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DEVELOPMENT OF 3-DIMENSIONAL DYNAMIC ANALYSIS PROGRAM FOR SATURATED POROUS MEDIA

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ABSTRACT

A three-dimensional dynamic analysis program for saturated porous-rocks and soils (MPDAP-3D) is developed in this study. The theoretical formulations incorporated in the proposed computer program are the extension of Biot's two-phase theory to nonlinear region. Numerical study for typical verification problems is carried out to show the validation of the computational algorithms of the computer program MPDAP-3D. It is shown that the computer program MPDAP-3D is a useful tool for the analysis of the structural safety evaluation of underground openings in natural geologies during construction and post-construction period.

KEYWORDS

Consolidation; dynamic response; finite element analysis; generalized Hoek-Brown model; porous media; pore-water; radioactive waste disposal; two-phase theory; underground opening; wave propagation

INTRODUCTION

Increasing demand for energy from nuclear resources has forced scientists and engineers to focus much attention on the safety of radioactive waste disposal systems. Long-term safety design of radioactive waste isolation storage system is a national concern because environmental exposure to radionuclide has both direct and long-term implications on public health. Thus the use of underground rock caverns for radioactive waste disposal has been considered in Korea. Proper structural safety analyses, however, are required to reduce the potential risks associated with such disposal systems in natural geologies (Kim et al., 1991).

Recent research in geomechanics is mainly focused on transient phenomena occurring in earthquakes and explosive loading. For all of these the coupling between the deformation of the geomaterials such as soils and rocks and the motion of the pore water is of primary importance (Zienkiewicz and Shiomi, 1984; Prevost, 1986).

In this study nonlinear two-phase theory is reviewed and a three-dimensional multi-phase dynamic analysis program for saturated porous-rocks and soils (MPDAP-3D) is developed. Then, numerical study for typical verification problems is carried out to show the validation of the computational algorithms of the computer program MPDAP-3D.

DEVELOPMENT OF THREE-DIMENSIONAL TWO-PHASE CODE

Biot introduced fundamental analytical work describing the behavior of saturated porous media in a series of papers extending over many years (Biot, 1956). Other investigators have applied Biot's analytic results using techniques which approximate his equations with varying degrees of accuracy and sophistication (Ghaboussi and Wilson, 1972). Theoretical formulations incorporated in the code MPDAP-3D are the extension of Biot's two-phase theory to nonlinear region. These nonlinear two-phase theories have been developed over a decade (e.g. Blouin and Kim, 1984; Kim et al., 1986 and 1990).

In this section, the fundamental equations and the three-dimensional elasto-plastic matrix implemented in the code MPDAP-3D are described.

Finite Element Formulation of Nonlinear Two-Phase Medium

Field equations representing fundamental mechanism of a two-phase medium include principle of effective stress, constitutive equation for skeleton deformation, continuity equation of pore fluid flow, equation of motion for the bulk mixture and equation of motion for pore fluid (Kim, 1993).

Principle of effective stress:

$$\sigma_{ij} = \sigma'_{ij} + \delta_{ij} \cdot \pi \tag{1}$$

where σ_{ij} = total stress, σ'_{ij} = effective stress, δ_{ij} = Kronecker's delta, and π = pore water pressure.

Constitutive equation for skeleton deformation:

$$\{d\sigma'\} = [\ D^{ep}\ ] \cdot \left(\{d\varepsilon\} - \frac{1}{3 \cdot K_g} \cdot \{1\} \cdot d\pi \right) \tag{2}$$

where $[\ D^{ep}\ ]$ = elasto-plastic stress-strain matrix for skeleton, $\{\varepsilon\}$ = skeleton strain, and K_g = bulk modulus of solid grain.

