Paper No. 35. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

PRINCIPAL/SECONDARY UPPER BOUND AND EQUIVALENT STATIC SOLUTION IN SEISMIC DESIGN

L.J. WANG
Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, USA

and

Q.X. WANG
Institute of Engineering Mechanics, State Seismological Bureau, Harbin 150080, P.R. China

ABSTRACT

The second author has recently proposed an innovative method to calculate upper bounds on the seismic response of structures with stiffness uncertainty. It has been shown that there exist two upper bounds on the seismic response, the principal upper bound (PUB) and the secondary upper bound (SUB), and two associated modes, the principal mode (PM) and the secondary mode (SM), which correspond to the two most unfavorable stiffness distributions. The PM and SM have similar shapes to the conventional first and second modes yet are not identical to them. Although a complete but non-unique set of modes can be generalized by the orthogonality condition from the PM and SM, the higher order modes (third, fourth, and so on) have zero modal participation factors and thus do not participate in modal response. The seismic design of a complicated multi-degree-of-freedom system can therefore be made on the basis of a simplified two-degree-of-freedom system.

Through several examples in this paper, it is further shown that the difference between the PUB and SUB is just the equivalent static solution (ESS) if the acceleration response spectrum is taken as the input ground motion. A formula for calculating the PUB and SUB of the seismic bending moment for high-rise buildings is provided.

KEYWORDS

Modal participation factor; modal analysis; principal and secondary upper bounds of seismic response; principal and secondary modes; response spectrum.

INTRODUCTION

An innovative method was proposed to calculate upper bounds on the seismic response on the structures with stiffness uncertainty (Wang *et al.*, 1990, 1991, 1993, 1994). For a spatial elastic system with lumped masses, the response of the internal force F under an earthquake is (Wang *et al.*, 1994)

$$F = [\sum_i m_i \sum_j I_{ij} d_{ji} \eta] A = \psi(d) A \tag{1}$$

where m_i is the lumped mass at node i, d_{ji} is the modal displacement of node i in j direction, I_{ij} is the influence coefficient of node i in j direction, A is the acceleration response spectrum, and η is the modal participation factor (MPF) given by

$$\eta = \frac{\sum_i m_i d_{ji}}{\sum_i m_i \sum_j d_{ji}^2} \tag{2}$$

Note that the stiffness of the system does not enter (1). Note also that the modal response of the internal force F is only a function of modal displacement d_{ji} if the acceleration response spectrum A is known. The upper bounds on F can therefore be found by maximizing $\psi(d)$ with respect to d.

It has been shown that (Wang *et al.*, 1990) there exist two upper bounds on the seismic response -- the principal upper bound (PUB) and the secondary upper bound (SUB), and two associated modes -- the principal mode (PM) and the secondary mode (SM), which correspond to the two most unfavorable stiffness distributions. The PM and SM have similar shapes to the conventional first and second modes yet are not identical to them. Although a complete but non-unique set of modes can be generalized by the orthogonality condition from the PM and SM, the higher order modes (third, fourth, and so on) have zero MPFs and thus do not participate in modal response. The seismic design of a complicated multi-degree-of-freedom system can therefore be made on the basis of a simplified two-degree-of-freedom system.

Through several examples in this paper, it is further shown that the difference between the PUB and SUB is just the equivalent static solution (ESS) if the acceleration response spectrum is taken as the input ground motion.

EXAMPLE I: SIMPLE LUMPED MASS SYSTEM

Consideration is first given to a simple two dimensional lumped mass system with unknown stiffness (see Figure 1). From (1), the modal responses of the shear force Q and bending moment M are

$$\left.\begin{aligned} Q_{01} &= \eta(-m_1 u_1 - m_2 u_2)A \\ Q_{12} &= \eta(-\tfrac{4}{5} m_2 u_2 + \tfrac{3}{5} m_2 v_2)A \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} M_0 &= \eta(-4m_1 u_1 - 8m_2 u_2 + 3m_2 v_2)hA \\ M_1 &= \eta(-4m_2 u_2 + 3m_2 v_2)hA \end{aligned}\right\} \tag{4}$$

