

## STUDY ON THE INDEX OF SEISMIC MOTION FOR FRAGILITY ANALYSIS

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### ABSTRACT

Reliability design can lead to more reasonable structures in seismically sensitive areas. In many reliability analyses of structures, peak ground acceleration is employed as an index of seismic motion. However, if the fragility curve proves to be steeper with another index, the index is considered better suited to express the fragility of structures. The ground motion indices that would be employed in fragility analysis were analyzed using the Monte Carlo simulation technique. We have proposed a new measurement, 'R,' as the ratio of the number of simulations in the ambiguous section of the fragility curve to the total number of simulations in order to compare the suitability of indices. The following two indices were found to be suitable for expressing fragility: the average spectral acceleration from 1.0 to 3.0 Hz, and the energy input. The energy input properly expresses the fragility when the natural period of the structure is calculated appropriately.

### KEYWORDS

probabilistic risk analysis; fragility curve; Monte Carlo simulation; seismic motion index; average spectral acceleration; artificial earthquake motion; nonlinear response.

### INTRODUCTION

The reliability analysis of structures subjected to seismic loading consists of a seismic hazard analysis and a fragility analysis. The former estimates the annual probability that seismic motion will exceed certain magnitudes. And the latter estimates the conditional failure probability for a given intensity. Seismic motion indices will be studied from the aspect of fragility analysis.

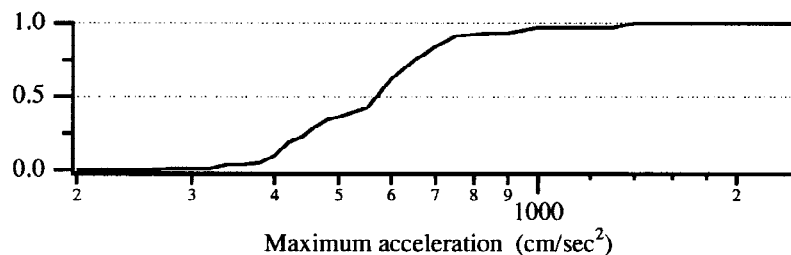


Fig. 1. Example of a fragility curve

Figure 1 shows an example of a fragility curve. The curve consists of 3 zones; fewer structures fail, some structures fail and almost all structures fail. If the curve is steep and the second zone is narrow, it can be concluded definitely whether or not a structure failed, and the ambiguity of fragility analysis can be reduced. In many reliability analyses of structures, the peak ground acceleration is employed as an index of seismic

motion. However, if the fragility curve is steeper with an index other than that for peak ground acceleration, that index is more suitable for expressing the fragility of structures.

In this paper, the suitability of ground motion indices that are employed in fragility analysis will be analyzed using the Monte Carlo simulation technique (MCS).

## SEISMIC MOTION INDICES

Peak ground acceleration (PGA) has often been used for the probabilistic risk analysis. PGA may not be suitable to express the fragility of a structure under earthquake loading, because PGA represents the instantaneous value of time history. PGA may not be able to express a system's fragility sufficiently, because it cannot express its characteristics when the structure is damaged and its natural period becomes longer. Six other seismic motion indices were considered as to whether they would be suitable for expressing fragility. Table 1 shows the seismic motion indices that were investigated in this study, along with their abbreviations.

Table 1. Indices of seismic motion and their abbreviations

Seismic motion index	Abbreviation
Peak ground acceleration	PGA
Peak ground velocity	PGV
Average spectral acceleration (3.0 - 8.5Hz)	SA-1
Average spectral acceleration (1.0 - 3.0Hz)	SA-2
Integration of squared acceleration	PW-1
Integration of squared velocity	PW-2
Energy input into a structure	E

### *Average spectral acceleration (SA-1, SA-2)*

First, the average value of the acceleration response spectrum over the specified frequencies was considered. Two average spectral acceleration indices were defined for each frequency interval. SA-1 was defined for a structure and its facilities. The natural period of the structure model that will be described below is 0.3 seconds. SA-2 was defined for a damaged structure. When a structure is damaged and its dynamic behavior becomes elasto-plastic, the natural period becomes longer and damping ratio larger. When certain elements of a structure fail, its natural period also becomes longer. The frequency interval of SA-2 covers the initial natural period and the varied natural period.

