minimum.
7) Increment p, and if $p \le r$, then go to step 3). After exiting from this searching layer loop, one set of control forces $\{ \bar{u}^*_s \}$ as the quasi-optimal control force vector can be decided.

By using the *algorithm-II*, the total combinations of control forces can be reduced into $r \times N$ (Multi-layer searching method). For the case that $< \bar{u}_j >$ are considered as inter-story forces which can be generated via active braces (as shown in Fig.1), a couple of control forces $\bar{u}_j$ and $-\bar{u}_j$ acts on upper and lower floors of the story equipped the j-th control device. It can be empirically considered that the control force of the j-th device may be transmitted and affected to the lower stories (the top edge of the structure is free). So, it is reasonable to denote the ordered series for searching sequence as the downward sequence (Step-down search),

$$gp = r - p + 1, \quad p = 1, 2, \cdots, r, \tag{8}$$

or as the upward sequence (Step-up search),

$$gp = p, \quad p = 1, 2, \cdots, r. \tag{9}$$



x_0 : Displacement (Basement),
x_i : Displacement,

u_i : Control force,
f_i : Virtual control force,

m_i : Mass,
k_i : Stiffness,

(i = 1, 2, 3).

Fig. 1 Multi-located aseismic active control system.

IV. NUMERICAL SIMULATIONS

In order to assure the effectiveness of the digital-optimizing control method, numerical simulations are executed. A three degrees-of-freedom system which is installed three active brace devices in every stories is adopted here (as shown in Fig.1). Structural properties of this testing system are shown in Fig.1, and those constants are approximately adjusted to the survey values of the experimental model [Tachibana, 1994]. El Centro (1940) NS and JMA-Kobe (1995) NS †1 are used as input ground motions, and those wave records are scaled down to the maximum acceleration amplitude of 30 $\mathrm{cm/s^2}$ according to the scale of the structural model. In the following case studies, the set of trial control forces is defined as a same-step type as follows.

$$\bar{u}_{j,a} = u_{max,j} \times (a - L) / L, \quad a = 0, 1, \cdots, N,$$

$$< \bar{u}_{j,a} > = < \bar{u}_{j,0}, \cdots, \bar{u}_{j,2L} > = < -u_{max,j}, \cdots, 0, \cdots, u_{max,j} >, \tag{10}$$

in which L is an integer number. To find out the suitable number of trial control forces and capacities of control forces, control performances by introducing the digital-optimizing control methods are investigated. The reductions of response and the required control forces in the case of EL Centro are shown in Fig.2, 3 and 4. Those figures are corresponding to the Universal-search (Fig.2), the Step-down search (Fig.3) and the Step-up search (Fig.4) algorithms of the digital-optimizing control method, respectively. In those figures, (a), (b) and (c) show the cases that the number of trial control forces N are 3, 5 and 7, respectively. As the estimation indexes of the reduced response and the required control forces, the DRMS and the FRMS are defined as follows.

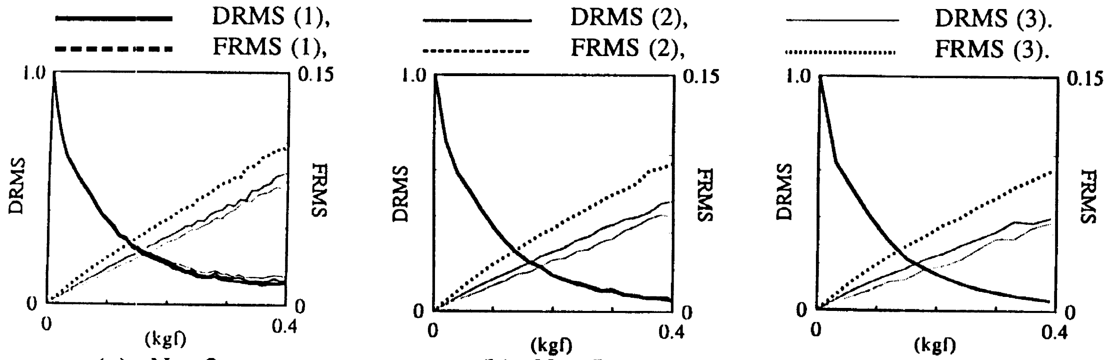
$$\mathrm{DRMS}\,(i) = x_{rms}(i) \,/\, x'_{rms}(i),$$

