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BASIC PROBLEMS OF THE EXPLICIT NUMERICAL METHOD FOR ANALYZING THE EARTHQUAKE RESPONSE OF INHOMOGENEOUS LOCAL SITE

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ABSTRACT

In this paper, an explicit finite element–finite diference method for analyzing the effects of viscoelastic inhomogeneous local site is introduced, the method is a hybridized method of finite element method in space and explicit finite difference method in time. Several concerned problems are discussed in detail, which are: the Generalized Multi–Transmitting Boundary; the high–frequency instability of Multi–transmitting Boundary; the drift instability of Multi–transmitting Boundary; the determination of the inward propagating wave field in artificial boundary area.

KEYWORDS

Explicit Method; Inhomogenous Local Site; Viscoelastc; Multi–Transmitting Boundary; High–Frequency Instability; Drift Instability; Inward Propagating Wave

INTRODUCTION

The analyses of the effects of local site condition on earthquake ground motion play important roles in the aseismic design of constructions. The effects of local site condition on earthquake ground motion include the topographic effects of local site and the inhomogeneous and nonlinear effects of medium. For the problem of the topographic effects, because the medium is considered as a linear and homogeneous one, there are little restricted conditions for selecting analyzing methods, and many kinds of semi–analytic methods and numerical methods are applicable, such as boundary element method (Du, et al., 1993), semi–analytic boundary element method (Zhao, et al., 1993), finite element method, finite element–finite difference method (Liao, 1984; Li, et al., 1995). For the problem of the regular topographic site, for example, the topographic form of concave and convex spherical surface, analytic method is effective. But for the problem considering the inhomogeneous and nonlinear effects of site, the applicable method must be numerical method. The previous researches and the practical experience in engineering show that explicit step–by–step complex local site, especially, if considering the nonlinear properties of medium. The explicit finite element–finite difference method (Liao, 1984) is a numerical method based on finite element method in space and center finite difference method in time, but it is no longer an explicit method for the problem of visco–elastic local site. An explicit high accuracy formula of the explicit finite element–finite difference method is proposed by author recently (Li et al., 1995) for the problem of visco–elastic local site.

In this paper, the basic formulas for the method applying to analyzing the effects of viscoelastic inhomogeneous local site is introduced, and several concerned problems are discussed in detail.

THE EXPLICIT FINITE ELEMENT–FINITE DIFERENCE METHOD

The Computing Model of Site

The two–dimensional model of viscoelastic inhomogenous local site is our analytic model. The analytic model with artificial boundaries is as shown in Figure 1.

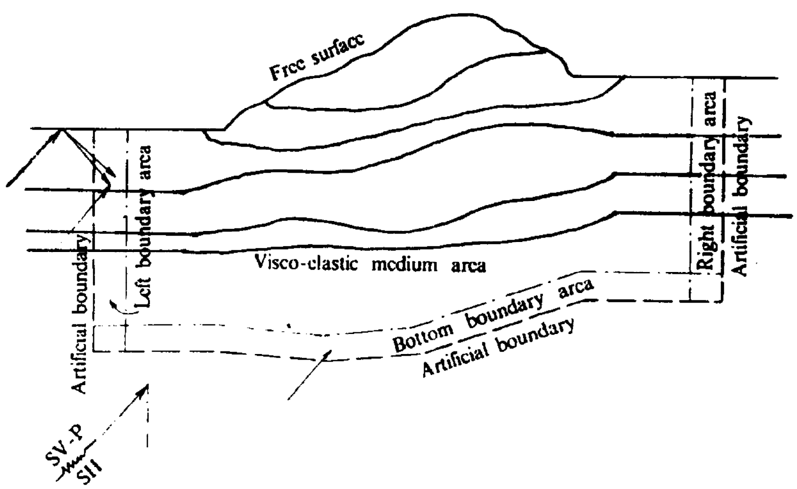

Fig. 1. Sketch map of two–dimensional model of viscoelastic inhomogenous local site

The finite element discrete model in space about the computing area inside the artificial boundaries can be obtained by finite element technique.