### *Integration of squared acceleration and velocity (PW-1, PW-2)*

The integrated value of squared acceleration and velocity with respect to time over the entire duration was secondarily considered. They indicate all the energy of the seismic wave and are related to all the power of the seismic wave using its duration. Although they are related to the peak value of time history when dealing with random vibrations, they were handled independently.

### *Energy input (E)*

Finally, the energy input into a structure was considered. This is the total sum of physical work done on a structure by the kinematic force induced by seismic motion. A seismic design methodology based on energy input was proposed by Housner (1956). It has been studied by many researchers, who then tried to construct a systematic design methodology. It is thought that energy input appropriately expresses the fragility of structures, because it corresponds to the damage of the structures. E is defined with the following equation.

$$E = - \int \ddot{y}(t) \dot{x}(t, \omega) dt \quad (1)$$

$\ddot{y}(t)$ : ground acceleration,  $\dot{x}(t, \omega)$ : velocity response of the structure  
 $\omega$ : the natural frequency of the structure model

# MONTE CARLO SIMULATION

## Input Motion

One hundred artificially generated earthquake motions were used as input motions for response analyses (Kai, 1994). Each artificial earthquake motion is generated in such a way as to match one target response spectrum. The duration is 30 seconds. Figure 2 shows the average of target response spectra and their fluctuations. Figure 3 shows the distribution of seismic motion indices of 100 input motions normalized to  $800 \text{ cm/sec}^2$  of PGA.