Contiuity equation of pore fluid flow:

$$d\pi = \overline{m}_2 \cdot d\varepsilon_v + \overline{m} \cdot n \cdot (d\varepsilon_F - d\varepsilon_v) \tag{3}$$

where $\overline{m} = \dfrac{1}{\left[\dfrac{1}{K_m} - \dfrac{K_s^{ep}}{K_g^2} \right]}$, $\overline{m}_2 = \left[1 - \dfrac{K_s^{ep}}{K_g} \right] \cdot \overline{m}$, ε_F = volumetric diffusion of pore water, ε_v= skeleton volumetric strain, n = porosity, K_m = bulk modulus of soil-water mixture with zero effective stress, and K_s^{ep} = elasto-plastic bulk modulus of skeleton.

Equation of motion for the bulk mixture:

$$\sigma_{ij,j} = \rho \cdot \ddot{u}_i + \rho_f \cdot \ddot{w}_i \tag{4}$$

where $\ddot{u}$ = skeleton acceleration, $\ddot{w}$ = apparent water acceleration relative to the solid skeleton, ρ = mass density of mixture, and ρ_f = mass density of pore water.

Equation of motion for the pore fluid:

$$\pi_{,i} = \frac{\mu}{\alpha} \cdot \dot{w}_i + \frac{\rho_f}{\beta} \cdot \dot{w}_i^{\,2} + \rho_f \cdot \ddot{U}_i \tag{5}$$

where μ = dynamic viscosity of the water, α, β = flow coefficients that are properties of the porous skeleton only, and $\ddot{U}$ = absolute water acceleration.

These field equations are described in terms of nodal values and expressed in incremental form. Within each finite element, variables in these field equations can be expressed in terms of element nodal variables using the shape functions.

$$\{\Delta u\} = [\ N\ ]\cdot\{\Delta \underline{u}\}_e$$

$$\{\Delta \varepsilon\} = [\ B\ ]\cdot\{\Delta \underline{u}\}_e$$

$$\{\Delta w\} = [\ N\ ]\cdot\{\Delta \underline{w}\}_e \tag{6}$$

$$\Delta w_{i,i} = \{1\}^T\cdot[\ N\ ]\cdot\{\Delta \underline{u}\}_e$$

Stress vector at time step n can be expressed as:

$$\{\sigma_n\} = \{\sigma_{n-1}\} + \{\Delta\sigma'\} + \{1\}\cdot\Delta\pi \tag{7}$$

Combining Eqs (1), (2), (3) and (6) yields:

$$\{\Delta\sigma\} = (\,[\ D^{ep}\ ]\cdot[\ B\ ] + \overline{m}_1\cdot\{1\}\cdot\{1\}^T\cdot[\ B\ ]\,)\cdot\{\Delta\underline{u}\} + \overline{m}_2\cdot\{1\}\cdot\{1\}^T\cdot[\ B\ ]\,)\cdot\{\Delta\underline{w}\} \tag{8}$$

where $\overline{m}_1 = \left[\ 1 - \frac{K_s^{ep}}{K_g}\right]^2\cdot\overline{m}.$ (9)

Global equilibrium equations for the two-phase medium are formulated by principle of virtual work and then linearized to be solved by linear equation solver.

Global equilibrium equation at time step n can be expressed in the simpler form:

$$[\ M\ ]\cdot\{\ \ddot{\underline{d}}_n\} + [\ D\ ]\cdot\{\ \dot{\underline{d}}_n\} + [\ K\ ]\cdot\{\Delta\ \underline{d}_n\} = \{P_n\} - \{R_{n-1}\} \tag{10}$$

