

## DYNAMIC INSTABILITY PHENOMENON IN BIDIRECTIONAL SYSTEMS

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### ABSTRACT

Unidirectional simplifications of three dimensional systems generally lack the ability to adequately represent the structural rotational capacity. Variations of the factor of safety against dynamic instability are assessed from the independent and simultaneous application of ground motion components to two single story three-dimensional structural systems. The effect of the ground motion angle of incidence in the structure's collapse characteristics is also analyzed. Collapse intensity values and mechanism profiles are seen to be strongly dependent on the relative orientation of the ground motion in all cases. There is a strong correlation on the level of the collapse intensity values and their associated mechanisms for different orientations of the same ground motion. The capacity of the structure to stably withstand the ground motion is overestimated from 20 to 70% when the ground motion components are independently considered. Additionally, the results show that the mechanism profile associated to the smallest collapse intensity at any orientation is fairly independent of the applied ground motion. An energy based simplified method to assess dynamic instability in three dimensional single story systems is presented and tested in a number of cases, and its potential application to multistory structures is discussed.

### KEYWORDS

Torsion; dynamic instability; bidirectionality; collapse; mechanisms; P- $\Delta$  effects; gravity.

### INTRODUCTION

The destabilizing effect of gravity loads on buildings subjected to severe ground motions can lead to catastrophic collapse when the structure is forced to develop significant inelastic excursions during the response. Although prevention of collapse is a fundamental objective of seismic design, there is significant uncertainty on the suitability of current procedures to provide adequate margin of safety against dynamic instability (*Recommended...* 1991). Early studies on dynamic instability of SDOF systems have indicated that the effect of gravity on inelastic response is typically small, except in those cases where the resistance of the system is close to that for which complete collapse occurs (Husid, 1967; Jennings and Husid, 1968; Sun *et al.*, 1973; Takizawa and Jennings, 1980; Bernal, 1992).

Recent studies on dynamic instability in MDOF planar structures have pointed out the decisive role that the failure mechanism plays in the safety of structures against collapse (Nakajima *et al.*, 1990; Bernal, 1990;

Bernal, 1992). It is then expected, and evidenced by recent destructive earthquakes, that rotational collapse mechanisms may be critical in the overall stability of a real three dimensional building. Although potential for failure due to torsional instability has long been recognized, there are relatively few studies that directly address this issue. One of the most outstanding pioneering investigations is reported by Shibata *et al.* (1969), where simple single storey systems are analyzed under the action of half sine acceleration pulses. Their results indicate the significant influence of rotational inelastic response in reducing the stability threshold of such systems. They also highlight the importance on the simultaneous consideration of the ground motion components for the adequate assessment of dynamic instability. In a similar study carried out by Morino and Uchida (1980), the importance of the multidirectional consideration of the ground motion in dynamic instability analyses has been further substantiated. Due to the importance of the problem, there is a strong need to develop rational simplified methodology to assess the safety margin against dynamic instability of buildings that account for the three dimensional nature of the problem.

In this paper, a thorough examination on the collapse characteristics of two specific bidirectional single storey systems is carried out. The importance of the bidirectional action of the ground motion on the controlling mechanisms is discussed. Additionally, a simplified energy approach to assess the margin of safety against dynamic instability is described. The methodology is based on the comparison of the maximum capacity that the structure can provide in terms of energy dissipation with the maximum energy demand likely to be imposed by critical ground motions. The main advantage of utilizing energy concepts in the study of three dimensional dynamic instability bears on reducing the problem to the evaluation of two scalars, i.e. the energy capacity and the energy demand. On these grounds, important features of the three dimensional behaviour near collapse are highlighted, intending to provide a better understanding of torsional instability in structures. Finally, an exploratory implementation of the method to multistory structures is discussed.

### EXPLORATORY EXAMPLE.

A thorough examination on the collapse characteristics of two bidirectional single storey systems (models A and B) when subjected to the simultaneous action of two arbitrarily oriented orthogonal components of ground excitations is carried out. The general characteristics of these two models are illustrated in Fig. 1 .

