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THE FUZZY-PROBABILITY METHOD FOR SOIL LIQUEFACTION HAZARD EVALUATION

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ABSTRACT

On the base of the stipulations related to soil liquefaction potential judgment and liquefaction hazard evaluation method in consideration of randomness and fuzziness of the soil liquefaction phenomenon has been put forward in this paper. In comparison of the previous study on this problem, the new method proposed in the paper possesses following specialties: 1. The analysis of the liquefaction probability is refereed to the whole soil profile rather than to a specified soil lager at given depth; 2. The classification of the liquefaction hazard degree is established on the base of fuzzy set theory by use of the suggested membership function relations; 3. The liquefaction hazard evaluation for the whole soil profile is carried out through applying fuzzy-probability method that provides a possibility of incorporating the influences of various uncertainties. The results in this paper have offered some fundamental materials for setting up the seismic design method for building foundation on the base of reliability theory.

KEYWORDS

Fuzzy-probability; Liquefaction; Membership function; Probability distribution.

THE PROBABILISTIC PREDICTION OF LIQUEFACTION POTENTIAL JUDGMENT

In order to sufficiently reflect the uncertainties involved in liquefaction potential evaluation in seismic design code we define following liquefaction potential index, *LPI*,

$$LPI = N_{63.5} / N_{cr} \tag{1}$$

$$N_{cr} = N_0\left[0.9 + 0.1\left(d_s - d_w\right)\right]\sqrt{3/\rho_c} \tag{2}$$

where N_0 and N_{cr} is respectively the basic and critical value of standard penetration resistance (SPR) for evaluating liquefaction, d_s is depth of penetration point of saturated soil (m), d_w is underground water table depth (m), ρ_c is the percentage of clay particle content.

According to the provisions stipulated by seismic design code it is easy to find out that *LPI* possesses following features

LPI<1 liquefaction will occur
LPI=1 the sand soil is in critical status of liquefaction
LPI>1 liquefaction will not occur

Therefore *LPI* not only satisfies the evaluation requirement of the seismic code but also gives the quantitative description of the various possible status from non-liquefaction to fully liquefaction.

Since the liquefaction evaluation method stipulated in seismic design code is based on given seismic intensity, it is assumed that the probabilistic distribution of seismic intensity or numerical characteristics is known when we do probabilistic prediction according to Eq.(1). In fact it is proved that the probabilistic distribution of seismic intensity obeys III-type extreme value distribution,

$$F_T(I) = exp\left[-T\left(\frac{W-I}{W-\varepsilon}\right)^k\right] \tag{3}$$

where W is upper limit of seismic intensity which can be taken as 12; ε is model intensity (most likely occurring intensity); k is shape parameter. These results can be directly used. On the basis of above requirements and available conditions and following total probability formula it is easy to find out a probability value of *LPI* less than a given value I in the period of T years,

$$P_T(LPI < i) = \int P(LPI < i|I) f_T(I) dI \tag{4}$$

where $f_T(I)$ is probability density function of seismic intensity in years of T that can be calculated from Eq.(3), $P(LPI<i|I)$ is conditional probability distribution provided that the intensity is given. Under hypothesis of log-normal distribution we have

$$P(LPI < i|I) = \int_0^i \frac{1}{\sigma\sqrt{2\pi u}} exp\left[-\frac{(ln u - m)^2}{2\sigma^2}\right] du \tag{5}$$

where m and σ is respectively log-mean value and log-standard-deviation of random variation u, usually can be determined by following formula through calculation of example average value μ and variance $V^2(f)$,

$$\begin{aligned} m &= ln\,\mu - ln\sqrt{1+V^2(f)} \\ \sigma &= \sqrt{ln\left(1+V^2(f)\right)} \end{aligned} \tag{6}$$

Based on the evaluation formula and associated stipulations in seismic design code, and also due to the measurement and testing of the variations while influence *LPI* in condition of given intensity are carrying out independeftly; the variance of *LPI* can be approximately considered in consist of following contents,

$$V^2(f) = V_{63.5}^2 + V_{d_w}^2 + V_{d_s}^2 + V_{\rho_c}^2 \tag{7}$$

where $V_{63.5}$, V_{dw}, V_{ds} and V_{ρ_c} are the variance of standard penetration resistance, under ground water table depth, buried depth of liquefied soil and clay particle content respectively. In normal circumstance the values of V_{ds} and V_{ρ_c} are very small, so that as an approximation they may be ignored. Generally speaking the values of $V_{63.5}$ and V_{dw} might be given by means of statistical analysis of large number of measuring and testing data. when the testing data are not sufficient it is suggested that $V_{63.5}$ be equal to 0.3 and V_{dw} to 0.021.