Introducing a time integration method which incorporates both Newmark's β method and Wilson's θ method (Bathe, 1982), we can obtain the following linearized global equilibrium equations which can be solved simultaneously at each time step:

$$[\ \hat{K}\ ]\cdot\{\Delta\ \underline{d}_n\} = \{\ \overline{P}_n\} \tag{11}$$

where the generalized stiffness matrix is given by

$$[\ \hat{K}\ ] = C_1\cdot[\ M\ ] + B_1\cdot[\ D\ ] + [\ K\ ] \tag{12}$$

and the generalized force vector is given by

$$\{\ \hat{P}_n\} = \{P_n\} - \{R_{n-1}\} - [\ M\ ]\cdot(C_2\cdot\{\ \dot{\underline{d}}_{n-1}\} + C_3\cdot\{\ \ddot{\underline{d}}_{n-1}\})\ \ [\ D\ ]\cdot(B_2\cdot\{\ \dot{\underline{d}}_{n-1}\} + B_3\cdot\{\ \ddot{\underline{d}}_{n-1}\}). \tag{13}$$

Three-Dimensional Elasto-Plastic Matrix and Implementation of Theoretical Formulations

The program MPDAP-3D uses the Generalized Hoek and Brown Model (Hoek and Brown, 1982) to represent the skeleton constitutive relations of soils or porous materials. This skeleton model can be used for concrete, steel, soils and rock mass. As one of the useful features, the model includes the empirical data base which is tabulated for several different rock types as a function of rock quality. The three-dimensional elasto-plastic matrix is derived for the Generalized Hoek and Brown Model in this study. The model is elastic below the failure surface and perfectly plastic along the failure surface with the volumetric and deviatoric behaviors dependent upon one another once the failure surface is reached.

Implementation of the three-dimensional theoretical formulations described in the previous section will be the major development effort in this study. The existing two-dimensional program MPDAP (Multi-Phase Dynamic Analysis Program) (Blouin et al., 1990) is modified to extend into three-dimensional coordinate system. The major modifications involve the implementation of mass, dissipation, and stiffness matrices in three-dimensional coordinate system.

VERIFICATION PROBLEMS

Results of numerical test for three verification problems is presented in this paper. The objective of this numerical test is to demonstrate the accuracy and validity of the computer code MPDAP-3D.

The first verification problem concerns fully coupled undrained uniaxial strain response of saturated porous linear elastic medium as shown in Figure 1. The exact solution for the undrained stress response is given by Blouin and Kim (1984). The exact ratio of pore water pressure (π_o) to applied total vertical stress is 0.4592, and the exact ratio of effective vertical stress (σ_v') to applied total vertical stress (σ_v) is given by 0.5408. Figure 2 shows predicted undrained uniaxial stress response compared with an exact solution. As shown in Figure 2, the predicted response by program MPDAP-3D is identical to the exact solution.

The second verification problem concerns Terzaghi's linear consolidation with initial triangular distribution of excess pore water pressures. As initial conditions, it is assumed that soil liquefied and pore water takes all the weight. The exact solution for the excess pore water pressure (π_e) is given by

$$\pi_e = \sum_{m=1,3}^{\infty} \left(\frac{8\gamma' H}{m^2 \pi^2} \right) \left(\sin \frac{m\pi}{2} \right) \left(\sin \frac{m\pi}{2H} y \right) e^{-\frac{m^2 \pi^2}{4} T} \tag{14}$$

Figure 3 shows profiles of pore water pressures at 0.05 and 0.5 seconds. And Figure 4 shows profiles of effective vertical stresses at 0.05 and 0.5 seconds. MPDAP-3D calculations are very close to the exact solution.

The third verification problem is to check overall two-phase dynamic equations implemented in the program MPDAP-3D. A vertically propagating planar compression wave through idealized saturated soil is considered. The input loading, as shown in Figure 5, is a short rise time triangular pulse with a peak stress of 5,000psi and a positive phase duration of 10msec. The loading pulse is applied to the saturated sand having the properties listed in Figure 5. The load is applied to an impermeable boundary at the ground surface. Computed profiles of pore water pressure and effective vertical stress at 20msec are shown in Figure 6 and 7, respectively. The closed-form solution for this problem is not available. So, the same problem has been solved by the existing two-dimensional version of MPDAP for direct comparison. These MPDAP-2D results are not shown in Figure 6 and 7, but they are identical to the MPDAP-3D results.