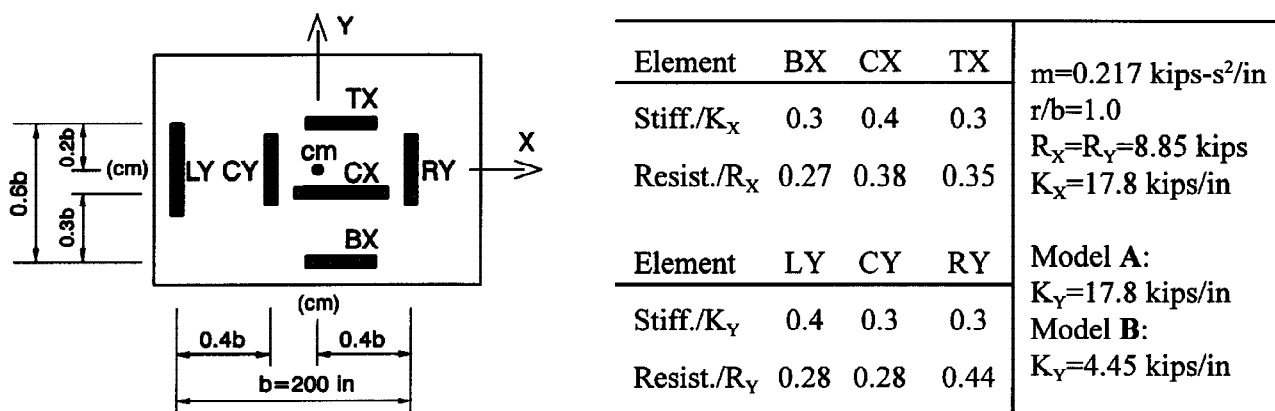
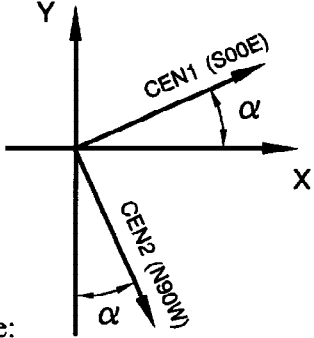


Fig. 1. General characteristics for the exploratory cases (Models A and B).

Let us define the *Collapse Intensity*  $I_c$  as the scale factor needed to be applied to the ground motion accelerations in order to obtain incipient collapse in the structure. It is worthnoting that  $I_c$  is then equivalent to the factor of safety against collapse of the structure for that particular ground motion. The collapse intensities associated with model A under the simultaneous action of the two horizontal components of El Centro, 1940 record were computed through an iterative dynamic analysis of the MDOF system for 0% viscous damping. The results thus obtained for different input directions of the ground motion are shown in table 1, together with those obtained from the separate application of each one of the components.

Table 1. Collapse Intensities ( $I_C$ ) and associated mechanisms for model A under the action of El Centro, 1940 record (CEN1=S00E component ; CEN2=S90W component).

$\alpha$ (deg.)	CEN1 & CEN2		CEN1		CEN2	
	$I_{C12}$	Mech. <sup>(*)</sup>	$I_{C1}$	Mech. <sup>(*)</sup>	$I_{C2}$	Mech. <sup>(*)</sup>
0	0.77	TX-RY	1.00	X-Trans	1.82	Y-Trans
15	0.68	TX-RY	0.98	X-Trans	1.01	CX-RY
30	0.61	CX-CY	0.66	CX-CY	1.06	CX-CY
45	0.69	CX-CY	<b>0.56</b>	CX-RY	1.02	CX-CY
60	0.61	CX-CY	0.61	CX-RY	1.00	CX-CY
75	0.56	CX-RY	0.62	CX-RY	1.18	X-Trans
90	<b>0.45</b>	CX-RY	1.00	Y-Trans	1.14	X-Trans
105	0.49	CX-RY	0.69	CX-RY	1.18	X-Trans
120	0.58	CX-RY	0.60	CX-RY	1.32	X-Trans
135	<b>0.45</b>	CX-CRY	0.61	CX-RY	1.06	CX-CRY
150	0.55	CX-CY	0.83	TX-RY	<b>0.90</b>	CX-RY
165	0.83	TX-RY	1.02	X-Trans	<b>0.90</b>	CX-RY



(\*) Note:  
The mechanism shape is defined by the elements that remain elastic at collapse (*pivoting*) for rotational modes, or indicating the direction of collapse for translational mechanisms.

From the examination of table 1, the mechanism mode developed under the single component ground motion seems to be very sensitive to its input orientation. When the structure is subjected to the simultaneous action of both horizontal components, certain mechanisms are induced through a wider orientation range. This behaviour is explained from the significant variation in the inertial forces direction during the response under the action of the bidirectional excitation, which increases the likelihood for the mechanisms with a relatively small energy dissipation capacity to be formed. In addition, the level of intensity leading to collapse is strongly correlated to the particular induced mechanism, stressing the idea that the collapse intensity is directly related to the inherent work capacity associated with the mechanism. The dependency of the collapse intensity levels on the unidirectional component selected is also apparent.

