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SEISMIC RESPONSE OF COMBINED PRIMARY-SECONDARY SYSTEMS VIA COMPONENT-MODE SYNTHESIS

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ABSTRACT

The evaluation of the response of a combined primary-secondary system requires the following fundamental steps: *i.* the solution of one or two eigenproblems with real solutions; *ii.* the application of a coordinate transformation which reduces, according to the so-called component-mode synthesis, the state variables; *iii.* the solution of an eigenproblem with complex eigenvalues and modal shapes. It follows that the evaluation of the response requires the determination of complex modes by numerical techniques, which are not as robust as techniques currently used for the solution of the real eigenvalue problem, and the use of complex algebra. In the present paper an unconditionally stable step-by-step procedure is presented to evaluate the response without using complex quantities. The method is based on the evaluation of the fundamental operator in approximate form of the numerical procedure. The great accuracy of the method is shown by performing accuracy tests.

KEYWORDS

Composite structures; component mode-synthesis; isolated structures; numerical solution; primary-secondary systems; transition matrix.

INTRODUCTION

Often, in dynamic analysis of structural system we have to evaluate the response of industrial or civil buildings which can be considered as composed by a primary structure connected with a secondary one (Chen et al.,1988). In many cases we encounter light secondary structures (HVAC, piping, equipment etc.) supported on heavy primary ones (nuclear power plant, building, offshore platforms, etc.). In the other cases we have heavy secondary substructures as in the problem of seismic isolation where the secondary substructure (seismically isolated building) is supported on the primary one (the base isolation system). In both cases the combined primary-secondary system possesses characteristics which are uncommon for the usual structures and the traditional analysis used for the latter is not feasible. Indeed for these combined structures we can have numerical problems in evaluating the response because of dissimilarities in the stiffness, damping and inertia properties of two substructures. It follows that usually the full eigensolution of combined subsystems is avoided and the traditional modal coordinate transformation is substituted by another one based on the so-called component- mode synthesis (Muscolino.,1990). By using the latter coordinate transformation we arise at a two-step eigenvalue problem: the two substructures eigenproblems which give real eigenproperties, and the transformed eigenproblem, which gives complex eigenproperties. Methods for the calculation of complex eigenproperties are not as robust as the techniques currently used for the solution of the usual eigenproblem. For this reason approximate numerical procedure have been proposed to avoid the complex eigenproblem (Singh et al.,1987; Spanos et al.,1988; Harichandran et al., 1989; Falsone et al.,1992). In this paper a numerical technique to evaluate the response of linear combined structures with both light and heavy secondary subsystems is proposed. The proposed method requires the evaluation in approximate form of the so-called transition matrix (Meirovitch., 1980; Muller et al., 1985) which represents the fundamental operator of the unconditionally step by step procedure here adopted.

GENERAL FORMULATION FOR STRUCTURAL SYSTEMS COMPOSED BY TWO SUBSTRUCTURES

The equations of motion of an n_s-degree-of-freedom (n_s-DOF) secondary subsystem supported on an n_p-DOF primary subsystem multiply connected to it and subjected to a seismic input can be written, in terms of 'total' or 'conventional' displacements, as follows:

$$\boldsymbol{M}_t\, \ddot{\boldsymbol{u}}_t(t) + \boldsymbol{C}_t\, \dot{\boldsymbol{u}}_t(t) + \boldsymbol{K}_t\, \boldsymbol{u}_t(t) = -\boldsymbol{M}_t \boldsymbol{\tau}\, \ddot{u}_g(t) \tag{1}$$

in which τ is the $(n\times1)$ influence vector, $\ddot{u}_g(t)$ is the ground motion acceleration, the dot, over a variable, denotes its time derivatives and the vector $\boldsymbol{u}_t(t)$ is the total displacement vector (of order $n = n_s+n_p$) given as :

$$\boldsymbol{u}_t(t) = \left[\boldsymbol{u}_s^T(t) \quad \boldsymbol{u}_p^T(t)\right]^T \tag{2}$$

where $\boldsymbol{u}_s(t)$ and $\boldsymbol{u}_p(t)$ are the displacement vectors with respect to the ground of the secondary and primary subsystems respectively. In this paper the properties associated with the primary and secondary substructures will be identified by the subscript p and s, respectively.
In equation (1), $\boldsymbol{M}_t$, $\boldsymbol{C}_t$ and $\boldsymbol{K}_t$ are the mass, damping and stiffness matrices of the composite system given as:

$$\boldsymbol{M}_t = \begin{bmatrix} \boldsymbol{M}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_p + \boldsymbol{M}_o \end{bmatrix} \quad ; \quad \boldsymbol{C}_t = \begin{bmatrix} \boldsymbol{C}_s & \boldsymbol{C}_{sp} \\ \boldsymbol{C}_{sp}^T & \boldsymbol{C}_p + \boldsymbol{C}_o \end{bmatrix} \quad ; \quad \boldsymbol{K}_t = \begin{bmatrix} \boldsymbol{K}_s & \boldsymbol{K}_{sp} \\ \boldsymbol{K}_{sp}^T & \boldsymbol{K}_p + \boldsymbol{K}_o \end{bmatrix} \tag{3}$$

where $\boldsymbol{M}_s$, $\boldsymbol{C}_s$, $\boldsymbol{K}_s$, $\boldsymbol{M}_p$, $\boldsymbol{C}_p$ and $\boldsymbol{K}_p$ are the mass, damping and stiffness matrices of the secondary and primary subsystems respectively, considered as fixed at their own bases (i.e. while the primary one is assumed to be fixed at ground ,the secondary one is assumed to be fixed at the multiple support points on the primary one as well as on the ground); $\boldsymbol{C}_{sp}$ and $\boldsymbol{K}_{sp}$ are matrices representing the physical coupling between the two subsystems and the matrices $\boldsymbol{M}_o$, $\boldsymbol{C}_o$ and $\boldsymbol{K}_o$ represent the increment to the mass, damping and stiffness matrices of the primary subsystem due to the presence of the secondary one.
As clarified in a recent paper (Muscolino.,1990) , the dynamic response of a combined system can be better represented when the so-called admissible coordinate transformation is adopted; this coordinate transformation is given as follows:

$$\boldsymbol{u}_t(t) = \Gamma_t\, \boldsymbol{q}(t) \tag{4}$$

where $\boldsymbol{q}(t)$ is the vector of modal coordinates while Γ_t is the transformation matrix, given respectively as (with argument omitted):

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}_s \\ \boldsymbol{q}_p \end{bmatrix} \quad ; \quad \Gamma_t = \begin{bmatrix} \Phi_s & \boldsymbol{N}\Phi_p \\ \boldsymbol{0} & \Phi_p \end{bmatrix} \tag{5}$$

In equation (5) $\boldsymbol{N}$ is the so-called pseudostatic influence matrix, Φ_s and Φ_p are the modal matrices of the two subsystems (of order n_s x m_s and n_p x m_p respectively, whit $m_s \leq n_s$ and $m_p \leq n_p$), normalised with respect to $\boldsymbol{M}_s$ and $\boldsymbol{M}_p$ respectively and obtained by solving the following eigenproblems:

$$M_s\Phi_s\Omega_s^2 = K_s\Phi_s \quad ; \qquad M_p\Phi_p\Omega_p^2 = K_p\Phi_p \tag{6}$$

where Ω_s and Ω_p are two diagonal matrices listing the natural circular frequencies ω_i of the secondary and primary subsystems respectively. The criterion for choosing the order of the modal matrices Φ_s and Φ_p in the use of this coordinate transformation can be based upon the natural frequencies of the two substructures taken separately. By using the admissible coordinate transformation defined in equation (4) and assuming that the two substructures are classically damped, equation (1) becomes a set of m differential equations (with $m=m_S+m_P$ generally smaller than the number of equations of the original system) which can be written as :

$$m\ddot{q} + c\dot{q} + kq = -\Gamma_t^T M_t \tau\, \ddot{u}_g(t) \tag{7}$$

where m , c and k are symmetric and positive definite matrices, given respectively as :

$$m = \begin{bmatrix} I_{m_s} & \Phi_s^T\left(M_{sp} + M_s N\right)\Phi_p \\ \Phi_p^T\left(M_{sp}^T + N^T M_s\right)\Phi_s & I_{m_p} + \Phi_p^T \Delta M \Phi_p \end{bmatrix} \;;\; c = \begin{bmatrix} c_s & 0 \\ 0 & c_p \end{bmatrix} \;;\; k = \begin{bmatrix} \Omega_s^2 & 0 \\ 0 & \Omega_p^2 + \Phi_p^T \Delta K \Phi_p \end{bmatrix} \tag{8}$$

In equations (8) I_m is the identity matrix of order m, 0 is the zero matrix and ΔM and ΔK are given respectively as follows:

$$\Delta M = M_o + N^T M_s N \quad ; \quad \Delta K = K_o - K_{sp}^T K_s^{-1} K_{sp} \tag{9}$$

Note that, if the two subsystems are assumed individually classically damped, the matrix c is a diagonal one because of the matrices c_s and c_p are diagonal ones, and ΔK is a full matrix that becomes a zero one for secondary subsystems mono-connected to primary one.