where u_i and v_i are the horizontal and vertical modal displacements at node i, and h is the unit of length. Under horizontal and vertical earthquakes, the MPFs are

$$\eta_u = \frac{m_1 u_1 + m_2 u_2}{m_1 u_1^2 + m_2 u_2^2 + m_2 v_2^2} \tag{5}$$

$$\eta_v = \frac{m_2 v_2}{m_1 u_1^2 + m_2 u_2^2 + m_2 v_2^2} \tag{6}$$

Allowing small deformations only, v_i can be expressed in terms of the independent coordinate u_i

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 3/4 & -3/4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{7}$$

Let $u_1 = x$ and $u_2 = 1$. Then, if $m_1 = m_2 = m$

$$\eta_u = \frac{16(x+1)}{25x^2 - 18x + 25} \tag{8}$$

$$\eta_v = \frac{12(x-1)}{25x^2 - 18x + 25} \tag{9}$$

If $m_1=2m$ and $m_2=m$

$$\eta_u = \frac{16(2x+1)}{41x^2-18x+25} \tag{10}$$

$$\eta_v = \frac{12(x-1)}{41x^2-18x+25} \tag{11}$$

The upper bounds on the shear force Q and bending moment M can be obtained by substituting (8)-(11) into (3) and (4) and finding the extrema of Q and M (see Table 1).

The same result can be reached by choosing different independent coordinates, for example, the increments of slopes of bars 0-1 and 1-2 in Figure 1, e_1 and e_2.

u and v may be expressed in terms of e_1 and e_2 as

$$\begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{Bmatrix} = h \begin{bmatrix} 4 & 0 \\ 4 & 4 \\ 0 & 0 \\ 0 & -3 \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} \tag{12}$$

Let $e_1=y$ and $e_2=1$, then the MPFs become

$$\eta_u = \frac{8y+4}{h(32y^2+32y+25)} \qquad \left(\frac{m_1}{m_2}=1\right) \tag{13}$$

$$\eta_v = \frac{-3}{h(32y^2+32y+25)} \qquad \left(\frac{m_1}{m_2}=1\right) \tag{14}$$

$$\eta_u = \frac{12y+4}{h(48y^2+32y+25)} \qquad \left(\frac{m_1}{m_2}=2\right) \tag{15}$$

$$\eta_v = \frac{-3}{h(48y^2+32y+25)} \qquad \left(\frac{m_1}{m_2}=2\right) \tag{16}$$

Similarly, the upper bounds on the shear force Q and bending moment M can be obtained by substituting (13)-(16) into (3) and (4) and finding the extrema of Q and M (see Table 2).

The ESSs of shear force and bending moment are listed in Tables 1 and 2 for comparison. The difference between the PUB and SUB is just the ESS.

EXAMPLE II: BUILDING FRAME

The interstory shear force Q and bending moment M in the building frame with unknown stiffness (see Figure 2) may be solved as

$$\begin{Bmatrix} Q_{BC} \\ Q_{CD} \\ Q_{DF} \\ Q_{FG} \end{Bmatrix} = \begin{bmatrix} -4/5 & -36/125 & -48/125 & 0 & 0 \\ 3/5 & 27/125 & 36/125 & 0 & 0 \\ -4/3 & -4/3 & 0 & -4/3 & -4/3 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{Bmatrix} X_B \\ X_C \\ Y_C \\ X_D \\ X_F \end{Bmatrix} \tag{17}$$