The last verification problem is to check the implementation of the 3-dimensional formulation of elasto-plastic matrix derived for the Generized Hoek and Brown Model. In this problem, the plane strain response of a tunnel subjected to axisymmetric loading as calculated using MPDAP-3D is compared to a previous semi-analytical solution developed by Piepenburg et al. (1986). Figure 8 shows a schematic section view of a 60 inch diameter circular tunnel subjected to a hydrostatic loading of 2800psi. The surrounding rock is assumed to be linear elastic beneath the failure surface and to follow the Drucker-Prager plasticity model upon reaching the failure surface. The elastic and strength properties of the rock are listed in Figure 8. By symmetry, only a quadrant of tunnel cross section is modeled as shown in Figure 9. Figure 10 shows tunnel deformed shape. Figure 11 shows stresses along the 4.5 from the X-axis. And Figure 12 shows stresses along the 85.5° from the X-axis. The computed stress profiles agree well with the semi-analytical solution in both the plastic and elastic zones of deformation surrounding the tunnel.

CONCLUSIONS

A 3-Dimensional Multi-Phase Dynamic Analysis Program (MPDAP-3D) has been developed by extending the existing 2-dimensional MPDAP. The computer program MPDAP-3D is a useful tool for the geomechanical analysis since it can solve static, consolidation and dynamic problems in dry, partially saturated or fully saturated soils and porous rock mass. The program considers material, geometric, and boundary condition nonlinearities.

Results of verical problems demonstrate the accuracy and validity of the computational algorithms implemented in the program MPDAP-3D. First three verification problems are mailly performed to check static, consolidation and dynamic analyses features of two-phase medium. The last verification problem is conducted to check the elasto-plastic response of the Generalized Hoek and Brown Model.

Specific applications related to the design of radioactive waste disposal repository include:

- o Structural safety of disposal system subjected to explosive loads from drill and blast excavation during the construction stages.
- o Prediction of ground water flow through the repository.
- o Prediction of stresses and deformations during the multi-staged repository excavations such as NATM.

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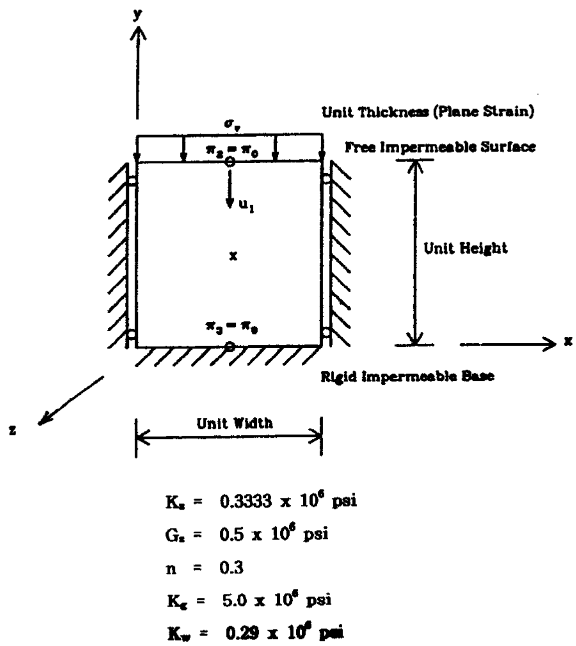


Figure 1 A cubic element subjected to undrained uniaxial strain loading.

