ALGORITHM FOR SEISMIC HYDRODYNAMIC RESPONSE SPECTRA

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ABSTRACT

Numerical methods are discussed to compute the hydrodynamic seismic response using the adimensional Kotsubo-Bessel integral due to the water compressibility; it is similar to the Duhamel integral for the structural response, and allows to obtain the Fourier Series pressure participation coefficient at the vertical dam face of a semi-infinite rectangular reservoir. Only maximum values of the time dependent coefficients series are presented. It is proposed that the Bessel function kernel effect can be obtained related to the undamped velocity structural response. It is stated that water does not have any equivalent linear clasical damping due to the non linear Bessel differential equation. This shows how water alone, a saturated sand or a saturated compressible soil, with pore pressure, stores and amplifies the seismic response as in zones of subsoil in México City or in coastal Cities with a bay.

KEY WORDS

hydrodynamic, pressure, compressibility, fourier series, bessel functions, algorithm, response spectra

HYDRODYNAMIC PRESSURE DURING EARTHQUAKES

Numerical methods are discussed to compute the hydrodynamic adimensional Kotsubo-Bessel integral (Kotsubo, 1959)

$$S(w\cdot t)=\int_{o}^{t}[1/g]\cdot A\ (\ t-u\ )\cdot Jo(\ w\cdot u\ )\ w\cdot du \qquad (1)$$

where A = seismic horizontal acceleration of the dam base
H = vertical depth of the reservoir
Jo= Bessel function of zero order
a = 1438.4456 m/seg = velocity of sound in water
g = 9.81 m/seg² = gravity constant
n = 1, 2, 3, ...
t = time
u = integer variable
α = máximun acceleration seismic factor related to g
w= wn= $\mu n\cdot a/H$= natural reservoir oscilation frequency
$\mu n = \pi\cdot(\ 2\cdot n - 1\ )/2$
π = 3.141592654

which allows to obtain the seismic pressure p(z,t) on a vertical dam face at depth z using the series:

$$p(z,t)/(\alpha \cdot \tau o \cdot H) = \sum_{n=1}^{\infty} \alpha n/\mu n \cdot S(wn \cdot t) \cdot \cos(\mu n \cdot z/H) \qquad (2)$$

where z = resevoir depth

$\alpha n = 2(-1)^{n+1}$

τo= volumetric weight of water

Methods

In ec (1), $w = 2 \cdot \pi/T$ is the natural circular frequency and T is the natural circular period of the liquid media, in seconds; Jo( ) can be expressed as: (Abramowitz, et al, 1965)

$$Jo(z) = 1 - \frac{(\frac{1}{4}z^2)}{(1!)^2} + \frac{(\frac{1}{4}z^2)^2}{(2!)^2} - \frac{(\frac{1}{4}z^2)^3}{(3!)^2} + \cdots = \sum_{k=o}^{\infty} [-\tfrac{1}{4}z^2]^k/(k!)^2 \qquad (3)$$

and with asymptotic approximations when z is large enough. However the use of eq. (3) to compute eq. (1) is cumbersome because it is necessary to repeat the computation for each t. This happens because Bessel is a non linear differential equation

$$\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} + w^2y = o \qquad (4)$$

and its homogeneous solution is:

$$y = C1 \cdot Jo(wt) + C2 \cdot Yo(wt) \qquad (5)$$

where C1 and C2 are constants and Yo() is the Bessel Function of second kind of a zero order. This solution is the kernel of eq. (1) with C2= o and this eq. (4) seems to have the equivalent non linear damping effect of $1/(2 \cdot w \cdot t)$, altough Newmark and Rosenblueth (1971) consider that water in dams seems to behave with a classical linear structural damping between .1 to .2 on actual earthquakes despite the term 1/t in eq. (4).

Direct integration algorithms

A numerical method to compute eq. (1) is to use linear variation in A( ) between two consecutive earthquake instants as a pulse and compute his mathemathical effect on t. The results obtained using digital computers available in 1965 are indicated, with xS in Table 1, and obtained by the author and published by Newmark and Rosenblueth (1971). Other method, and used here, is to compute the effect of a linear pulse with

$$\delta S = \delta t \cdot [\ Ai \cdot (Jo<i> + 2 \cdot Jo<i+\tfrac{1}{2}>) + Aj \cdot (2 \cdot Jo<i+\tfrac{1}{2}> + Jo<j>)\ ]/6 \qquad (6)$$

where Ai and $Jo<i>$ are computed at time ti, Aj and $Jo<j>$ at time ti+1 and $Jo<i+\tfrac{1}{2}>$ is obtained at half interval. Also $\delta t = ti+i - ti$ is not greater than $w \cdot \delta t = \pi/32$. For this computation a personal computer was used (1995). Results obtained for the first 12 sec data of El Centro earthquake, may 18 of 1940, are in Table 1 as Smax. The values obtained with each method are similar.