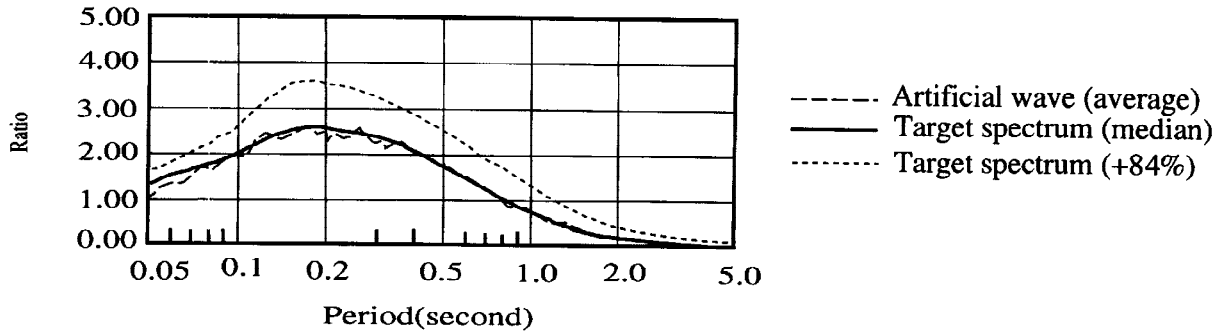


Fig. 2. Average of response spectra and their fluctuations

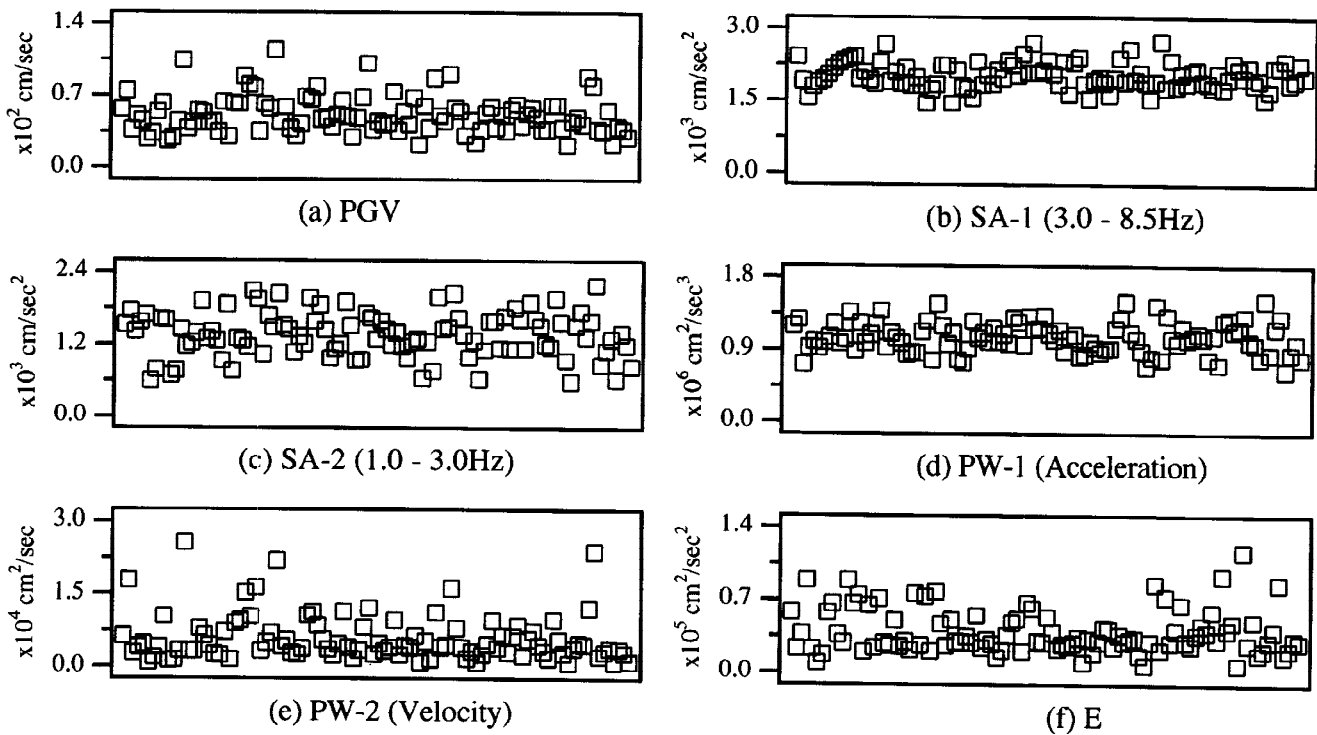


Fig. 3. Distribution of seismic motion indices of 100 input motions ( $\text{PGA} = 800 \text{ cm/sec}^2$ )

## Simulation Model

Figure 4 illustrates an SDOF simulation model that consists of 2 elements. Each element is modeled as a shear spring. Table 2 shows the model parameters employed; stiffness ratio and strength ratio. The failure of an element is defined by the excess of its ultimate displacement, and the failure of the system is defined as the intersection of failures of the two elements. Three types of behavior are also considered; elastic response, brittle response and ductile response. A peak-oriented hysteretic loop has been used for ductile response analysis before each element reaches its ultimate state. Figure 5 and 6 illustrate each stress-strain relation of an element used for each response analysis.

Table 2. Parameters

model	stiffness ratio	strength ratio
A	$k_1:k_2 = 2:1$	$F_{y1}:F_{y2} = 2:1$
B	$k_1:k_2 = 2:1$	$F_{y1}:F_{y2} = 1:1$
C	$k_1:k_2 = 4:1$	$F_{y1}:F_{y2} = 1:1$

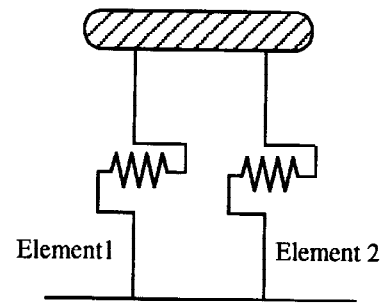


Fig. 4. Simulation model of structure

*Response Analyses*

Seventy-two PGA levels were uniformly defined between  $200\text{cm/sec}^2$  and  $12,000\text{cm/sec}^2$  on a logarithmic axis. One hundred input motions were normalized to corresponding PGA levels. In the case of PGA evaluations, response analyses were performed using these 100 normalized artificial earthquake motions at each level. Then the fragility, or failure ratio, was computed. In the other cases, the seismic motion index for each input motion was calculated and its fragility curve was simply generated from the PGA fragility curve without performing the Monte Carlo Simulation.