It is to be emphasized that a quite different component-mode synthesis coordinate transformation with respect to the one previously described is required for seismically isolated structures. These structures can be considered as composed by a primary one (the base isolation system) and the secondary one (the isolated building). For such composed structure the damping of the isolator is usually referred to the total mass of the combined structures which is evaluated lumping the mass of the secondary system to the mass of the primary one. It follows that the coordinate transformation (4) as to be substituted by the following one:

$$u_t(t) = \overline{\Gamma}_t \overline{q}(t) \quad ; \quad \overline{\Gamma}_t = \begin{bmatrix} \Phi_s & N\overline{\Phi}_p \\ 0 & \overline{\Phi}_p \end{bmatrix} \tag{10}$$

with $\overline{\Phi}_p$ modal matrix solution of the following eigenproblem

$$\left(M_p + N^T M_s N\right)\overline{\Phi}_p \overline{\Omega}_p^2 = \left(K_p + \Delta K\right)\overline{\Phi}_p \tag{11}$$

normalised with respect to the matrix $\left(M_p + N^T M_s N\right)$. By using equation (10) we can rewrite equation (7) in the form:

$$\overline{m}\ddot{\overline{q}} + \overline{c}\dot{\overline{q}} + \overline{k}\overline{q} = -\overline{\Gamma}_t^T M_t \tau\, \ddot{u}_g(t) \tag{12}$$

where $\overline{m}$ is a full matrix while $\overline{c}$ and $\overline{k}$ two diagonal one, given respectively as follows

$$\overline{m} = \begin{bmatrix} I_{ms} & \Phi_s^T M_s N \overline{\Phi}_p \\ \overline{\Phi}_p N^T M_s \Phi_s & I_{mp} \end{bmatrix} \;;\; \overline{c} = \begin{bmatrix} c_s & 0 \\ 0 & \overline{c}_p \end{bmatrix} \;;\; \overline{k} = \begin{bmatrix} \Omega_s^2 & 0 \\ 0 & \overline{\Omega}_p^2 \end{bmatrix} \tag{13}$$

SOLUTION OF MOTION EQUATIONS

Equations (7) and (12) represent two sets of coupled differential equations which cannot be decoupled in other subspaces because of the relationships $\boldsymbol{k}\,\boldsymbol{m}^{-1}\boldsymbol{c} \neq \boldsymbol{c}\,\boldsymbol{m}^{-1}\boldsymbol{k}$ and $\bar{\boldsymbol{k}}\,\bar{\boldsymbol{m}}^{-1}\bar{\boldsymbol{c}} \neq \bar{\boldsymbol{c}}\,\bar{\boldsymbol{m}}^{-1}\bar{\boldsymbol{k}}$ hold (Caughey et al., 1965). It follows that the solution of equation (7) and (12) can be obtained by the $2m$-dimension state vector approach. By means of the approach equation (7) can be written as follows

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\tau\,\ddot{u}_g(t) \tag{14}$$

where:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} \;;\; \boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{m}^{-1}\boldsymbol{k} & -\boldsymbol{m}^{-1}\boldsymbol{c} \end{bmatrix} \;;\; \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{m}^{-1} \end{bmatrix} \Gamma_t^T \boldsymbol{M}_t \tag{15}$$

In order to obtain the solution of equation (14) it needs to evaluate the inverse of matrix $\boldsymbol{m}$. Due to the positive definite of matrix $\boldsymbol{m}$ we can calculate alternatively the inverse by means of the Cholesky decomposition of matrix $\boldsymbol{m}$ obtaining the lower triangular matrix $\boldsymbol{T}$ as:

$$\boldsymbol{T}\boldsymbol{T}^T = \boldsymbol{m} \tag{16}$$

It follows that by means of the following coordinate transformation:

$$\boldsymbol{q}(t) = \boldsymbol{T}^{-T}\boldsymbol{y}(t) \tag{17}$$

we can write equation (7) as:

$$\ddot{\boldsymbol{y}}(t) + (\Xi + \Delta\Xi)\dot{\boldsymbol{y}}(t) + (\Omega^2 + \Delta\Omega^2)\boldsymbol{y}(t) = -\boldsymbol{T}^{-1}\Gamma_t \boldsymbol{M}_t \tau\,\ddot{u}_g(t) \tag{18}$$

where Ξ and Ω^2 are diagonal matrices where elements are the elements on the principal diagonal of the matrices $\boldsymbol{T}^{-1}\boldsymbol{c}\,\boldsymbol{T}^{-T}$ and $\boldsymbol{T}^{-1}\boldsymbol{k}\,\boldsymbol{T}^{-T}$ respectively, while $\Delta\Xi$ and $\Delta\Omega^2$ are matrices having zero diagonal elements and the corresponding off-diagonal elements of $\boldsymbol{T}^{-1}\boldsymbol{c}\,\boldsymbol{T}^{-T}$ and $\boldsymbol{T}^{-1}\boldsymbol{k}\,\boldsymbol{T}^{-T}$ respectively. Notice that the inverse of the triangular matrix $\boldsymbol{T}$ can be evaluated in a close form solution once the matrix $\boldsymbol{T}$ is calculated. Equation (18) represents a set of second order coupled differential equations whose solution can be obtained by means of the $2m$ dimension state vector approach. For this purpose, introducing the vector $\boldsymbol{z}(t)$ of the 2m state variables equation (18), can be written a follows:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{D}\boldsymbol{z}(t) + \boldsymbol{V}\boldsymbol{f}(t) \tag{19}$$

where:

$$\boldsymbol{f}(t) = -\tau_t \ddot{u}_g(t) \;;\; \boldsymbol{D} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_m \\ -(\Omega^2 + \Delta\Omega^2) & -(\Xi + \Delta\Xi) \end{bmatrix} \;;\; \boldsymbol{z} = \begin{bmatrix} \boldsymbol{y} \\ \dot{\boldsymbol{y}} \end{bmatrix} \;;\; \boldsymbol{V} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{T}^{-1}\Gamma_t^T \boldsymbol{M}_t \end{bmatrix} \tag{20}$$

The vector solution $\boldsymbol{z}(t)$ of equation (19) can be written in integral form as follows (Muller et al., 1985; Borino et al., 1986):

$$\boldsymbol{z}(t) = \Theta(t - t_o)\,\boldsymbol{z}(t_o) + \int_{t_o}^{t} \Theta(t - \tau)\,\boldsymbol{V}\boldsymbol{f}(\tau)\,d\tau \tag{21}$$

where $\boldsymbol{z}(t_o)$ is the vector of initial condition in the modal state-vector space and $\Theta(t)$ is the so-called fundamental or transition matrix (Meirovitch., 1980; Muller et al., 1985), given respectively as follows:

$$\boldsymbol{z}(t_o) = \begin{bmatrix} \boldsymbol{T}^T\Gamma_t^{-1}\boldsymbol{u}_o \\ \boldsymbol{T}^T\Gamma_t^{-1}\dot{\boldsymbol{u}}_o \end{bmatrix} \;;\quad \Theta(t) = \Psi\,\exp[t\,\Lambda]\,\Psi^{-1} \;;\; \Gamma_t^{-1} = \begin{bmatrix} \Phi_s^T\boldsymbol{M}_s & -\Phi_s^T\boldsymbol{M}_s\boldsymbol{N} \\ \boldsymbol{0} & \Phi_p^T\boldsymbol{M}_p \end{bmatrix} \tag{22 a,b,c}$$

In equation (22 b) Ψ and Λ are complex matrices listing the eigenvectors and eigenvalues of $\boldsymbol{D}$ respectively, that is :

$$\boldsymbol{D}\,\Psi = \Psi\,\Lambda \tag{23}$$

In many cases of practical interest the convolution integral (21) cannot be computed in closed form and a numerical solution method has to must be applied. To this purpose let the time space be divided into small intervals of equal length Δt and let $t_0=0$, $t_1,\dots,t_{k-1},t_k,t_{k+1},..$ be the division times. By assuming that the equation (21) is satisfied at the above discrete time instants and adopting a piecewise linear forcing function vector in each interval, we can write the numerical solution of equation (21) as follows (Fiedler., 1986):