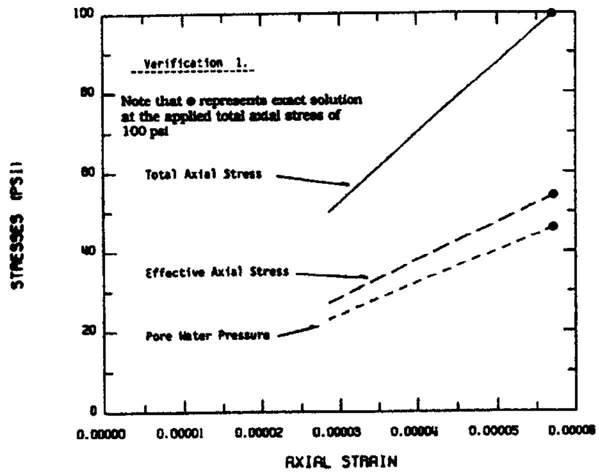


Figure 2 Computed undrained stress response compared with exact solution.

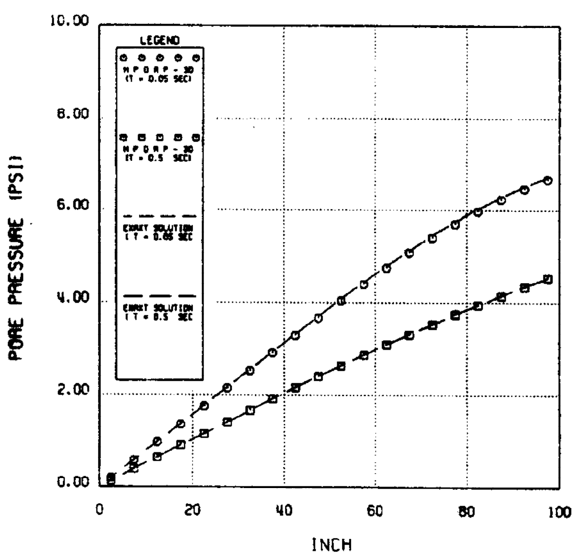


Figure 3 Pore water pressure profiles at 0.05 and 0.5 seconds.

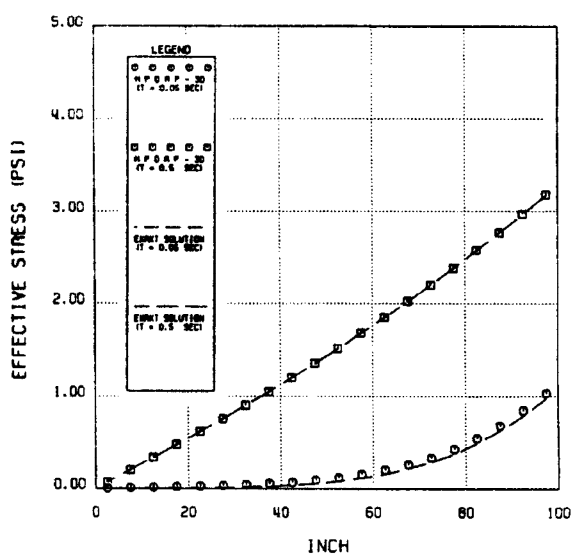
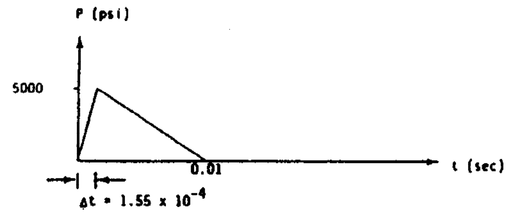


Figure 4 Effective vertical stress profiles at 0.05 and 0.5 seconds.



ASSUMED MATERIAL PROPERTIES

Pore Water		
	Bulk Modulus	0.29×10^6 psi
Solid Grains		
	Bulk Modulus	5.0×10^6 psi
	Specific Gravity	2.67
Drained Skeleton Properties		**Soil**
	Bulk Modulus	3000 psi
	Constrained Modulus	6000 psi
	Poisson's Ratio	0.20
	Porosity	0.35
Permeability		.001 in/s

Figure 5 Loading time history and material properties.