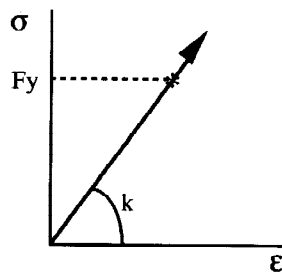


Fig. 5. Stress-strain relation of one element (elastic response)

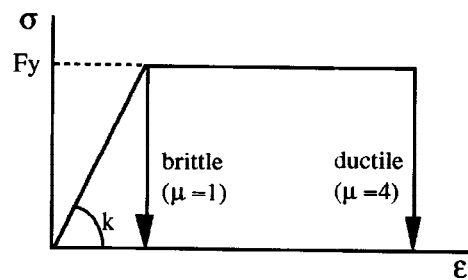


Fig. 6. Stress-strain relation of one element (brittle and ductile response)

METHOD FOR COMPARING INDICES

The effectiveness of a seismic motion index should be measured according to the ambiguity of its fragility curve. This means that an index that predicts clearly whether a structure will be destroyed or not is a good one. The ambiguity of the index is calculated using the length of the ambiguous section of a fragility curve. The ambiguous section means that it is difficult to predict whether or not a structure will be destroyed. In other words, the seismic motion index whose ambiguous section is short is suitable for expressing fragility. However, the length cannot be compared among different indices directly because their units differ. So the number of simulations in this section is used instead of their lengths.

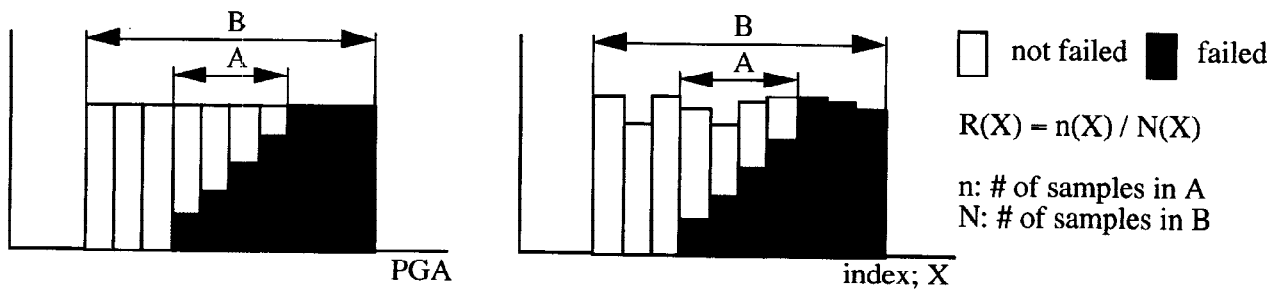


Fig. 7. Concept of suitable measure 'R' (Method for comparing seismic motion indices)

'R' is defined as the ratio of the number of simulations in the ambiguous section (n) to the total number of simulations (N). Figure 7 illustrates the concept of 'R'. The effectiveness of a seismic motion index can be

measured using 'R' value. If the 'R' value of an index is smaller than any other index, then it is better for expressing fragility.