$$\boldsymbol{z}(t_{k+1}) = \Theta(\Delta t)\,\boldsymbol{z}(t_k) + \gamma_o(\Delta t)\boldsymbol{f}(t_k) + \gamma_1(\Delta t)\boldsymbol{f}(t_{k+1}) \tag{24}$$

where:

$$\gamma_o(\Delta t) = -\left\{\frac{1}{\Delta t}\left[\Theta(\Delta t) - \boldsymbol{I}\right] - \Theta(\Delta t)\boldsymbol{D}\right\}\boldsymbol{D}^{-2}\boldsymbol{V} \quad ; \quad \gamma_1(\Delta t) = \left\{\frac{1}{\Delta t}\left[\Theta(\Delta t) - \boldsymbol{I}\right] - \boldsymbol{D}\right\}\boldsymbol{D}^{-2}\boldsymbol{V} \tag{25 a,b}$$

Equation (21) gives an unconditionally stable step-by-step procedure where the only source of numerical errors is in the modelling the forcing function vector as a stepwise linear function (Fiedler., 1986). Once the response in terms of state variable is evaluated, we can evaluate the conventional response $\boldsymbol{u}_t(t)$ and $\dot{\boldsymbol{u}}_t(t)$ as follows:

$$\boldsymbol{u}_t(t) = \Gamma_t\,\boldsymbol{T}^{-T}\boldsymbol{y}(t) \quad ; \quad \dot{\boldsymbol{u}}_t(t) = \Gamma_t\,\boldsymbol{T}^{-T}\dot{\boldsymbol{y}}(t) \tag{26}$$

where $\boldsymbol{y}(t)$ and $\dot{\boldsymbol{y}}(t)$ are the first m and second m element of vector $\boldsymbol{z}$(t). Similar procedure can be obtained applying the previously described solution to equation (12).

APPROXIMATE EVALUATION OF THE TRANSITION MATRIX FOR COMPOSED STRUCTURES

The main computational drawback, in evaluating the response of equation (19), is the evaluation of complex eigenproperties of the matrix $\boldsymbol{D}$ required for the evaluation of the transition matrix. It is well known that the solution of the eigenproblem (23) represents the main computational difficulty in the analysis of composed structures. Recently it has been shown that the computation of complex eigenproperties of $\boldsymbol{D}$ can be avoided by directly evaluating the transition matrix in approximate form by means of Taylor expansion leading to a conditionally stable step-by-step procedure (Falsone et al., 1992). In this section an alternative method recently proposed (Muscolino et al., 1995) is here extended to evaluate in approximated form the transition matrix of composed structures such that the step-by-step procedure remains an unconditionally stable one. In order to do this, we remember that the transition matrix $\Theta(t)$ is the solution of the following differential equation (Muller et al., 1985) :

$$\dot{\Theta}(t) = \boldsymbol{D}\Theta(t) \quad ; \quad \Theta(0) = \boldsymbol{I} \quad ; \quad \Theta(t) = \exp\left[\boldsymbol{D}\,t\right] \tag{27 a,b,c}$$

It follows that one can represent the transition matrix as a solution of equations (27 a) by the matrix exponential function. Setting

$$\boldsymbol{D} = \boldsymbol{D}_d + \boldsymbol{D}_f \quad ; \quad \boldsymbol{D}_d = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\Omega^2 & -\Xi \end{bmatrix} \quad ; \quad \boldsymbol{D}_f = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ -\Delta\Omega^2 & -\Delta\Xi \end{bmatrix} \tag{28 a,b,c}$$

and substituting the matrix $\boldsymbol{D}$, in the form defined in equation (28 a), into differential equation (27 a) we can write:

$$\dot{\Theta}(t) = \boldsymbol{D}_d\Theta(t) + \boldsymbol{D}_f\Theta(t) \tag{29}$$

In this equation the second term on the right-hand side, which takes into account of the coupling terms, is considered as pseudo-force matrix. It follows that the integral solution of equation (29), with the initial condition (27 b), can written as follows:

$$\Theta(t) = \exp\left[t\,\boldsymbol{D}_d\right]\Theta(0) + \int_0^t \exp\left[(t-\tau)\,\boldsymbol{D}_d\right]\boldsymbol{D}_f\,\Theta(\tau)\,d\tau \tag{30}$$