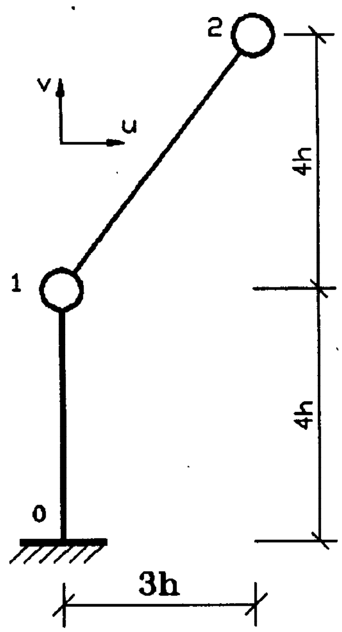


Figure 1. A folded cantilever in example I

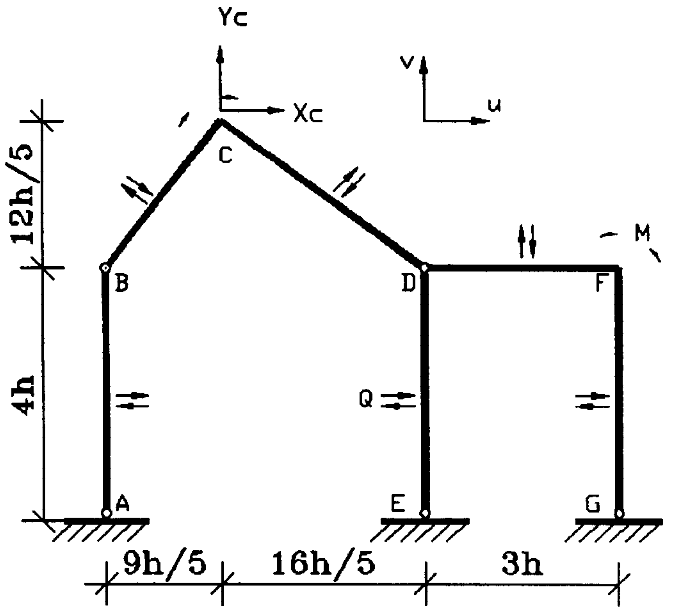
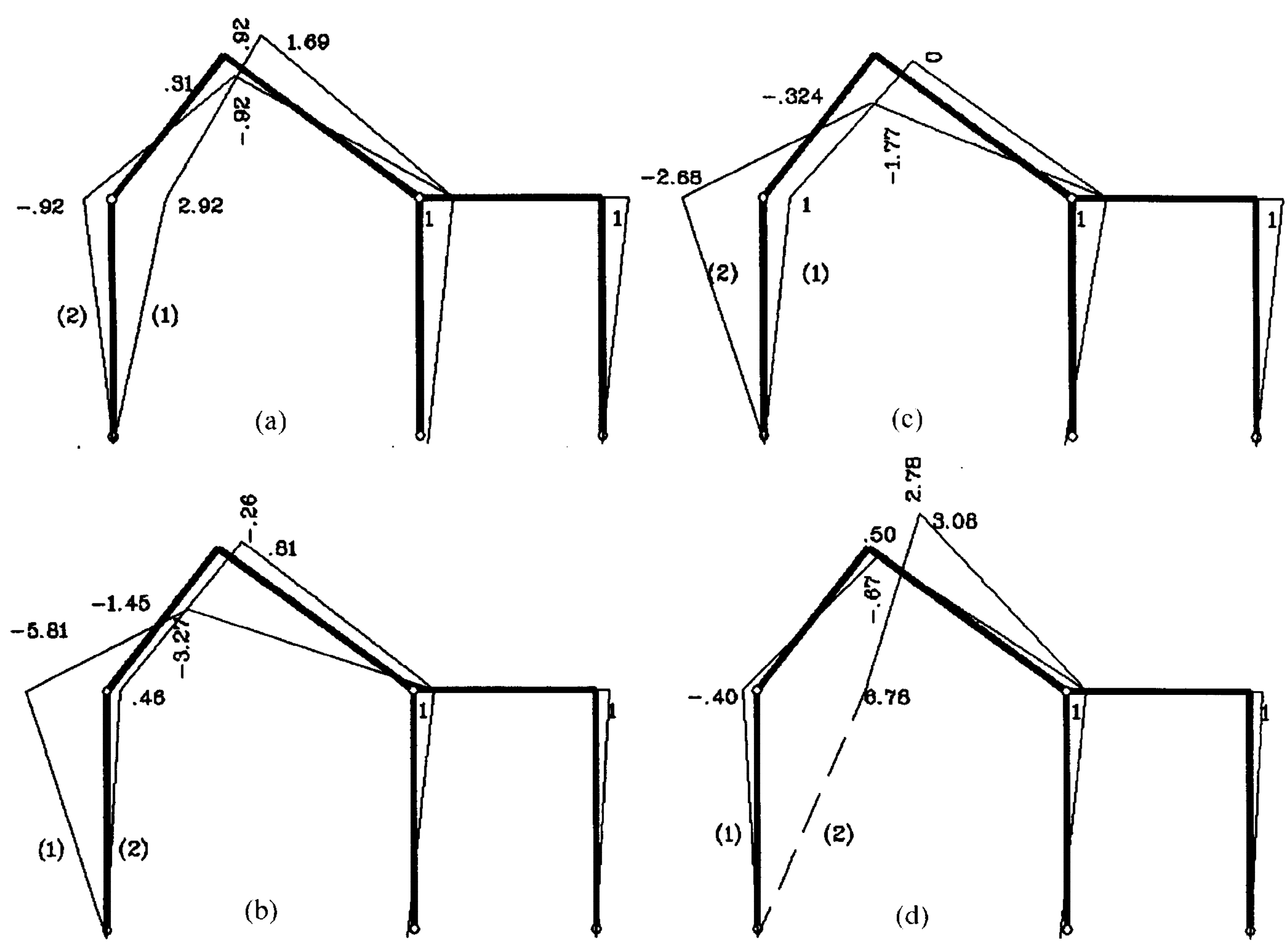