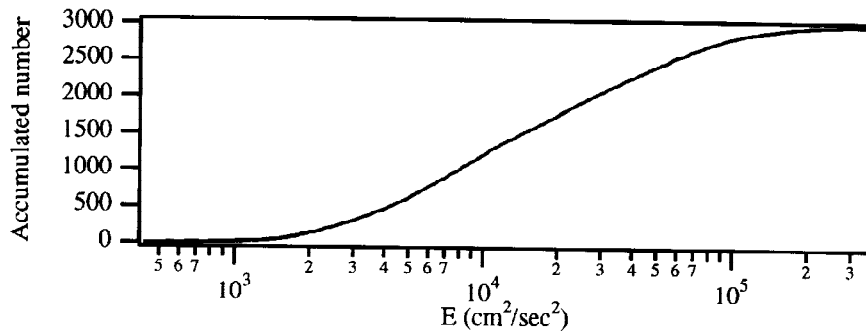


Fig. 8. Accumulated number of samples in relation to energy input

Samples of simulations should be distributed in relation to each seismic motion index uniformly in order to justify decisions based on 'R' values. It is not guaranteed, because the samples of simulations are defined only for MCS using PGA. The accumulated number of samples in relation to each seismic motion index was investigated to confirm this hypothesis. The accumulated number curve should be nearly linear along its main part and should be distributed uniformly on a logarithmic axis. Figure 8 shows the accumulated number of samples placed in relation to their input energies. The total number of samples is 3000. Almost all seem to be linear except for the two edges. It can be concluded that the samples are distributed uniformly in relation to input energy on a logarithmic axis, because the ambiguous section exists in the center of the distribution.

## RESULTS

Figure 9 shows the 'R' value for each index when considering the elastic response. In this case, E is the best index to express the system's fragility. SA-2 is the next best index. This is true for model A, model B, and model C.

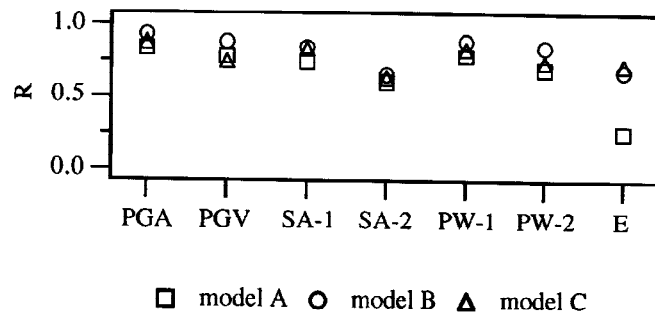
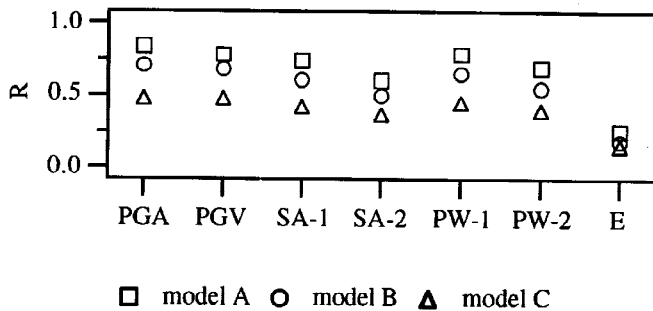


Fig. 9. R value of each index (elastic response)

Fig. 10. R value of each index (brittle response)

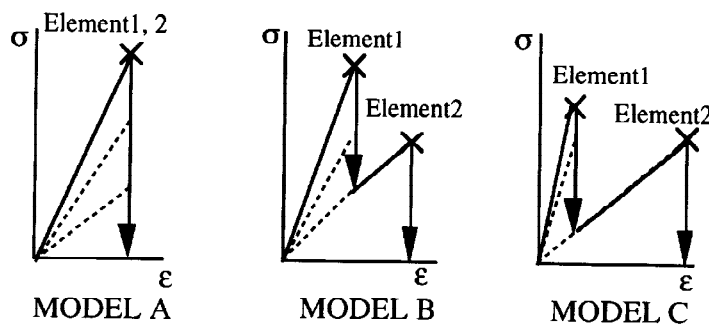


Fig. 11. Stress-strain relation of the system (brittle response)

Figure 10 shows the 'R' value for each index when considering the brittle response. For model A, the same tendency is observed as that with elastic response, since both elements fail simultaneously, as shown in figure

11. For models B and C, SA-2 is a suitable index. In these cases, element 1 fails before element 2 fails. When element 1 fails, only element 2 supports the structure. Thus, characteristics of the structure system vary. E, input energy, is calculated based on the initial natural period. After element 1 fails, the natural period of the system becomes longer and the input energy is not the same as E. SA-2 is suitable for model B and C, because SA-2 is calculated using the response spectrum that covers the initial natural period and the longer period.