By observing that $\exp\left[t\,\boldsymbol{D}_d\right]$ is the transition matrix of the composed structures, we can write a set of decoupled second order differential equations having damped matrix Ξ and stiffness matrix Ω^2 :

$$\exp\left[t\,\boldsymbol{D}_d\right] = \Theta_d(t) = \begin{bmatrix} -g(t)\Omega^2 & h(t) \\ -h(t)\Omega^2 & \dot{h}(t) \end{bmatrix} \tag{31}$$

where $g(t)$ and $h(t)$ are two diagonal matrices whose i-th element is given respectively as:

$$h_i(t) = \dot{g}_i(t) = \frac{1}{\bar{\omega}_i} e^{-\xi_i\omega_i t}\,sin(\bar{\omega}_i t) \quad ; \quad g_i(t) = -\frac{1}{\omega_i^2} e^{-\xi_i\omega_i t}\left[\cos(\bar{\omega}_i t) + \frac{\xi_i\omega_i}{\bar{\omega}_i} sin(\bar{\omega}_i t)\right] \tag{32 a,b}$$

in which $\omega_i = \sqrt{\Omega_{ii}^2}$, $\xi_i = \Xi_{ii}/2\omega_i$ and $\bar{\omega}_i = \omega_i\sqrt{1-\xi_i^2}$. The numerical step-by-step solution of equation (30) by assuming the elements of the transition matrix $\Theta(\tau)$ piecewise linear in each step, can be written in approximated form as follows (Muscolino et al., 1995):

$$\tilde{\Theta}(\Delta t) = \left[\boldsymbol{I} - \Gamma_1(\Delta t)\right]^{-1}\left[\Theta_d(\Delta t) + \Gamma_o(\Delta t)\right] \tag{33}$$

where $\tilde{\Theta}(\Delta t)$ is the approximation of the effective transition matrix defined in equation (30) and:

$$\Gamma_0(\Delta t) = \left[\Theta_d(\Delta t) - \frac{1}{\Delta t}\boldsymbol{L}_d(\Delta t)\right]\boldsymbol{D}_d^{-1}\boldsymbol{D}_f \quad ; \; \Gamma_1(\Delta t) = \left[\frac{1}{\Delta t}\boldsymbol{L}_d(\Delta t) - \boldsymbol{I}\right]\boldsymbol{D}_d^{-1}\boldsymbol{D}_f \tag{34}$$

$$\boldsymbol{L}_d(\Delta t) = \left[\Theta_d(\Delta t) - \boldsymbol{I}\right]\boldsymbol{D}_d^{-1} \tag{35}$$

By substituting the matrix $\Theta(\Delta t)$ given in equation (33) into equation (24), we can perform the numerical solution of equation (19) without solving any eingenproblem.

ACCURACY

In a very recent paper it has been shown that the numerical procedure based on the approximate transition matrix, evaluated in the previous section, leads to a unconditionally stable step-by-step procedure (Muscolino et al., 1995). Moreover it has been show that: i) a good accuracy of the numerical procedure is achieved assuming a time step lesser or equal to 1/8 of the minimum natural undamped period of the modes included in the modal analysis; ii) a bigger time steps, although does not imply stability problems, does not account for, in accurate form, the contributes to time history of the modes having period higher than $8\Delta t$. Extending these concepts to composed primary-secondary subsystems we can have a good accuracy if we known the lowest undamped period of the composite system exactly or alternatively in approximate form. This period can be evaluated once the highest eigenvalue of the matrix $\boldsymbol{G} = \Omega^2 + \Delta\Omega^2$ is knows. Obviously, for composite structures, having detuned eigenvalues for which the undamped eigenvalues of the composite system does not differ very much from the corresponding eigenvalues of the two subsystems we can evaluate the step of numerical procedure equal to 1/8 of the lowest period of the two substructures separately taken. In the other cases to have a good accuracy we have to evaluate the highest eigenvalue of the matrix $\boldsymbol{G}$. Approximately this eigenvalue can be evaluate by using the Georschgorin theorem (Fiedler., 1986) and consequently the step of the numerical procedure can be evaluated as follows:

$$\Delta t \le \pi\Big/\left(4\sqrt{\max(\bar{\omega}_i^2)}\right) \quad ; \quad \max(\bar{\omega}_i^2) \le \max\left(\Omega_{ii}^2 + \sum_{j\ne i}\left|\Delta\Omega_{ij}^2\right|\right) \tag{36}$$