Figure 2. A building frame in example II



(a) and (b): PM and SM for QBC, QCD and MC under horizontal and vertical earthquakes
(c) and (d): PM and SM for QDF, QFG and MF under horizontal and vertical earthquakes
Figure 3. Principal and secondary modes in example II

Table 1. Result summary of example I (part 1)

m_1/m_2	Quake	Internal Forces	Product Function	x_1	x_2	PUB	SUB	ESS
1	horizontal	Q_{01}	$16(x+1)^2/F_1(x)$	1	-1	2	0	2
		Q_{12}	$0.8(x+1)(9x-25)/F_1(x)$	0.469	-7.612	1.107	0.307	0.8
		M_0	$4(x+1)(7x+41)/F_1(x)$	0.727	-2.009	12.671	0.671	12
		M_1	$4(x+1)(9x-25)/F_1(x)$	0.469	-7.612	5.536	1.536	4
1	vertical	Q_{01}	$12(x-1)(x+1)/F_1(x)$	0.186	5.369	0.515	0.515	0
		Q_{12}	$0.6(x-1)(9x-25)/F_1(x)$	-0.377	1.540	0.664	0.064	0.6
		M_0	$3(x-1)(7x+41)/F_1(x)$	-0.046	2.505	4.932	1.932	3
		M_1	$3(x-1)(9x-25)/F_1(x)$	-0.377	1.540	3.319	0.319	3
2	horizontal	Q_{01}	$16(2x+1)^2/F_2(x)$	1	-0.5	3	0	3
		Q_{12}	$0.8(2x+1)(9x-25)/F_2(x)$	0.419	-2.593	1.266	0.466	0.8
		M_0	$4(2x+1)(23x+41)/F_2(x)$	0.713	-0.919	16.874	0.874	16
		M_1	$4(2x+1)(9x-25)/F_2(x)$	0.419	-2.593	6.330	2.330	4
2	vertical	Q_{01}	$12(x-1)(2x+1)/F_2(x)$	-36.635	0.235	0.586	0.586	0
		Q_{12}	$0.6(x-1)(9x-25)/F_2(x)$	-0.215	1.513	0.638	0.038	0.6
		M_0	$3(x-1)(23x+41)/F_2(x)$	0.065	3.852	4.966	1.966	3
		M_1	$3(x-1)(9x-25)/F_2(x)$	-0.215	1.513	3.191	0.191	3