Figure 12 shows the 'R' value for each index when the failure of the system is defined as the failure of element 1. The same tendency is observed as that in the case of the elastic response for all models. Before element 1 fails, the system behaves like an elastic response. Therefore, E, which represents characteristics of the initial system, is suitable for expressing the system's fragility. But after element 1 fails, the natural period of the system is the period of the system that contains only element 2. Then, SA-2, which represents the natural period of the initial system and that of the system that contains only element 2, is more suitable than E, which represents only the initial natural period.

Figure 13 shows the 'R' value for each index when considering the ductile response. SA-2 is the best index for every model. Although both elements fail simultaneously for model A, as shown in figure 14, the system is already not linear. The natural period and other characteristics of the system vary when the elements become plastic. The 'R' value of SA-2 is smaller than those of any other indices because SA-2 also represents the characteristics of the system after it becomes plastic. For models B and C, both the plastic effect and the plural elements effect influence the system characteristics, which means SA-2 is the best index.

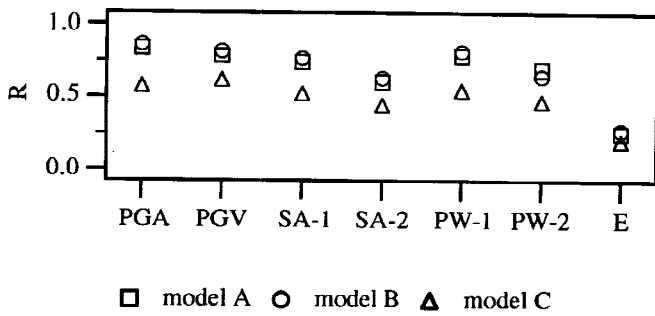


Fig. 12. R value of each index concerning damage of element 1 (brittle response)

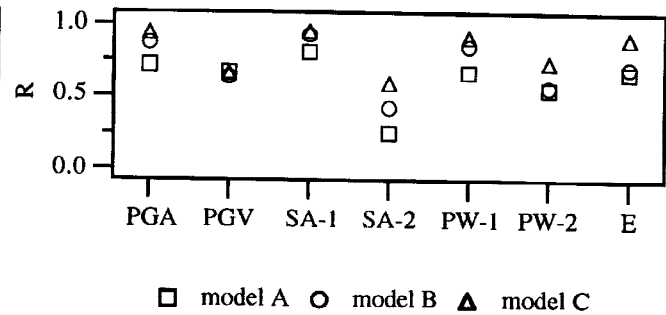


Fig. 13. R value of each index (ductile response)

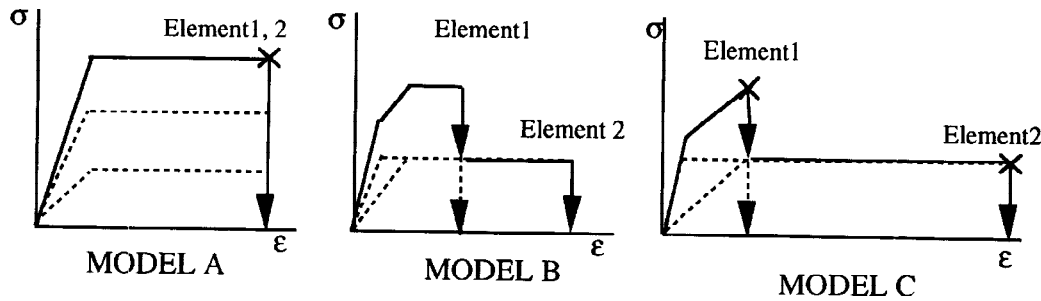


Fig. 14. Stress-strain relation of the system (ductile response)

It seems that the indices corresponding to velocity, such as PGV and PW-2, are more suitable than those corresponding to acceleration, such as PGA and PW-1, respectively. Because those velocity-oriented indices represent the characteristics at longer periods of the system than the periods that acceleration-oriented indices represent.

## DISCUSSIONS

Table 3 and figure 15 shows the ratio of the 'R' value of each index to the 'R' value of PGA. These indicate whether the index is more suitable than PGA.