In terms of the definition of *LPI* it is known that the probability of *LPI* is less than 1 in period of T-years. It is apparently that $P_T(LPI<1)$ is the probability of liquefaction occurrence, therefore, the probability of non-liquefaction can be obtained by using following formula,

$$P_T(LPI \geq 1) = 1 - P_T(LPI < 1) \tag{8}$$

From Eq.(4) it is easy to find out the probability of *LPI* which is very important to depict the uncertainties of the changing process of *LPI*. As it is known that Eq.(8) provides a certain kind of probability calibration for the liquefaction evaluation method stipulated in seismic design code. It also sets up a foundation for the liquefaction evaluation in terms of probability theory.

THE ESTIMATION OF PROBABILITY CHARACTERISTICS OF LIQUEFACTION INDEX FOR THE WHOLE PROFILE OF SOIL LAYERS

In the current seismic design code the comprehensive liquefaction index for the whole profile is

$$I_{LE} = \sum_{i=1}^{n}(1 - N_i/N_{cri})d_i W_i = \sum_{i=1}^{n}(1 - LPI_i)R_i \tag{9}$$

where $LPI_i=N_i/N_{cri}$, $R_i=d_iW_i$, d_i is soil thickness represented by i-point and W_i is weighted function of the i-th soil layer (m^{-1}). The value of W_i is adopted as follows, W_i=10 for $h_i < 5$m , $W_i=15-h_i$ for $5 < h_i < 15$m and W_i=0 for h_i>15m, where h_i is depth of the i-point.

Based on probability theory and Eq.(8), it is easy to obtain $P_T(LPI_i)$, the probability density function of LPI_i and corresponding mean value and variance. If I_{LE} obeys certain distribution within given depth, for instance log-normal distribution and it is supposed that LPI_i at different depths be mutually independent, then we can easily obtain the probability density function of I_{LE} in period of T-years from the numerical characteristics previously described

$$f_P(I_{LE}) = \begin{cases} \dfrac{1}{t\sqrt{2\pi}I_{LE}} exp\left[-\dfrac{(lnI_{LE}-k)^2}{2t^2}\right] & I_{LE} > 0 \\ 0 & I_{LE} \leq 0 \end{cases} \tag{10}$$

where k and t is log-mean value and log-standard deviation respectively, which can be calculated by following formulas,

$$\begin{aligned} k &= ln\Psi_{LE} - ln\sqrt{1+V^2(I_{LE})} \\ t &= \sqrt{ln\left(1+V^2(I_{LE})\right)} \end{aligned} \tag{11}$$

where $V(I_{LE}) = \sigma_{LE} / \Psi_{LE} = \sqrt{V_{LE}} / \Psi_{LE}$

THE FUZZY CLASSIFICATION OF LIQUEFACTION AND MEMBERSHIP FUNCTION

The current seismic design code divides the liquefaction hazard level into three categories in accordance with whether $I_{LE} \le 5$, $5 < I_{LE} \le 15$ or $I_{LE} > 15$. Actually liquefaction hazard level is a fuzzy conception. In order to reflect this sort of fuzziness we could convert the liquefaction hazard degree category into several fuzzy domains of the liquefaction category that can be depicted by comprehensive liquefaction index. If we use discourses $\underset{\sim}{A_1}$, $\underset{\sim}{A_2}$ and $\underset{\sim}{A_3}$ to represent the three fuzzy domains of liquefaction hazard degree. These domains are correspondent to different liquefaction hazard level. Therefore we have