$F_1(x) = 25x^2 - 18x + 25$

$F_2(x) = 41x^2 - 18x + 25$

Table 2. Result summary of example I (part 2)

m_1/m_2	Quake	Internal Forces	Product Function	y_1	y_2	PUB	SUB	ESS
1	horizontal	Q_{01}	$16(2y+1)^2/G_1(y)$	*	-0.5	2	0	2
		Q_{12}	$0.8(2y+1)(16y+25)/G_1(y)$	0.884	-0.884	1.107	0.307	0.8
		M_0	$4(2y+1)(48y+41)/G_1(y)$	2.668	-0.668	12.671	0.671	12
		M_1	$4(2y+1)(16y+25)/G_1(y)$	0.884	-0.884	5.536	1.536	4
1	vertical	Q_{01}	$12(2y+1)/G_1(y)$	0.229	-1.229	0.515	0.515	0
		Q_{12}	$0.6(16y+25)/G_1(y)$	-0.274	-2.851	0.664	0.064	0.6
		M_0	$3(48y+41)/G_1(y)$	-0.044	-1.665	4.932	1.932	3
		M_1	$3(16y+25)/G_1(y)$	-0.274	-2.851	3.319	0.319	3
2	horizontal	Q_{01}	$16(3y+1)^2/G_2(y)$	*	-0.333	3	0	3
		Q_{12}	$0.8(3y+1)(16y+25)/G_2(y)$	0.772	-0.722	1.266	0.466	0.8
		M_0	$4(3y+1)(64y+41)/G_2(y)$	2.479	-0.479	16.874	0.874	16
		M_1	$4(3y+1)(16y+25)/G_2(y)$	0.722	-0.722	6.330	2.330	4
2	vertical	Q_{01}	$12(3y+1)/G_2(y)$	0.306	-0.973	0.586	0.586	0
		Q_{12}	$0.6(16y+25)/G_2(y)$	-0.177	-2.948	0.638	0.038	0.6
		M_0	$3(64y+41)/G_2(y)$	0.069	-1.351	4.966	1.966	3
		M_1	$3(16y+25)/G_2(y)$	-0.177	-2.948	3.191	0.191	3

$G_1(y) = 32y^2 + 32y + 25$

$G_2(y) = 48y^2 + 32y + 25$

* very large number

$$\begin{Bmatrix} M_C \\ M_F \end{Bmatrix} = h\begin{bmatrix} -12/5 & -108/125 & -144/125 & 0 & 0 \\ -4 & -4 & 0 & -4 & -4 \end{bmatrix}\begin{Bmatrix} X_B \\ X_C \\ Y_C \\ X_D \\ X_F \end{Bmatrix} \tag{18}$$

where X_i and Y_i are the horizontal and vertical loads at node i. The MPFs under horizontal and vertical earthquakes are

$$\eta_u = \frac{m_B u_B + m_C u_C + m_D u_D + m_F u_F}{m_B u_B^2 + m_c(u_C^2 + v_C^2) + m_D u_D^2 + m_F u_F^2} \tag{19}$$

$$\eta_v = \frac{m_C v_C}{m_B u_B^2 + m_c(u_C^2 + v_C^2) + m_D u_D^2 + m_F u_F^2} \tag{20}$$

Expressing u_C, v_C, and u_F in terms of the independent coordinates u_B and u_D gives

$$\begin{Bmatrix} u_C \\ v_C \\ u_F \end{Bmatrix} = \begin{bmatrix} 0.36 & 0.64 \\ 0.48 & -0.48 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} u_B \\ u_D \end{Bmatrix} \tag{21}$$

Let $m_B=m_C=m_F=m$ and $m_D=2m$, and let $u_B=z$ and $u_D=1$, then (19) and (20) become

$$\eta_u = \frac{(1.36z + 3.64)}{1.36z^2 + 3.64} \tag{22}$$

$$\eta_v = \frac{(0.48z - 0.48)}{1.36z^2 + 3.64} \tag{23}$$

Replacing X_i and Y_i by $m_i u_i A$ and $m_i v_i A$, the PUB and SUB on the interstory shear force and bending moment are determined by the extremum condition of Q and M (see Table 3).