PGV is more suitable than PGA for brittle and ductile response. In particular, the improvement of ductile response for models B and C are remarkable. In these cases, the indices that represent the dynamic behavior of the system at longer periods than the initial natural periods are suitable because, after element 1 becomes

plastic, both the natural period and the damping of the system change, and after element 1 fails, the structure is supported by only element 2. PGV seemed to represent the characteristics of dynamic behavior of the system after it changes. For model A, the ratio of 'R' value of ductile response is not as small as those for models B and C, because both elements fail simultaneously in this case and only the plastic effect influences the system characteristics.

Table 3. Relation of the 'R' value of each seismic motion index to the 'R' value of PGA

Elastic response							Brittle response								
PGA	PGV	SA-1	SA-2	PW-1	PW-2	E	PGA	PGV	SA-1	SA-2	PW-1	PW-2	E		
A	1.000	0.935	0.888	0.724	0.938	0.830	0.306	A	1.000	0.935	0.888	0.724	0.939	0.830	0.306
B	1.000	0.964	0.856	0.706	0.922	0.781	0.257	B	1.000	0.942	0.908	0.704	0.948	0.905	0.729
C	1.000	0.994	0.875	0.758	0.935	0.825	0.317	C	1.000	0.850	0.950	0.721	0.949	0.845	0.824

Ductile response

PGA	PGV	SA-1	SA-2	PW-1	PW-2	E	
A	1.000	0.938	1.127	0.347	0.937	0.770	0.933
B	1.000	0.732	1.066	0.486	0.970	0.649	0.808
C	1.000	0.703	1.013	0.631	0.968	0.784	0.961