$$[\underset{\sim}{A_1}, \underset{\sim}{A_2}, \underset{\sim}{A_3}] = [\textit{slight liquefaction, moderate liquefaction, intensity liquefaction}] \tag{12}$$

in which each category $\underset{\sim}{A_i}\ (i = 1,2,3)$ is a sub-fuzzy-set in the discourses whose diagram of the membership function $\mu_{\underset{\sim}{A_i}}(X)$ are shown in Fig.1. In general we can adopt following membership function may be adopted,

$$\mu_{\underset{\sim}{A_1}}(I_{LE}) = \begin{cases} 1 & (0 \le I_{LE} < a_1) \\ 1/2\left[sin\left(\dfrac{I_{LE} - a_1}{a_2 - a_1} + 1/2\right)\pi + 1\right] & (a_1 \le I_{LE} < a_2) \end{cases}$$

$$\mu_{\underset{\sim}{A_2}}(I_{LE}) = \begin{cases} 1/2\left[sin\left(\dfrac{I_{LE} - a_1}{a_2 - a_1} - 1/2\right)\pi + 1\right] & (a_1 \le I_{LE} < a_2) \\ 1/2\left[sin\left(\dfrac{I_{LE} - a_2}{a_3 - a_2} + 1/2\right)\pi + 1\right] & (a_2 \le I_{LE} < a_3) \end{cases} \tag{13}$$

$$\mu_{\underset{\sim}{A_3}}(I_{LE}) = \begin{cases} 1/2\left[sin\left(\dfrac{I_{LE} - a_2}{a_3 - a_2} - 1/2\right)\pi + 1\right] & (a_2 \le I_{LE} < a_3) \\ 1 & (I_{LE} \ge a_3) \end{cases}$$

It can be seen from these sets of membership functions in Fig.1 that the membership degree at the crossing points between the two neighboring liquefaction categories equals 0.5 that is apparently reasonable. In Fig.1 a_i(i=1,2,3) is the representative value of I-th category.

From the above mentioned membership functions we can obtain the following membership functions of the liquefaction hazard level $\underset{\sim}{A_i^*}$ reaching category $\underset{\sim}{A_i}$ or more intensive ones(see Fig.2),

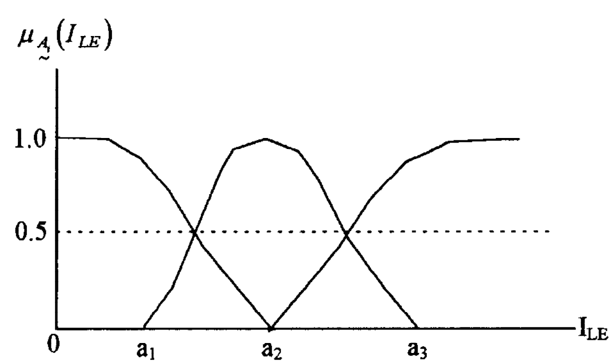

Fig.1

$$\mu_{\underset{\sim}{A_i^*}}(\mathrm{I_{LE}}) = \begin{cases} 0 & (I_{LE} < a_{i-1}) \\ 1/2\left[sin\left(\dfrac{I_{LE} - a_{i-1}}{a_i - a_{i-1}} - 1/2\right)\pi + 1\right] & (a_{i-1} \le I_{LE} \le a_i) \\ 1 & (I_{LE} > a_i) \end{cases} \tag{14}$$

In contrast, the membership function of liquefaction category $\underset{\sim}{A_i}$ or more intensive degree category not taking place can be abstained by mearns of the relationship of $\underset{\sim}{A_i^*}$ or its complementary set,

$$\mu_{\underset{\sim}{\bar{A}_i^*}}(I_{LE}) = 1 - \mu_{\underset{\sim}{A_i^*}}(I_{LE}) \tag{15}$$

The diagram of $\mu_{\underset{\sim}{\bar{A}_i^*}}$ is shown in Fig.3.