#### Bidirectional vs Unidirectional Ground Motion Excitation.

Let us now define the *Critical Collapse Intensity* ( $I_{CC}$ ) as the minimum collapse intensity obtained for any ground motion orientation. Table 2 shows the minimum  $I_{CC}$  values from those obtained from the independent application of the two orthogonal components of four ground motions to models A and B, as a ratio to the corresponding  $I_{CC}$  bicomponent values. 5% viscous damping is considered for the two first modes of vibration. The results show that unidirectional estimates lead to significant capacity overestimations.

Table 2. Comparison of  $I_{CC}$  from independent and simultaneous ground motion components application.

$I_{CCunidir} / I_{CCbidir}$	CEN	PAR	SOL	LAK
Model A	1.36 (CEN1)	1.19 (PAR2)	1.15 (SOL2)	1.62 (LAK2)
Model B	1.25 (CEN1)	1.09 (PAR2)	1.48 (SOL1)	1.58 (LAK2)

CEN ; El Centro, 05/18/1940 (CEN1=S00E ; CEN2=S90W)  
 PAR ; Parkfield record, 06/27/1966 (PAR1=N05W ; PAR2=N85E)  
 SOL ; 646 South Olive record, 02/09/1971 (SOL1=S53E ; SOL2=S37W)  
 LAK ; Lake Hughes No. 12, 02/09/1971 (LAK1=N21E ; LAK2=N69W)

Bidirectional ground motion orientation sensitivity.

As it was seen in the previous section, the level of the intensity of the bidirectional ground motion needed to collapse the structure depends on its orientation relative to the structure. In order to quantitatively assess this issue, the collapse intensities ( $I_C$ ) associated to models A and B (5% damping) when subjected to four bidirectional ground motions for different relative orientations are plotted in Fig. 2, normalized to their corresponding Critical Collapse Intensity ( $I_{CC}$ ). It is clear from the figure that variations in collapse intensities associated to different relative orientations are significant, particularly for model B, where the uncoupled translational periods do not coincide. Therefore, an adequate assessment of dynamic instability can not be performed without the explicit consideration of the critical ground motion orientation relative to the structure.

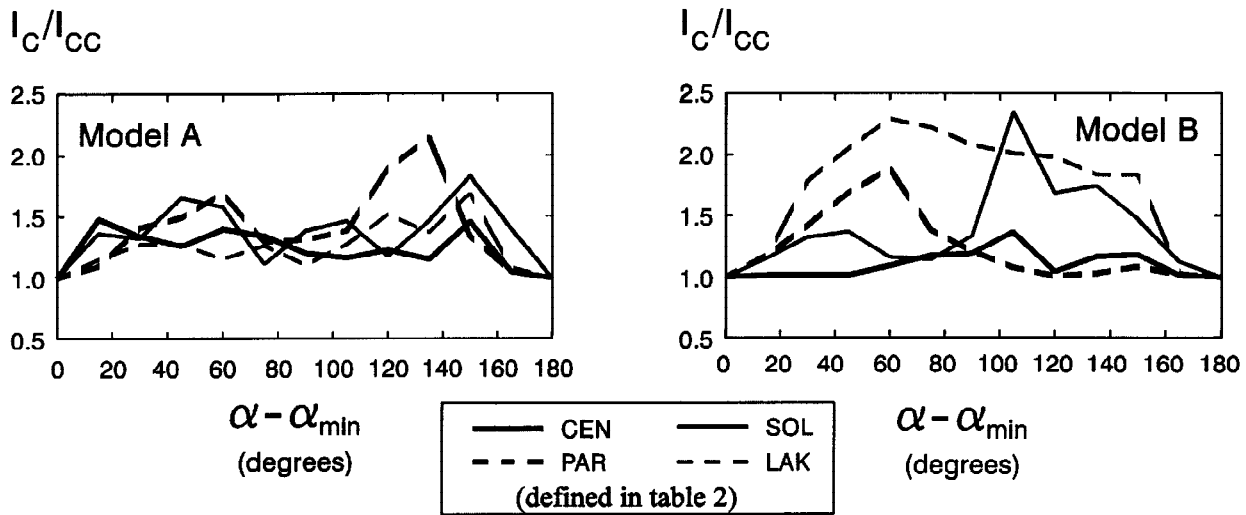


Fig. 2. Collapse Intensity values for different ground motion orientations. ( $\alpha$  is defined in table 1).