The accuracy tests are performed considering the structures depicted in Fig. 1 subjected to a sinusoidal seismic input. In particular the composite structure of Fig. 1.a is composed by a three degree-of-freedom primary substructure having $m_p = m_4 = m_5 = m_6 = 500 \ N\sec^2/cm$, $k_p/2 = 40000\,N/cm$, damping ratio for all modes $\xi_p = 0.05$ and by a three degree-of-freedom light secondary substructure having $m_s = m_1 = m_2 = m_3 = 4$ $N\sec^2/cm$, $k_s = 100\,N/cm$, damping ratio for all modes $\xi_s = 0.05$. The undamped natural circular frequencies are $\omega_{1p} = 5.63$ rad/sec, $\omega_{2p} = 15.77$ rad/sec, $\omega_{3p} = 22.79$ rad/sec, $\omega_{1s} = 3.83$ rad/sec, $\omega_{2s} = 7.07$ rad/sec, $\omega_{3s} = 9.24$ rad/sec. In the numerical test the seismic circular frequencies has been assumed equals to $\omega_f = 4$ rad/sec and $\omega_f = 6$ rad/sec. The corresponding percentage errors of the largest peak of the response of the structural masses evaluated by means of the proposed procedure are reported in table I. In Figure 2.a are depicted the responses evaluated by means of proposed numerical procedure and the exact one for the masses which present in table I the highest percentage errors. In Figure 1.b is depicted the isolated structure of the second example studied in this section. In particular the structure is a three degree-of-freedom with $m_s = m_1 = m_2 = m_3 = 500 \ N\sec^2/cm$, $k_s/2 = 40000\,N/cm$, damping ratio for all modes $\xi_s = 0.05$, $\omega_{1s} = 5.63$ rad/sec $\omega_{2s} = 15.77$ rad/sec, $\omega_{3s} = 22.79$ rad/sec while the isolator system has $m_I = 2.5 \ N\sec^2/cm$, $k_I = 8000\,N/cm$, $\xi_I = 0.1$. The numerical tests has been performed for $\omega_f = 2.5$ rad/sec and $\omega_f = 6$ rad/sec. The percentage errors hare reported in table II while in Fig. 2.b hare depicted the exact and approximate response in the two most unfavourable cases. The numerical tests described here show the great accuracy of the proposed procedure.

CONCLUSIONS

A numerical procedure has been presented in order to evaluate the seismic response of combined primary-secondary systems via component-mode synthesis method. It is well known that this method requires the solution of two eigenproblems with real solution and an eigenproblem with complex solution, the later in the reduced subspace, after the application of the so-called admissible coordinate transformation. The proposed procedure avoids the complex eigensolution evaluating the fundamental operator of the step-by-step solution in approximate form. The great accuracy of the method, which is unconditionally stable, has been shown in the accuracy section.

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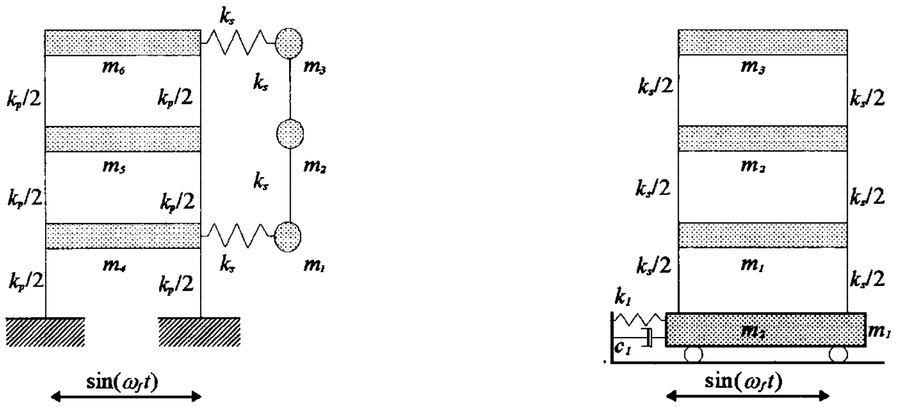


(*a*) (*b*)