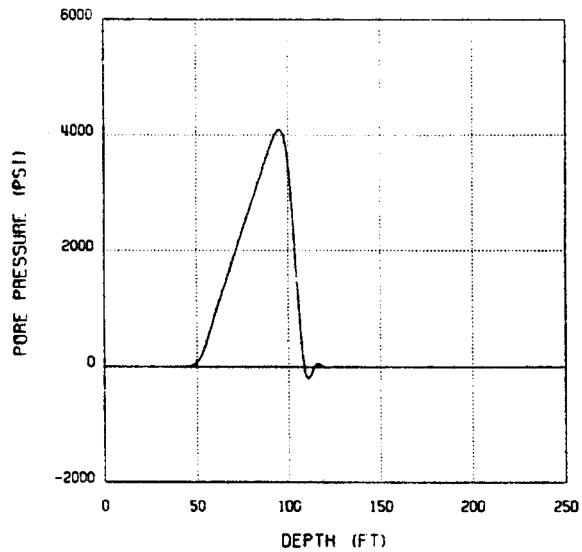


Figure 6 Profiles of pore water pressure at 20msec

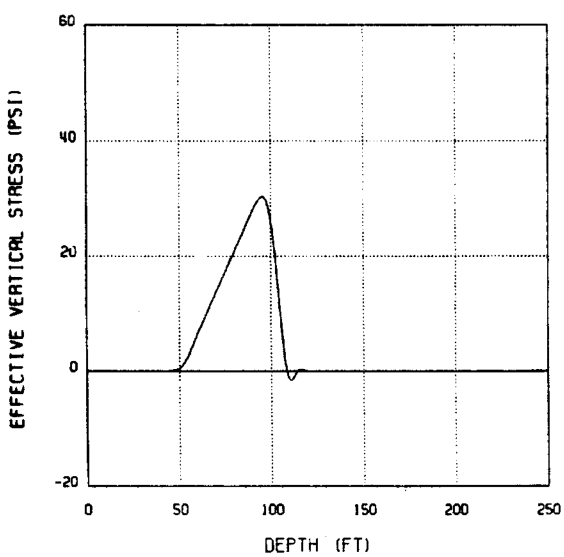
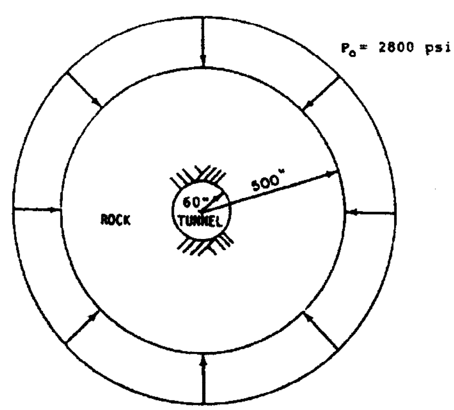


Figure 7 Profiles of effective vertical stress at 20 msec.



Material Model: Drucker-Prager Model

Rock Properties: E = 1,150,000 psi (Young's Modulus)
ν = 0.33 (Poisson's Ratio)
σ_c = 1,800 psi (Unconfined Strength)
ϕ = 18° (Friction Angle)

Figure 8 Circular tunnel subjected to axisymmetric loading.

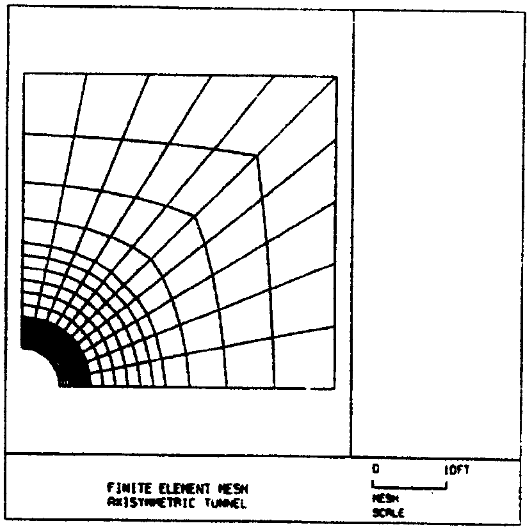


Figure 9 Finite element mesh on X-Y plane.