ENERGY APPROACH TO DYNAMIC INSTABILITY.

For detecting the key parameters affecting dynamic instability of three dimensional structures a capacity vs. demand approach is examined. The key issues involved are the definition of the supply and demand energy terms,  $W_S$  and  $W_D$  respectively. The examination was based on the premise that, with appropriate definitions, the intensity of the ground motion needed to drive the system to incipient collapse may be estimated as  $I_C = (W_S / W_D)^{1/2}$ . The definitions presented next lead to  $I_C$  values that are in good agreement with results obtained from inelastic time history analysis. A detailed discussion on the considerations used at arriving at these definitions can be found in Sordo and Bernal (1994).

Work Supply ( $W_S$ )

$W_S$  is defined as the external work needed to displace the structure to a position where it is in equilibrium (albeit unstable equilibrium from a static perspective) with the gravity loads only. With some exceptions that rarely govern and which are discussed by Sordo and Bernal (1994), the value of  $W_S$  is path independent and can be computed by inspecting the structure in the final configuration. Since  $W_S$  depends on the mechanism type, one must explore all possibilities and select the one leading to the smallest value.

## Work Demand ( $W_D$ )

Definition of the work demand is identical to that of the work supply except that it is not applied on the actual system, but on a replacement elasto-plastic SDOF oscillator. This oscillator has elastic parameters which are dependent on the properties of the structure and a yield strength that is the minimum required to ensure stable response under one of the ground motion components. It is appropriate to note that making  $W_D$  a function of the collapse threshold of a SDOF system does not introduce unwarranted complication, as this strength level can be readily estimated using the statistically based expressions presented by Bernal (1992).

The research results indicate that the SDOF period ( $T$ ), damping and stability coefficient ( $\theta$ ) can be taken as those corresponding to the elastic modes of the 3-D system.  $W_D$  is then selected as the largest value from all the modes. In performing these assessment, the component of the motion that leads to the largest work demand is used. A general expression for  $W_D$  can be obtained:

$$W_{Di} = \frac{(1-\theta_i)}{8 \pi^2 \theta_i} \Gamma_i^2 S_{ACi}^2 T_i^2 \quad (1)$$

where the subscript  $i$  refers to mode  $i$ , and  $\Gamma_i$  is the largest mass normalized participation factor associated with any direction for mode  $i$ . It can be easily shown that this maximum can be computed as the square root of the sum of the squares of the participation factors in any two orthogonal directions.  $S_{AC}$  is the minimum resistance per unit mass to prevent collapse in the considered SDOF system. The stability coefficient can be computed as  $\theta_i = 1 - (T_i / T_{iP-\Delta})^2$ , where  $T_{iP-\Delta}$  is the modal period considering P- $\Delta$  effects.

## Collapse intensity prediction. Sample case.

The methodology to estimate the Critical Collapse Intensity ( $I_{CC}$ ) is illustrated next for the structure labeled as model A, whose general characteristics were depicted in Fig. 1. When subjected to El Centro 1940 record, the value for  $I_{CC}$  for 0% damping was shown to be 0.49 (table 1), associated to mechanism CX-RY.

Work supply for models A and B. The work capacities ( $W_S$ ) associated to different mechanism shapes for models A and B are shown in table 3. It must be pointed out that, from all feasible mechanisms, only those corresponding to the rotational pivoting about one element in each direction and the two translational mechanisms were considered for simplicity. As it was shown in table 1, the mechanisms most frequently induced on model A under the bidirectional action of El Centro record were CX-CY and CX-RY. These mechanisms correspond to those with the smallest associated work capacities, as shown in table 3.

Table 3. Work capacities ( $W_S$ ) in kips-in for different mechanisms.

Pivoting elements	MODEL A			MODEL B		
	LY	CY	RY	LY	CY	RY
TX	44.4	35.1	39.5	35.6	29.5	33.4
CX	36.5	27.4	31.6	27.3	21.7	24.9
BX	45.4	36.1	40.5	36.5	30.5	34.2
Translation	100.4 (X) ; 100.1 (Y)			99.8 (X) ; 92.8 (Y)		

The work supply for model A is then taken as  $W_{SA} = 27.4$  kips-in, associated to mechanism CX-CY. It is worth noting that the work capacity associated with the actual observed collapse mechanism at the critical intensity (CX-RY) is only 15% larger than this value.