It is evident that the difference between the PUB and SUB is just the ESS. The PUB and SUB are orthogonal to each other and are shown in Figure 3.

Table 3. Result summary of example II

Quake	Internal Forces	Product Function	z_1	z_2	PUB	SUB	ESS
horizontal	Q_{BC}	$1.088z(1.36z+3.64)/H(z)$	2.917	-0.917	1.587	0.499	1.088
	Q_{CD}	$0.816z(1.36z+3.64)/H(z)$	2.917	-0.917	1.190	0.374	0.816
	M_C	$3.264z(1.36z+3.64)/H(z)$	2.917	-0.917	4.761	1.497	3.264
	Q_{DF}	$1.333(1.36z+3.64)^2/H(z)$	1	-2.677	6.667	0	6.667
	Q_{FG}	$(1.36+3.64)^2/H(z)$	1	-2.677	5	0	5
	M_F	$4(1.36z+3.64)^2/H(z)$	1	-2.677	20	0	20
vertical	Q_{BC}	$1.088z(0.48z-0.48)/H(z)$	-5.813	0.460	0.417	0.033	0.384
	Q_{CD}	$0.816z(0.48z-0.48)/H(z)$	-5.813	0.460	0.313	0.025	0.288
	M_C	$3.264z(0.48z-0.48)/H(z)$	-5.813	0.460	1.251	0.099	1.152
	Q_{DF}	$1.333(1.36z+3.64)(0.48z-0.48)/H(z)$	-0.395	6.781	0.719	0.719	0
	Q_{FG}	$(1.36z+3.64)(0.48z-0.48)/H(z)$	-0.395	6.781	0.539	0.539	0
	M_F	$4(1.36z+3.64)(0.48-0.48)/H(z)$	-0.395	6.781	2.157	2.157	0

$H(z) = 1.36z^2 + 3.64$

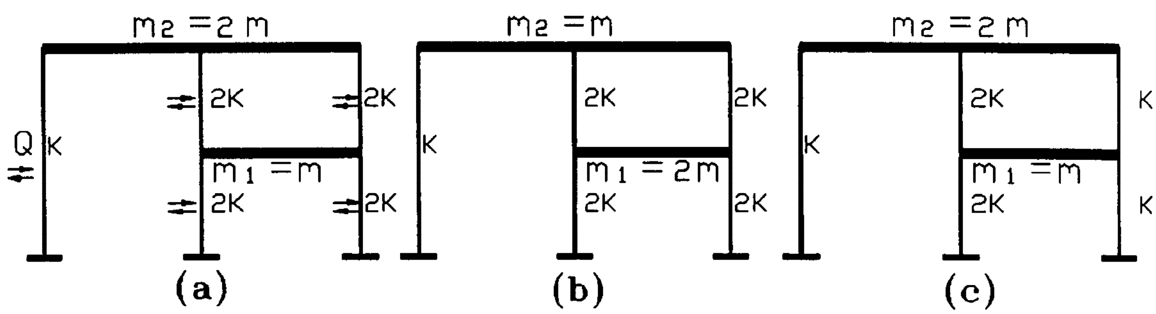


Figure 4. An irregular two-story frame

EXAMPLE III: IRREGULAR TWO-STORY FRAME

The interstory shear force Q of the irregular two-story frame with known stiffness in Figure 4a under a horizontal floor load X may be solved as

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{bmatrix} 1/6 & 1/3 \\ -1/6 & 2/3 \\ 5/6 & 2/3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \tag{24}$$

Let $u_1=1$ and $u_2=p$, then the MPF is

$$\eta = \frac{m_1 u_1 + m_2 u_2}{m_1 u_1^2 + m_2 u_2^2} = \frac{2p+1}{2p^2+1} \tag{25}$$