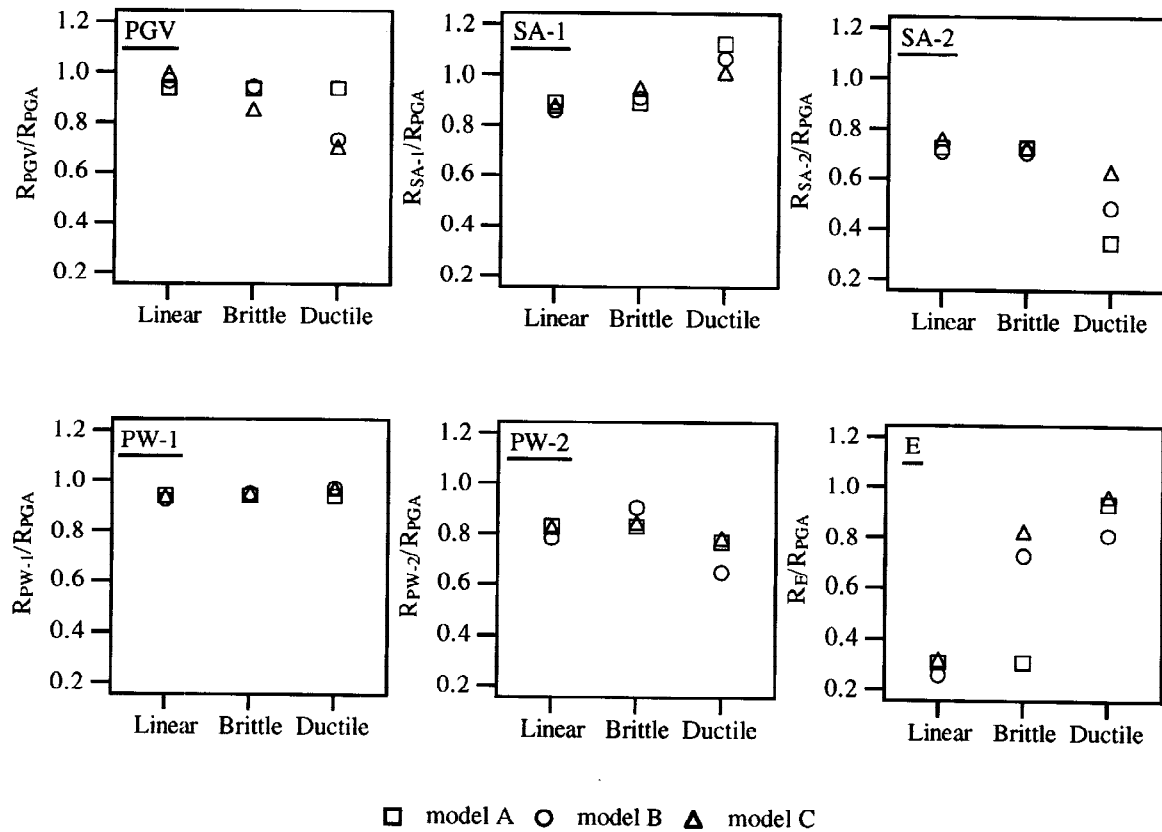


Fig. 15. Comparison of ratio of the 'R' value of each index to 'R' value of PGA

In the case of elastic response, the ratio of 'R' value of SA-1 is smaller than 1.0. This means that SA-1 is more suitable than PGA for elastic response. On the other hand, the ratio of 'R' value of SA-1 is larger than 1.0 for ductile response. SA-1 is not suitable for ductile response, because the period range of SA-1 does not cover the natural period of the damaged structure.

The ratio of 'R' value of SA-2 is smaller than 0.8 in all cases. This means that SA-2 is more suitable than PGA for any types of response and models. In particular, the ratio of 'R' value of SA-2 is smaller than any

other seismic motion index for ductile response. When dealing with elastic, brittle, and ductile response, SA-2 is the best seismic motion index.

The ratio of 'R' value of PW-1 is nearly equal to 1.0. Therefore, the fragility expression cannot be improved using PW-1. The ratio of 'R' value of PW-2 is smaller than 0.8 for ductile response and is smaller than 0.9 for elastic and brittle responses, with the exception of brittle model B. Therefore, PW-2 is suitable for ductile response and can improve the fragility expression for elastic and brittle responses.

The ratio of 'R' value of E is about 0.3 for elastic response and brittle response model A. This means that E is the most suitable index for elastic response, because the brittle response model A is considered an elastic response. On the other hand, the improvement of fragility expression for inelastic response is not as large as that for elastic response. Energy input should be a suitable seismic motion index, because the input energy corresponds to the damage of the system. E is a suitable index when the natural period of the system is clear and fixed and, therefore, the energy input of a system corresponds to its damage. On the other hand, E is not a suitable index when the natural period of the system varies and the energy input of the system cannot be calculated accurately.

In a practical structure, not all of the elements fail simultaneously. If one element is damaged or fails, it becomes plastic and the natural period of the system becomes longer. In such cases, SA-2, which represents both the initial structure characteristics and the varied structure characteristics, is the most suitable index.

## CONCLUSIONS

The 6 indices of seismic ground motion that would be employed in fragility analysis were investigated using Monte Carlo simulation technique. The 'R' value is proposed in order to compare these indices. 'R' is defined as the ratio of the number of simulations in the ambiguous section to the total number of simulations. The results may be summarized as follows:

- (1) The suitability of an index depends on the types of stress-strain relation and on the model of the structure.
- (2) The spectral acceleration average between 1.0 and 3.0 Hz is the most suitable index for fragility analyses in inelastic cases (brittle and ductile).
- (3) The energy input can properly express the fragility in elastic cases.

The following points remain to be studied further:

- (4) The influence of duration and shape of the input motion on the suitability of seismic indices.
- (5) When concluding probabilistic risk analysis using the results of seismic hazard and fragility analyses, the conclusion mentioned above should be studied with regard to ways it can improve probabilistic risk analysis.
- (6) From the aspect of seismic hazard analysis, it is very important whether or not an attenuation equation of a seismic motion index can be obtained. PGA, PGV and the average acceleration response spectrum are being investigated very actively in seismic hazard analysis. Although some types of attenuation equations for these indices have been proposed based on observed strong motion data, an attenuation equation for input energy has not proposed yet. Therefore, an attenuation equation for input energy needs to be constructed if energy input is to be used as a seismic motion index.

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