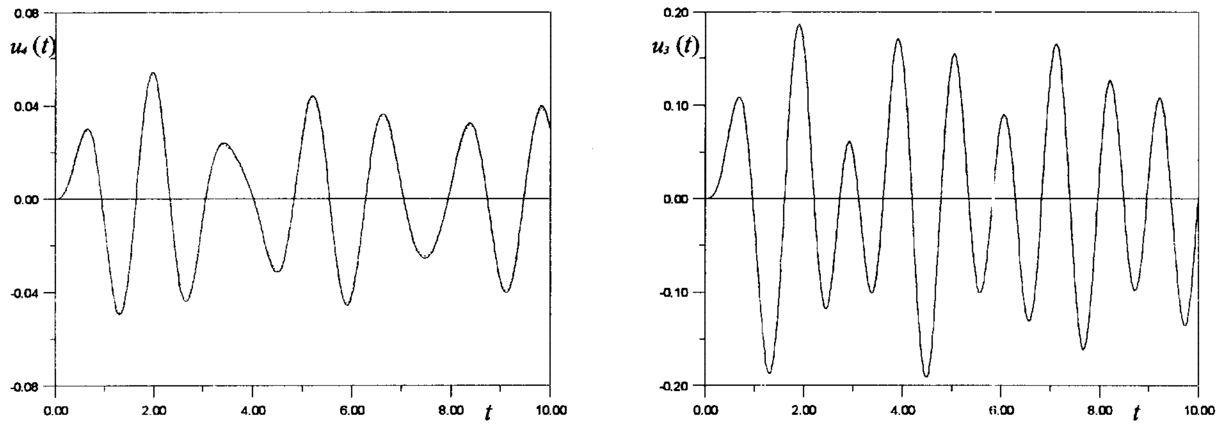
Fig. 1. Structures for accuracy tests; *a*) composite structure; *b*) isolated structure

Tab. I. Percentage errors of the largest peak of the responses of the composite structure

Mass	ω_f = 4 rad/sec	ω_f = 6 rad/sec
1	0.0968	0.1882
2	0.1300	0.0316
3	0.1575	0.7212
4	1.5680	0.4546
5	0.2130	0.0660
6	0.4698	0.1222

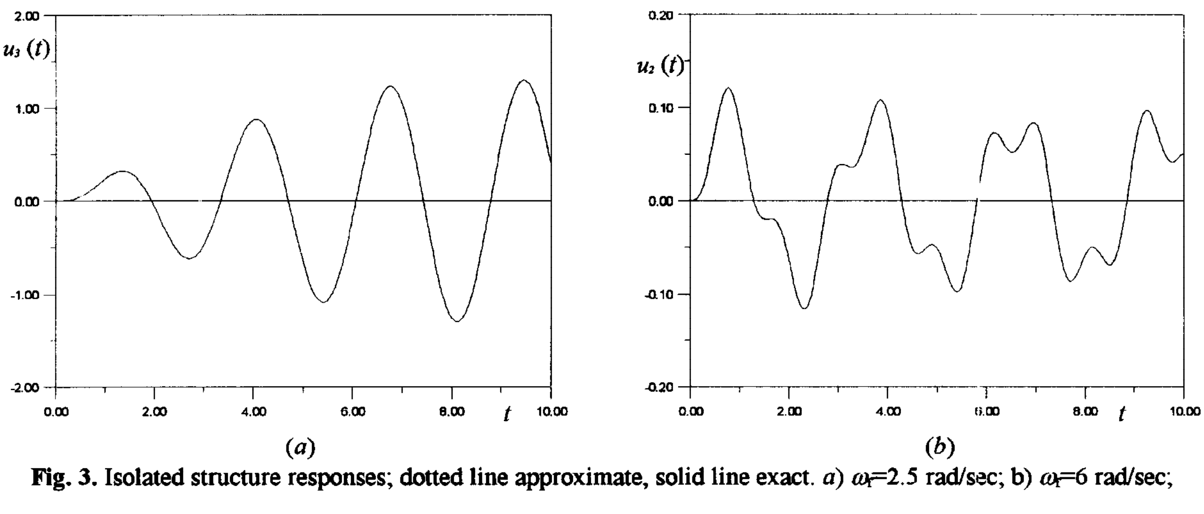
Tab. II. Percentage errors of the largest peak of the responses of the isolated structure

Mass	ω_f = 2.5 rad/sec	ω_f = 6 rad/sec
1	0.1136	0.0278
2	0.1179	0.0329
3	0.1191	0.0269
4	0.1078	0.0143



(*a*) (*b*)

Fig. 2. Composite structures responses: dotted line approximate, solid line exact. *a*) ω_f=4 rad/sec; b) ω_f=6 rad/sec;



(*a*) (*b*)

Fig. 3. Isolated structure responses; dotted line approximate, solid line exact. *a*) ω_f=2.5 rad/sec; b) ω_f=6 rad/sec;