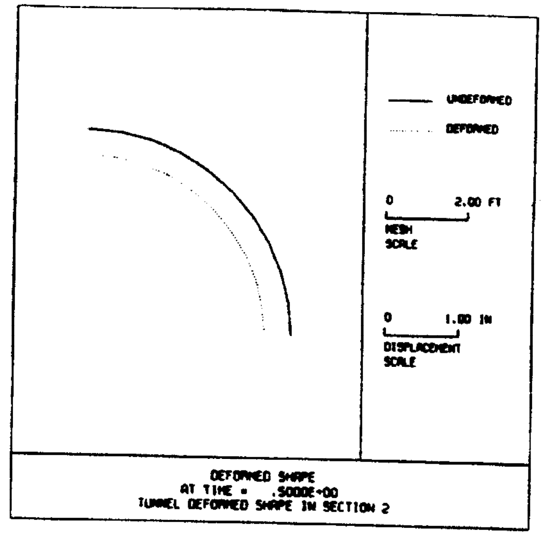


Figure 10 Tunnel deformed shape

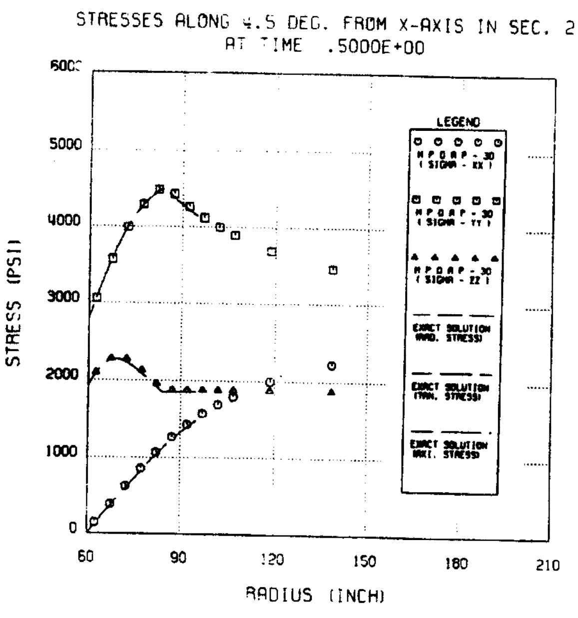


Figure 11 Stresses along 4.5 degree from X-axis

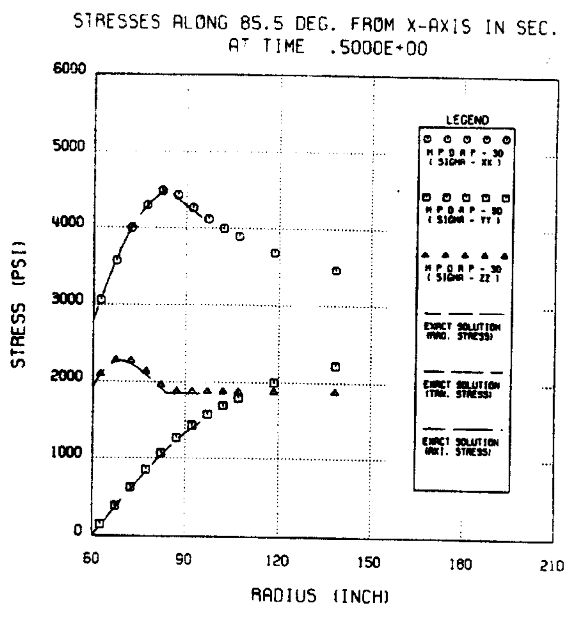


Figure 12 Stresses along 4.5 degree from X-axis