$$\mathrm{FRMS}\,(i) = u_{rms}(i) \,/\, (k_i \times 1), \quad i = 1, 2, 3, \tag{11}$$

in which $x_{rms}(i)$ and $x'_{rms}(i)$ mean the RMS (root-mean-square) values of the i-th story's controlled and non-controlled displacements, respectively. FRMS (i) is defined as the ratio of the RMS value of the i-th story's control force $u_{rms}(i)$ by the i-th story's resistant force for the unit deformation of 1 (cm). As seen

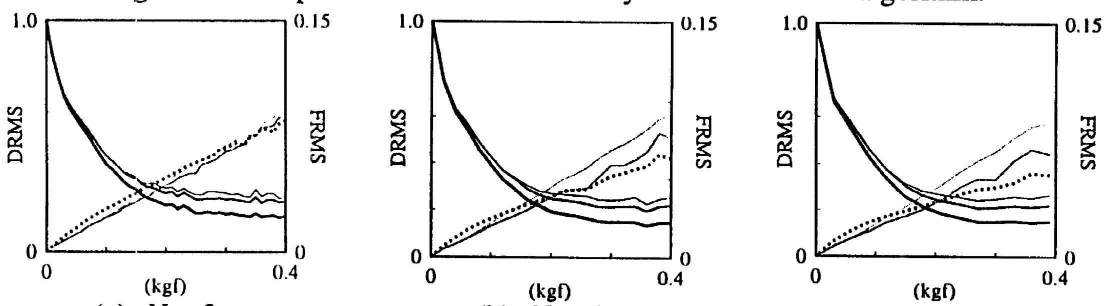
†1 Ground acceleration record which was observed at the Kobe station of the Japan Weather Association on January 17, 1995 (the Hyogo-Ken Nanbu Earthquake, Japan).

in Fig.2, 3 and 4, it appeared that effective reductions of response are gained by selecting only a few kind of trial control forces, when the enough capacity of control force is introduced into each device. Moreover, by comparing control performances of the *algorithm-I* (Universal search) with those of the *algorithm-II*, the case of the Step-down search shows good control performances which are very similar with the case of the Universal search.



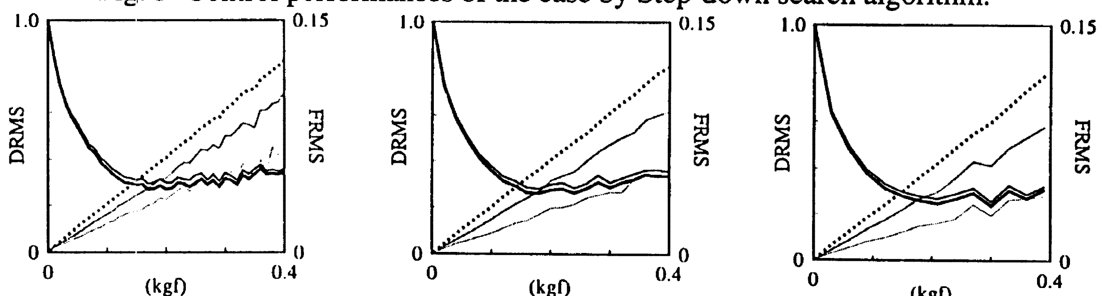
(a) $N = 3$, (b) $N = 5$, (c) $N = 7$,

Fig. 2 Control performances of the case by Universal search algorithm.



(a) $N = 3$, (b) $N = 5$, (c) $N = 7$,

Fig. 3 Control performances of the case by Step-down search algorithm.



(a) $N = 3$, (b) $N = 5$, (c) $N = 7$,

Fig. 4 Control performances of the case by Step-up search algorithm.