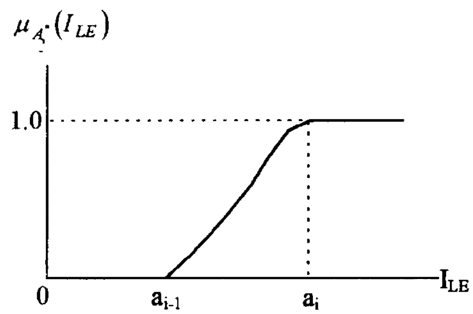

Fig.2

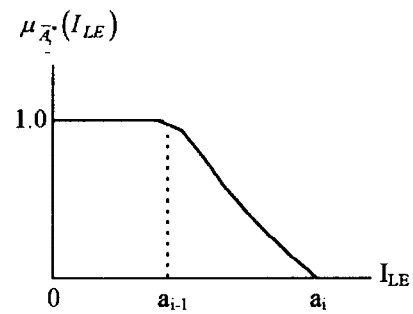

Fig.3

FUZZY PROBABILITY CALIBRATION OF SEISMIC LIQUEFACTION HAZARD CATEGORY

Based on the probability density function $f_T(I_{LE})$ of comprehensive liquefaction index I_{LE} and the membership functions that have been obtained in previous paragraph, we can find out the fuzzy probability of seismic liquefaction hazard category equals to or greater than a certain threshold in period of T-years. In accordance with the calculation method of the fuzzy event in fuzzy-set theory, the pre-mentioned fuzzy probability can be obtained using following formula,

$$P(\underset{\sim i}{\bar{A}^*}(I_{LE})|T) = \int_a^b f_T(I_{LE}) \cdot \mu_{\underset{\sim i}{A^*}}(I_{LE})\, dI_{LE} \tag{16}$$

Substituting Eq.(10) and (14) into Eq.(16), we can obtain

$$P\left(\underset{\sim i}{A^*}(I_{LE})|T\right) = 1 + \varphi\left(-\frac{k}{t}\right) + 1/2\left[P_1 - \varphi\left(\frac{l_n a_{i-1} - k}{t}\right)\right] - \varphi\left(\frac{l_n a_i - k}{t}\right) \tag{17}$$

in which $\varphi(\ )$ is standard normal probability distribution

$$P_1 = \int_{a_{i-1}}^{a_i} \frac{1}{\sqrt{2\pi} I_{LE}} exp\left[-\frac{(ln I_{LE} - k)^2}{2t^2}\right] sin\left[\frac{I_{LE} - a_{i-1}}{a_i - a_{i-1}} - 1/2\right] \pi dI_{LE}$$

The above integral is not easy to be expressed by primary functions but can be integrated by numerical method.

Based on Eq.(15), similar to getting Eq.(17) we can obtain fuzzy probability of seismic liquefaction hazard category A_i or more intensive ones not taking place at a prespecified site in service period of T-years,

$$P(\overline{\underset{\sim i}{A}}^*(I_{LE})|T) = 1 - P(\underset{\sim i}{A}^*(I_{LE})|T) = 1/2[\varphi(\frac{l_n a_i - k}{t}) + \varphi(\frac{l_n a_{i-1} - k}{t}) - P_1] - \varphi(-\frac{k}{t}) \quad (18)$$

Using Eq.(17) and (18) it is easy to obtain the quantitative information of the uncertainties involved in liquefaction hazard level estimation. These formulas also provides basic data for estimating the reliability of the liquefaction evaluation.

NUMERICAL EXAMPLE

In order to illustrate the real calculation procedure by use of the suggested method in this paper, the following example is discussed.

Suppose it is known that the probability distribution of seismic intensity consists with the law described by Eq.(3) from the seismic hazard analysis results for a prescribed site. The values of parameters involved in Eq.(3) are w=12, ε=5.54, k=8.718.

The geologic columnar section of typical layer profile is given in Tab.1 to calibrate the fuzzy probability for various liquefaction hazard categories at the site.