In the case of Figure 4b, the Q-X relation is (24), but the MPF is

$$\eta = \frac{p+2}{p^2+2} \tag{26}$$

In the case of Figure 4c, the MPF is (25), but the Q-X relation is

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{bmatrix} 1/7 & 3/7 \\ -1/7 & 4/7 \\ 6/7 & 4/7 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \tag{27}$$

Replacing X_i by $m_i u_i A$, the PUB and SUB on the interstory shear force can be obtained by maximizing Q with respect to p (see Table 4). Again, the difference between the PUB and SUB is just the ESS.

Table 4. Result summary of example III

Figure 4	Internal Forces	Product Function	p_1	p_2	PUB	SUB	ESS	First Mode
(a)	Q_1	$(2p+1)(4p+1)/6(2p^2+1)$	1.366	-0.366	0.849	0.016	0.833	0.844
	Q_2	$(2p+1)(8p-1)/6(2p^2+1)$	3.158	-0.158	1.413	0.246	1.167	1.360
	Q_3	$(2p+1)(8p+5)/6(2p^2+1)$	0.893	-0.560	2.173	0.006	2.167	2.018
(b)	Q_1	$(p+2)(p+1)/3(p^2+2)$	1.414	-1.414	0.687	0.020	0.667	0.681
	Q_2	$(p+2)(2p-1)/3(p^2+2)$	4.450	-0.450	0.779	0.446	0.333	0.428
	Q_3	$(p+2)(2p+5)/3(p^2+2)$	0.897	-2.230	2.339	0.006	2.333	2.298
(c)	Q_1	$(2p+1)(6p+1)/7(2p^2+1)$	1.569	-0.319	1.039	0.039	1	1.017
	Q_2	$(2p+1)(8p-1)/7(2p^2+1)$	3.158	-0.158	1.211	0.211	1	1.206
	Q_3	$(2p+1)(8p+6)/7(2p^2+1)$	0.814	-0.614	2.020	0.020	2	1.659

CONCLUSIONS

The PUB and SUB on the bending moment M (similar result applies to the interstory shear forces Q) for high-rise buildings are

$$(M_i)_{PUB} = \frac{A}{2}\left(\sqrt{\sum_{j=1}^{n} m_j \sum_{j=i+1}^{n} m_j \Delta h_{ij}^2} + \sum_{j=i+1}^{n} m_j \Delta h_{ij}\right) \tag{28}$$

$$(M_i)_{SUB} = \frac{A}{2}\left(\sqrt{\sum_{j=1}^{n} m_j \sum_{j=i+1}^{n} m_j \Delta h_{ij}^2} - \sum_{j=i+1}^{n} m_j \Delta h_{ij}\right) \tag{29}$$

where A is the acceleration response spectrum, m_j is the lumped mass at story j, and Δh_{ij} is the interstory height between stories i and j.

Subtracting (29) from (28) yields

$$(M_i)_{PUB} - (M_i)_{SUB} = A \sum_{j=i+1}^{n} m_j \Delta h_{ij} = (M_i)_{ESS} \tag{30}$$

Hence, the difference between the PUB and SUB on the interstory shear force and bending moment is just the ESS.

REFERENCES

Wang, Q.X. and C.G. Liu (1990). Seismic loads on supporting frames of gas pipelines. *ASME-PVP*, Vol. 187, pp35-46, Nashville, USA.

Wang, Q.X. and C.G. Liu (1991). Seismic loads on structures with unidentified stiffness distribution. *SMiRT-11 Transaction*, Vol. K, pp. 237-242, Tokyo, Japan.

Wang, Q.X. and Y.H. Zhang (1993). Upper bound seismic loads on high rise structures. *Proceedings of Workshop on Seismic Design Code*, China.

Wang, Q.X. and Y.H. Zhang (1994). Seismic margin identification of engineering structures. *ASME-PVP*, Minneapolis, USA.