It is very interested in the investigations about where the acting point of each trial control force $\bar{u}_j$ is arranged apparently. In general, the trial control force $\bar{u}_j$ is regarded as the each control force u_j which acts as the inter-story force in the case of the active brace system (as shown in Fig.1), namely,

$$\{\bar{u}\} = \{u\}. \tag{12}$$

When the trial control force $\bar{u}_j$ is supposed as the body force f_j acting on the each mass,

$$\{\bar{u}\} = \{f\} = [U]\{u\}, \tag{13}$$

in which $[U]$ means the position matrix of the control forces. In the next case studies, those control force systems of the *Case-I* (a 'dependent control force system') defined as the Exp. (12) and the *Case-II* (an 'independent control force system') defined as the Exp. (13) are investigated. In the *Case-II*, it should be paid attentions that the all discrete component of the control forces of the active braces $\{u\}$ (which are determined from the all components of the trial control forces $\{\bar{u}\}$ by the transformation $[U^{-1}]$) should not be over its capacities in the digital-optimizing control method. Since the transformation $[U^{-1}]$ is defined as,

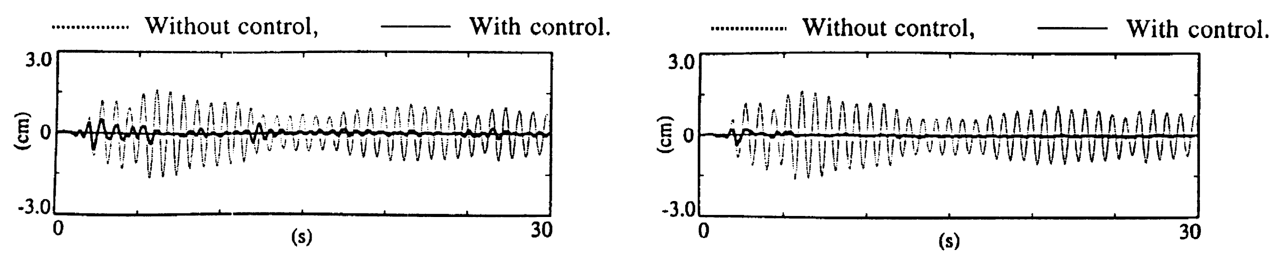
$$[U^{-1}] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (14)$$

the capacity of the control force of the lowest story's active brace in the *Case-II* is the three times as much as that in the *Case-I.* So, it is reasonable that the capacities of the control forces of the active braces should be selected as to be 3 : 1 between the *Case-I* and the *Case-II.* In the next comparing studies, the set of the trial control forces for the *Case-II* should be selected as follows (replacing the Exp. (10) into the Exp.(15)).

$$\bar{u}_{j,\,a} = (u_{max,\,j} / 3) \times (a - L) / L, \quad a = 0, 1, \cdots, N,$$

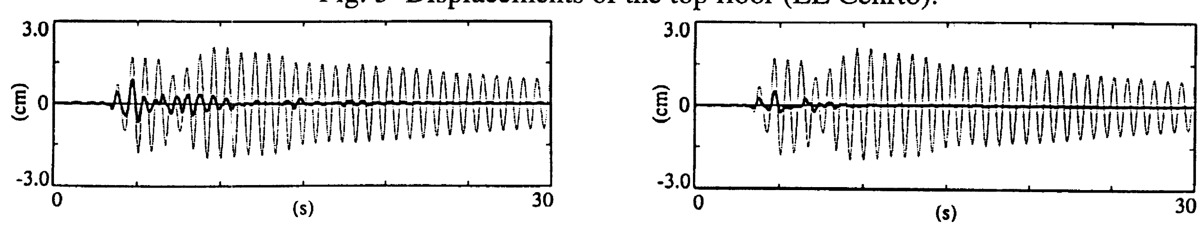
$$< \bar{u}_{j,\,a} > = < \bar{u}_{j,\,0}\,, \cdots, \bar{u}_{j,\,2L} > = < -u_{max,\,j} / 3, \cdots, 0, \cdots, u_{max,\,j} / 3 >. \qquad (15)$$