Double integration methods

It is proposed here to compute eq. (1) using an integral of the type:

$$Jo(\sqrt{(z^2-\underline{a}^2)})=\frac{2}{\pi}\int_o^{\frac{1}{2}\pi}\cosh(\ \underline{a}\cdot\cos\Theta\ )\cdot\cos(\ z\cdot\sin\Theta\ )\cdot d\Theta \qquad (7)$$

McLachan(1961) p 191, to apply any usual computational method for linear structural response of Duhamel integral which has sin() in eq. (1) instead of Jo. Thus, from eq.(7) with $\underline{a}$=o it can be used in eq. (1)

$$Jo(z)=\frac{2}{\pi}\int_o^{\frac{1}{2}\pi}\cos(\ z\cdot\sin\Theta\ )\cdot d\Theta = \frac{2}{\pi}\int_o^{\frac{1}{2}\pi}\cos(\ z\cdot\cos\Theta\ )\cdot d\Theta \qquad (8)$$

$$Jo(z)=\frac{2}{\pi}\int_o^{1}\frac{\cos(\ z\cdot x\ )}{\sqrt{(1-x^2)}}\,dx \qquad = \frac{2}{\pi}\int_1^{\infty}\frac{\cos(\ z\cdot v\ )}{\sqrt{(v^2-1)}}\,dv \qquad (9)$$

(8a) and (9a) are related using $x=\sin\Theta$ and $\sqrt{(1-x^2)}=\cos\Theta$.

Thus, with $z=w\cdot u$ in eq. (8) then eq. (1) can be stated as

$$S(w\cdot t)=2/\pi g\int_o^{t}A(\ t\ -\ u\ )\cdot\int_o^{\frac{1}{2}\pi}\cos(\ w\cdot u\cdot\sin\Theta\ )\cdot d\Theta\cdot w\cdot du \qquad (10)$$

interchanging the integration sequence and using $1=\sin\Theta/\sin\Theta$

$$S(w\cdot t)=2/\pi g\int_o^{\frac{1}{2}\pi}(1/\sin\Theta)[\int_o^{t}A(t-u)\cdot\cos(w\cdot\sin\Theta\cdot u)\cdot w\cdot\sin\Theta\cdot du]\cdot d\Theta \qquad (11)$$

the following advantages and interpretations are obtained:

Thus, with $\phi=\sin\Theta$, allows the use of adimensional velocity response of undamped single linear system in the following form:

$$wV(w\cdot\phi\cdot t)=1/g\int_o^{t}A(\ t\ -\ u\ )\cdot\cos(\ w\cdot\phi\cdot u\ )\cdot w\cdot\phi\cdot du \qquad (12)$$

This eq. (12) gives the usual advantages of traditional numerical computing methods to obtain the time velocity response of undamped single system in an actual earthquake and obtain:

$$S(\ w\cdot t\ )=2/\pi\int_o^{\frac{1}{2}\pi}[wV(\ w\cdot t\cdot\sin\Theta\ )/\sin\Theta\ ]\cdot d\Theta \qquad (13)$$

It is always finite for any actual earthquake because this eq. (13), with $r=\sin\Theta$ and $\sqrt{(1-r^2)}=\cos\Theta$, can be stated as

$$S(w\cdot t)= \frac{2}{\pi}\int_o^1 \frac{wV\{ w\cdot t\cdot r \}}{r\cdot\sqrt{(1-r^2)}}\, dr = \frac{2}{\pi}\int_o^1 \frac{wV\{ w\cdot t\cdot\sqrt{(1-x)} \}}{x\cdot\sqrt{(1-x^2)}}\, dx \qquad (14)$$

and with r=o or x= 1−r= 1 must be:

$$wV(o)= o \qquad (15)$$

at the beggining of the earthquake. And when r=1, eq. (14) it is finite because in many mathematical texts it can be found:

$$\int_o^1 \frac{dr}{\sqrt{(1-r^2)}} = \text{arc sin } 1= \frac{\pi}{2} \qquad (16)$$

this ensures finite values and results of S() although not necessarily small, because ec. (14) has no limit with wV= constant.