Work demand for model A. In table 4, the work demand values computed from the different bilinear reference SDOF systems subjected to El Centro, 1940 record (0% damping) are illustrated. The value selected for the final work demand is highlighted in the table.

Table 4. Work Demand ( $W_D$ ) for the reference SDOF systems associated to model A when subjected to El Centro, 1940 record.

Reference SDOF (0% damp.)	MODE 1 T=1.50 sec $\theta=0.100$ $\Gamma_1=0.12^*$	MODE 2 T=0.69 sec $\theta=0.021$ $\Gamma_1=0.47^*$	MODE 3 T=0.68 sec $\theta=0.020$ $\Gamma_1=0.45^*$	
$W_{Dmax}$	5.97 Kips-in	111.2 Kips-in	106.6 Kips-in	
$(\alpha_{W_{Dmax}})$	(145°)	(160° - 165°)	(155° - 165°)	

\* The mass normalized modal participation factors ( $\Gamma_i$ ) are those associated to the system's direction for which they are maximized

**Critical Collapse Intensity prediction for model A.** A predictive estimate of the Critical Collapse Intensity is computed as  $I_{CC} \approx (W_s / W_D)^{1/2} = (27.4 / 111.2)^{1/2} = 0.50$ , which turns out to be quite acceptable (10% larger than the MDOF critical collapse intensity, as given in table 1).

#### BIDIRECTIONAL CASE STUDIES.

The described methodology is to be assessed through the study of the systems depicted in Fig. 1 when subjected to a set of four ground motion records and 5% damping. The characteristics for the different reference SDOF systems utilized in the computation of the work demand are summarized in table 5. For these systems, the maximum work demands corresponding to any unidirectional component of each of the ground motions in study were computed, their values being shown in table 5. The critical work capacities for models A and B are, from table 3;  $W_{s(A)}=27.4$  kips-in (CX-CY) and  $W_{s(B)}=23.17$  kips-in (CX-CY). From the previous values for the work capacities and demands the  $I_{CC}$  estimates are computed and also shown in table 5.

Table 5. Critical Collapse Intensity estimates for models A and B.  
Four ground motion records and 5% damping are considered here.

Ground Motion record (refer to table 1) :		CEN	PAR	SOL	LAK
<b>MODEL A</b> ( $W_s = 27.4$ kip-in) $T_1=1.50s$ $\theta_1=0.10$ $\Gamma_1^2=0.014$ $T_2=0.69s$ $\theta_2=0.021$ $\Gamma_2^2=0.22$ $T_3=0.68s$ $\theta_3=0.020$ $\Gamma_3^2=0.20$	$W_{Dmax}$ (Contr. mode)	62.2 (2)	17.6 (2)	10.7 (2)	3.94 (2)
	$I_{CCest} = (W_s / W_{Dmax})^{1/2}$	0.67	1.3	1.6	2.6
	$I_{CC MDOF}$ (Mech.)	0.68(CX-RY)	1.3(CX-RY)	1.7(CX-RY)	2.5(CX-RY)
	$I_{CCest} / I_{CC MDOF}$	0.98	0.98	0.93	1.07
<b>MODEL B</b> ( $W_s = 23.2$ kip-in) $T_1=2.30s$ $\theta_1=0.24$ $\Gamma_1^2=0.018$ $T_2=1.35s$ $\theta_2=0.082$ $\Gamma_2^2=0.20$ $T_3=0.69s$ $\theta_3=0.021$ $\Gamma_3^2=0.22$	$W_{Dmax}$ (Contr. mode)	63.3 (3)	17.7 (3)	17.9 (3)	4.72 (2)
	$I_{CC est} = (W_s / W_{Dmax})^{1/2}$	0.61	1.2	1.1	2.2
	$I_{CC MDOF}$ (Mech.)	0.64(CX-CY)	1.4(CX-CY)	1.0(CX-CY)	2.0(CX-CY)
	$I_{CC est} / I_{CC MDOF}$	0.94	0.8	1.13	1.12

The Critical Collapse Intensities ( $I_{CC}$ ) associated to models A and B when subjected to the four ground motion records in study and 5% damping were then computed from iterative step by step analyses of the MDOF system, being the results also indicated in table 5. As it can be seen in this table, the estimated values of  $I_{CC}$  are in very good agreement with those obtained through the analysis of the MDOF system.