Tab.1 The geologic columnar section of typical layer profile

No	Columnar section	depth (m)	depth of SP (m)	No. of SPR N_i	Critical (SPR) N_{cri}
1	filled soil	1.0			
2	silt clay	2.1	1.4	2	5.64
3	silt clay	3.3			
4	fine send		5.0	8	7.8
5	fine send		7.0	12	9
6	clay	8.0			

Probability Prediction of LPI

From Eq.(5) we can obtain the probability distribution (as shown in Fig.4) of liquefaction index *LPI* at the liquefaction points assigned by soil columnar section listed in Tab.1. It can be seen from Fig.4 that No.2 liquefaction probability is the highest then No.4 layer's and No.5 layer's in turn. The above is consistent in tendency with determinate evaluation result that predict No.2 layer will liquefy, No.4 layer will reach critical status of liquefaction and No.5 layer's liquefaction potential will quite low.

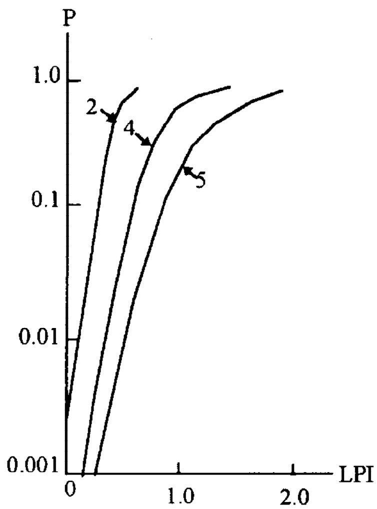

Fig.4

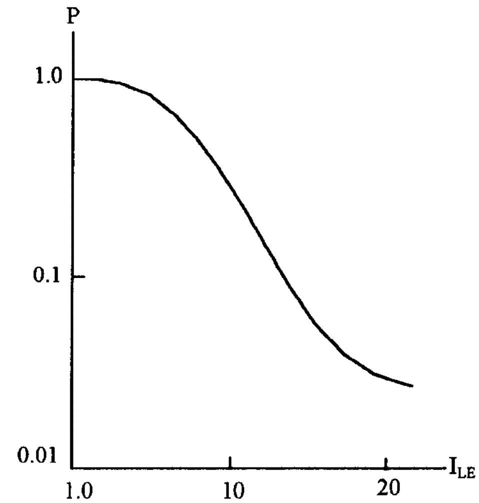

Fig.5

The Probability Distribution of the Comprehensive Liquefaction Index of Soil Columnar Section

Based on Eq.(10) we can gain the probability distribution of the whole soil columnar section. The results are shown in Fig.5. According to the deterministic liquefaction evaluation method stipulated in seismic design code it has been calculated that the liquefaction index corresponding to basis intensity VII is about 7, and the probability of the liquefaction index corresponding to 7 is about 66%. These results show that the possibility of liquefaction index greater than 7 at the site is quite large on the one hand and that the deterministic evaluation based on seismic design code is just at the level of statistical mean value on the other hand.

Fuzzy Probability Calibration of Liquefaction Category

Based on Eq.(17) we find out the fuzzy probabilities of liquefaction categories I, II and III at the given site in service period of 50 years as shown in Tab.2. It can be seen from the values shown in Tab.2 that, for the given site, the fuzzy probability of the liquefaction hazard degree category II is the highest, then that of category I and that of category III is the lowest. This result is consistent with the results from deterministic method in which the liquefaction category of the site belongs to II but inclines to category I.

Tab.2 Fuzzy probabilities of liquefaction categories I, II and III

Liquefaction Category	I	II	III
Fuzzy Probability	0.18	0.717	0.071

It is easy to find out that the suggest method not only provides a possibility for transferring the liquefaction evaluation method stipulated in current seismic design code onto basis of reliability theory but also gives a approach for estimating the probability distribution of the site liquefaction index and the fuzzy probability of the liquefaction category, those probabilistic information are able to back up the further research and decision making analysis.

CONCLUDING REMARKS

This paper put forwards a sort of method for calculating the site liquefaction index probability distribution and fuzzy probability of the liquefaction category. This method is closely related with the contents of seismic design code concerning liquefaction potential judgment and evaluation of liquefaction hazard level, and at present the accuracy and reliability for both is roughly the same, but following the theory frame offered by this paper it is possible to sufficently use the uncertainty data and associated parameters to determine the probability distribution and fuzzy probability of the whole soil columnar section so that to turn the liquefaction potential judgment and hazard level evaluation to a unified basis of reliability degree.

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