Other interpretation for a computing method is to use the equation:

$$Jo(z)= \cos z + 2\cdot J2(z) - 2\cdot J4(z) + 2\cdot J6(z) + \cdots \qquad (17)$$

which implies the use of ec. (12) with ϕ= 1 and result

$$S(w\cdot t)= wV(w\cdot t) - 2/g\sum_{n=1}^{\infty}(-1)^n \int_o^t A(t - u)\cdot J2n(w\cdot u)\cdot w\cdot du \qquad (18)$$

The previous equations imply no structural equivalent damping on the interactive or reflective dynamic stress waves at the wall face or bottom and also states no upper limit to the pressure resulting from a resonant earthquake's natural effects. This phenomena means that water has no equivalent linear structural damping and its effects are related to the undamped structural response. This discloses that in cities with soft soil and water saturation, the pore pressure can develop strong effects as in Mexico City and also in coastal cities with a bay. In addition to the pressure of seismic waves, water saturation in sand induces the problem of soil liquefaction due to the dynamic increase of the internal liquid pore pressures. This shows that further research in water pore pressure in saturated soils, wave propagation and local soil amplification is needed. This also explains why an upper bound for the seismic design coefficients has not been found or stated in recent years. The results obtained here calls for research and the use of the complex cosine part of the Fast Fourier Transform, FFT, to obtain the velocity response related to water or pore dynamic pressure.

STRUCTURAL UNDAMPED VELOCITY RESPONSE

To compute eq. (12) the Newmark β Factor or a similar method can be used and this method can be stated in the following six steps:

1. At t= ti, the acceleration A= Ai, the velocity V= Vi and the displacement D= Di are known at time ti.
   Let δ= ti+1 − ti, assume an A'= Ai+1 and then
2. Compute $V'= \frac{1}{2}\cdot(A + A')\cdot\delta$

Table 1. Response Smax and w·V/g max to El Centro earthquake, may 18, 1940

T sec	w=2π/T	Smax	t sec	H m	xSmax	t sec	w·V/g	t sec
.025	251.327	.4686	2.007	5.7	.	.	.410	12.028
.05	125.664	.5189	1.924	11.4	.	.	.573	5.498
.1	62.832	.5664	2.45	22.9	.356	2.32	1.50	9.766
.15	41.888	.4494	2.45	34.3	.483	4.83	2.753	4.901
.2	31.416	.6695	2.652	45.8	.560	2.65	1.875	3.150
.25	25.133	.5204	2.007	57.2	.433	2.01	1.297	2.948
.3	20.944	.5401	2.007	68.7	.453	2.01	.912	8.617
.35	17.952	.5338	2.27	80.1	.491	2.39	1.155	2.519
.4	15.708	.6234	2.27	91.5	.543	2.27	2.148	11.919
.45	13.963	.5999	2.27	103.0	.615	2.27	2.135	11.808
.5	12.566	.6602	2.27	114.4	.657	2.27	1.367	2.976
.55	11.424	.5229	2.27	125.9	.563	2.32	1.937	5.343
.6	10.472	.4244	2.32	137.4	.452	2.32	1.445	11.988
.65	9.666	.3404	1.855	148.8	.	.	.827	2.320
.7	8.976	.3436	1.855	160.2	.	.	1.189	9.123
.75	8.378	.3367	1.855	171.7	.	.	.825	12.133
.8	7.854	.3233	1.855	183.1	.	.	.633	11.207
.85	7.392	.3079	1.855	194.6	.	.	1.113	12.113
.9	6.981	.2905	1.855	206.0	.	.	1.080	11.227
.95	6.614	.2721	1.855	217.5	.	.	1.229	12.113
1.0	6.283	.2538	1.855	228.9	.	.	.841	4.618
1.05	5.984	.2362	1.855	240.4	.	.	.723	4.665
1.1	5.712	.2194	1.855	251.8	.	.	.546	4.876
1.15	5.464	.2208	2.708	263.3	.	.	.498	9.053
1.2	5.984	.2208	2.708	274.7	.	.	.504	11.780
1.5	4.189	.2008	2.708	343.3	.	.	.362	11.808
2.0	3.142	.1629	5.454	457.7	.	.	.186	11.808
3.0	2.094	.1342	5.51	686.7	.	.	.303	11.434
4.0	1.571	.0913	4.416	915.6	.	.	.134	4.416
5.0	1.257	.0753	2.893	1144.4	.	.	.076	4.97

3. Compute $D' = D + \delta \cdot V + (\frac{1}{2}-\beta)\cdot\delta^2\cdot A + \beta\cdot\delta^2\cdot A'$
4. Compute new $A' = -2\cdot\mu\cdot w\cdot V' - w^2\cdot(D' - Do) - As$
5. If the new A′ differs from previous, repeat steps 2 to 4
6. Now make $A_{i+1} = A'$, $V_{i+1} = V'$ and $D_{i+1} = D'$ and increase t.