## Forced mechanisms.

An important consequence of the approximate methodology postulated for the study of bidirectional single storey structures is that the resistance distribution will only affect the collapse intensity associated to the system by modifying the total work capacity of each mechanism. In contrast, the work demanded from the ground motion is unaffected, as long as the modal characteristics remain the same. In order to check this point, model A was forced to collapse in three different mechanisms (X-transl., Y-transl. and BX-LY) by conveniently increasing the resistance of the elements desired to remain elastic. The collapse intensity estimations should then vary proportionally to the square root values of the work capacities of the different mechanisms (table 3). As can be seen from table 6, the energy method accurately predicts that the ratio in which collapse intensity is modified depends only on the change of energy dissipation capacity of the developed mechanism.

Table 6.  $I_{CC}$  predictions for model A when forced to develop a predefined mechanism.

Forced Mechanism: Ground Motion:	X - Translation				Y - Translation				BX-LX			
	CEN	PAR	SOL	LAK	CEN	PAR	SOL	LAK	CEN	PAR	SOL	LAK
$I_{C\ MDOF}$	1.31	2.53	3.05	5.47	1.39	2.82	3.62	5.80	0.742	1.74	2.09	3.82
$I_{CC\ est} = \lambda \cdot I_{CC\ est}^{(*)}$	1.27	2.39	3.08	5.04	1.27	2.39	3.08	5.04	0.858	1.61	2.08	3.41
$I_{CC\ est} / I_{C\ MDOF}$	0.97	0.94	1.01	0.92	0.91	0.85	0.85	0.87	1.16	0.93	1.00	0.89

(\*)  $I_{CC\ est} = I_{CC}$  estimate for original model A, obtained from table 5

$$\begin{aligned} \lambda &= (W_{S\ (Forced\ mech.)} / W_{S\ (CX-CY)})^{0.5} = (100.4 / 27.4)^{0.5} = 1.91 \text{ (For X-translation)} \\ &= (100.1 / 27.4)^{0.5} = 1.91 \text{ (For Y-translation)} \\ &= (45.4 / 27.4)^{0.5} = 1.29 \text{ (For pivoting about BX-LX)} \end{aligned}$$

## MULTISTORY STRUCTURES.

Studies on dynamic instability in three dimensional multistory structures are extremely scarce. An exploratory study of dynamic instability on a multistory structure subjected to an unidirectional ground motion has been recently reported by Sordo and Bernal (1992). They conclude that the three dimensional shape of the failure mechanism is likely to play a fundamental role in the safety against instability of multistory structures. A more recent examination of dynamic instability on a L-shaped five story building subjected to the effect of different unidirectional and bidirectional ground motions has been carried out by Sordo and Bernal (1993). In that study, the importance of considering the complete multidirectional ground motion in the analyses, particularly when rotational failures are prone to be developed, is highlighted. Therefore, it becomes of great interest to examine the applicability and potential benefits of the proposed methodology in the study of such complex systems.

An exploratory study on the suitability of the proposed method to multistory structures was recently conducted by the writers (Sordo and Bernal, 1994), with encouraging results. A relatively simple procedure was developed in their research, where the collapse intensity and mechanism for a five story L-shaped structure was predicted with a surprising accuracy. In this case, the different feasible vertical shapes of the mechanism makes necessary to introduce an incremental mass proportional load vector to detect the potential controlling mechanism, consistently to the observations indicated by Bernal (1992) for planar systems. Despite these promising results, further research is still needed to adequately validate this methodology for multistory structures.

## CONCLUSIONS.

The general characteristics of the response at the instability threshold for three dimensional single storey systems subjected to bidirectional ground motions have been examined. A general energy based methodology to predict collapse intensities has been formulated and tested in a number of cases. It was found that collapse intensity predictions from the energy method are in good agreement with those obtained from iterative dynamic analyses for the studied cases. The method is also adequate to detect the mechanisms most susceptible to lead to dynamic instability. From the results obtained, the following observations can be made.

- Application of the ground motion at different angles of incidence leads to important variations on the collapse intensity values. Then, analytical studies on bidirectional systems must account for uncertainties in the expected angle of incidence through the assessment of the most unfavorable relative orientation.
- Nonlinear dynamic analyses based on the application of arbitrary unidirectional ground motion components can in some cases lead to excessively unconservative assessment of dynamic instability.
- The instability failure modes detected during the dynamic analysis do not appear to be dependent of the particular ground motion characteristics, as it has been previously detected for planar framed systems (Bernal, 1992). This observation supports the use of a methodology based on the separate treatment of the structural work capacity and the work demanded by the ground motion.
- There are indications that the methodology can be implemented for multistory systems with some adequations. Nevertheless, further research is needed to adequately assess this issue.

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