By selecting the sets of trial control forces $< \bar{u}_j >$ for the *Case-I* and the *Case-II* as 5 kinds of set which are < -0.36, -0.18, 0, 0.18, 0.36 (kgf) > and < -0.12, -0.06, 0, 0.06, 0.12 (kgf) >, respectively, comparing studies are executed through numerical simulations. The weight matrices of response in the Exp. (6) are set as diag.$[Q_d]$ = 100 and diag.$[Q_v]$ = 0.01. The Step-down search algorithm is used for the digital-optimizing control procedures. Displacements of the top floor are shown in Fig.5 (EL Centro) and Fig.6 (JMA-Kobe). In those figures, the results of the *Case-I* are shown in (a) and the results of the *Case-II* are shown in (b). By comparing those cases, the more effective reductions of response are observed in the *Case-II.*



(a) *Case-I,* (b) *Case-II,*

Fig. 5 Displacements of the top floor (EL Cenrto).



(a) *Case-I,* (b) *Case-II,*

Fig. 6 Displacements of the top floor (JMA-Kobe).

The DRMS and FRMS values for those results are shown in Table 1 and Table 2, respectively. In those tables, $\overline{\text{DRMS}}$ and $\overline{\text{FRMS}}$ mean the average values. By considering those tables, it is found that remarkable reductions of response are gained by small control forces in *the Case-II.* The control performances of the *Case-II* is fairly superior to the *Case-I.* The required control forces of the lowest story are quite similar values in bose cases but, in the *Case-II*, the control forces of the upper stories are going on decreasing. Those results show that the control forces for lower stories are important in the active brace system as far as earthquake motions are supposed as the external excitations.

Table 1 DRMS of each control force system (%).

Input motion	Case	DRMS (1)	DRMS (2)	DRMS (3)	$\overline{DRMS}$
EL Centro	I	13.41	19.67	22.46	18.51
	II	7.05	6.89	6.72	6.89
JMA-Kobe	I	11.47	16.07	19.09	15.54
	II	6.57	6.39	6.33	6.43

Table 2 FRMS of each control force system (%).

Input motion	Case	FRMS (1)	FRMS (2)	FRMS (3)	$\overline{FRMS}$
EL Centro	I	5.82	6.78	8.24	6.95
	II	5.32	3.04	2.73	3.70
JMA-Kobe	I	5.82	7.23	8.15	7.07
	II	4.87	2.77	2.78	3.47

V. CONCLUDING REMARKS

A practical aseismic active vibration control algorithms named digital-optimizing control method is presented. This algorithm aims the active control system using multi-devices, especially for the active tendon system or the active brace system. In this algorithm, control forces can be determined by a simple way. In order to save CPU times, multi-layer searching method are also proposed. Moreover, two kinds of control force systems producible by active braces are investigated. Through numerical simulations, it appeared that the digital-optimizing control method is very effective for reductions of structural responses under earthquake excitations.

VI. ACKNOWLEDGEMENTS

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VII. REFERENCES

1. Inoue, Y., Tachibana, E. and Mukai, Y., 1993, "Recent Developments in Active Structural Control of Buildings in Japan", *Proc. of Int. Workshop on Structural Control*, pp. 239-247.
2. Yang, J. N., Akbarpour, A. and Ghaemmaghami, P., 1987, "New Optimal Control Algorithms for Structural Control", *Journal of EM*, ASCE, Vol. 113, No. 9, pp. 1369-1386.
3. Friedland, B., 1986, *Control System Design - An Introduction to State-Space Method*, McGraw-Hill, New York.
4. Luenberger, D. G., 1973, *Introduction to Linear and Nonlinear Programming*, Addison-Wesley, Massachusetts.
5. Hatada, T., Sakakiyama, T., Tachibana, E. and Inoue, Y., 1989, "Generalized Optimal Control Algorithms for Civil Engineering Structures", *Proc. of 12th Canadian Congress of Applied Mechanics*, Vol. 2, pp. 770-771.
6. Tachibana, E., 1994, "Experimental Evaluations of Various Kinds of Active Control System for Structural Vibrations", *Knowledge-Based (Expert) System Applications in Power Plant and Structural Engineering*, EC Joint Research Center, pp. 301-311.
7. Tachibana, E., Mukai, Y. and Inoue, Y., 1994, "Structural Vibration Control Using Active Braces to Earthquake Excitations", *Proc. of the 1st World Conf. on Structural Control*, Vol. 3, pp. (FP5) 39-48.