Here μ is the damping coefficient, w the circular frequency and Do is the cuasi-static displacement due to a force time dependent. The β factor is a coefficient to obtain an área between A and A′ multyplied by its centroidal distance from t_{i+1}. β can have values between ¼ to 1/6, that is $\frac{1}{2}\cdot\frac{1}{2}$ to $\frac{1}{2}\cdot 1/3$, the last done is for linear relation between A and A′. Therefore $\beta = 1/6$ is used here. The response values at t_i and t_{i+1} are related in matrix form as

$$\begin{bmatrix} 1 & 2\mu w & w^2 \\ -\frac{1}{2}\delta & 1 & 0 \\ -\beta\delta^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} A' \\ V' \\ D' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2}\cdot\delta & 1 & 0 \\ (\frac{1}{2}-\beta)\delta^2 & \delta & 1 \end{bmatrix} \begin{bmatrix} A \\ V \\ D \end{bmatrix} + \begin{bmatrix} w^2 Do - As \\ 0 \\ 0 \end{bmatrix} \qquad (19)$$

Solving ec. (19) to eliminate iterations, for δ at ti to ti+1= ti + δ with Do= o and $\Omega= 1/[\ 1 + \delta\cdot w\cdot(\beta\cdot\delta\cdot w+\mu)\ ]$

$$\begin{bmatrix} A' \\ V' \\ D' \end{bmatrix} = \Omega \begin{bmatrix} -\delta w[\mu+(\tfrac{1}{2}-\beta)\delta w] & -2\mu w-\delta w^2 & -w^2 \\ \tfrac{1}{2}\delta[1-w^2\delta^2(\tfrac{1}{2}-2\beta)] & 1-(\tfrac{1}{2}-\beta)\delta^2 w^2 & -\tfrac{1}{2}\delta w^2 \\ \delta^2[(\tfrac{1}{2}-\beta)(1+\mu\delta w)-\beta\mu\delta w] & \delta+(1-2\beta)\mu\delta^2 w & 1+\mu\delta w \end{bmatrix} \begin{bmatrix} A \\ V \\ D \end{bmatrix} - \Omega \begin{bmatrix} 1 \\ \tfrac{1}{2}\delta \\ \beta\delta^2 \end{bmatrix} As \qquad (20)$$

and with no damping, i.e. μ= o, result $\Omega= 1/(\ 1+\beta\delta^2 w^2\ )$ and

$$\begin{bmatrix} A' \\ V' \\ D' \end{bmatrix} = \Omega \begin{bmatrix} -\delta^2 w^2(\tfrac{1}{2}-\beta) & -\delta w^2 & -w^2 \\ \tfrac{1}{2}\delta[1-w^2\delta^2(\tfrac{1}{2}-2\beta)] & 1-(\tfrac{1}{2}-\beta)\delta^2 w^2 & -\tfrac{1}{2}\delta w^2 \\ \delta^2(\tfrac{1}{2}-\beta) & \delta & 1 \end{bmatrix} \begin{bmatrix} A \\ V \\ D \end{bmatrix} - \Omega \begin{bmatrix} 1 \\ \tfrac{1}{2}\delta \\ \beta\delta^2 \end{bmatrix} As \qquad (21)$$

with β= 1/6, then $\Omega= 1/(\ 1 + \delta^2\cdot w^2/6\ )$ and can be used:

$$\begin{bmatrix} A' \\ V' \\ D' \end{bmatrix} = \Omega \begin{bmatrix} -\delta^2\cdot w^2/3 & -\delta w^2 & -w^2 \\ \tfrac{1}{2}\delta[1-w^2\cdot\delta^2/6] & 1-\delta^2 w^2/3 & -\tfrac{1}{2}\delta\cdot w^2 \\ \delta^2/3 & \delta & 1 \end{bmatrix} \begin{bmatrix} A \\ V \\ D \end{bmatrix} - \Omega \begin{bmatrix} 1 \\ \tfrac{1}{2}\delta \\ \delta^2/6 \end{bmatrix} As \qquad (22)$$

this eq. (22) allows to compute eq. (12) for different w and then obtain S with eq. (13), eq. (14) or eq. (18). Here only the values of Vmax with ec. (22) where obtained. They are shown in column $w\cdot V/g$ in Table 1 with its instant t of appearance. This also calls for further research.

CONCLUSIONS

- Computation of the hydrodynamic response can be done using a common personal computer.
- The hydrodynamic Bessel kernel Jo() is related mainly to a cosine harmonic function and implies that hydrodinamic response is relative to the usual velocity spectra without damping in about 1/3.
- Due that the equations are non linear, the usual damping effect is lost and there are no lower limit for seismic water response.
- It seems the liquid phenomena is related to subsoil compressibility and void relation in saturated media with actual pore pressure.
- Similar methods can be applied and used to obtain results on the problem of foundation response and soil structure interaction.

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