The Dynamic Equations

Assuming that the mediums in the computing area are visco–elastic isotropic bodies, the corresponding dynamic equations of the nodes in the finite element discrete model are

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{R(t)\} \qquad (1)$$

in Eq.(1), [M], [C] and [K] are the assembly mass matrix, damping matrix and stiffness matrix of the finite element system, respectively; {u(t)}, $\{\dot{u}(t)\}$and {u(t)} are the acceleration vector, velocity vector and displacement vector of the nodal motions, respectively; {R(t)} is the nodal loading vector. If the damping effects of the medium are considered by the Rayleigh damping theory, we have

$$[C] = \alpha[M] + \beta[K] \qquad (2)$$

where, α, β are scaling factors controlling the degrees of the damping effects.

For the finite element model with numbers of elements, excessive computer memory space and excessive computing time are needed to solve the dynamic equations if the matrix [M], [C] and [K] in Eq.(1) are formed directly by the traditional finite element method. Therefore, the special finite element method similar to finite difference method is introduced to set up the dynamic equations of nodal motions for the local nodal system in whole computing model.

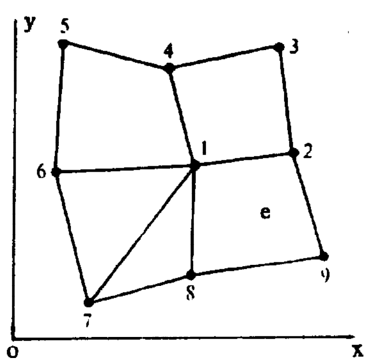

Fig. 2. Sketch map of the element connections in local nodal system

For the local nodal system as shown in Figure 2, a kind of lumped mass finite element method similar to finite difference method (Liao, 1984) is used to set up the dynamic equations of node No.1. The obtained dynamic equation is:

$$M_1 \ddot{u}_1 + \sum_{l=1}^{L}(\alpha M_{1l} + \beta K_{1l})\dot{u}_l + \sum_{l=1}^{L} K_{1l} u_l - R_1 = 0 \qquad (3)$$

in which

$$M_1 = \sum_{k=1}^{Ne} \sum_{j=1}^{Nd(k)} M_{1j}^{k} \qquad (4)$$

$$\sum_{l=1}^{L} M_{1l} = \sum_{k=1}^{Ne} \sum_{j=1}^{Nd(k)} M_{1j}^{k} \qquad (5)$$

$$\sum_{l=1}^{L} K_{1l} = \sum_{k=1}^{Ne} \sum_{j=1}^{Nd(k)} K_{1j}^{k} \qquad (6)$$

$$R_1 = \sum_{k=1}^{Ne} R_1^{k} \qquad (7)$$

In above equations, M_{1j}^k, K_{1j}^k, $R_1^k$are the elements of the mass matrax $[M_{1j}^k]$, the stiffness matrax $[K_{1j}^k]$ and the equivalent nodal loading vector $[R_1^k]$ of the finite element No.k in the local nodal system, respectively; Ne is the number of the finite elements containing the node No.1; Nd(k) is the number of the nodes of the finite element No.K.

On the analogy, the relations of the response quantities between each node and the related nodes can be obtained for finite element discrete model.

The Explicit Difference Solution of the Dynamic Equations

In order to get an explicit high accuracy step–by–step integration formula to analyzing the earthquake response of site, the explicit finite difference scheme for solving the visco–elasto–plastic dynamic equations proposed recently by the author (Li, et al., 1992; Li et al., 1995) is used. The explict finite difference solution of the dynamic equation (3) is

$$
\begin{aligned}
u_1^{p+1} &= \frac{\Delta t^2}{2M_1} R_1 + u_1^p + \Delta t \dot{u}_1^p - \frac{\Delta t^2}{2M_1} \sum_{l=1}^{L} [K_{1l} u_l^p + (\alpha M_{1l} + \beta K_{1l}) \dot{u}_l^p] \\
\dot{u}_1^{p+1} &= \frac{\Delta t}{2M_1} (R_1^{p+1} + R_1^p) + \dot{u}_1^p - \frac{1}{2M_1} \sum_{l=1}^{L} [K_{1l} \Delta t (u_l^{p+1} + u_l^p) \\
&\quad + 2(\alpha M_{1l} + \beta K_{1l})(u_l^{p+1} - u_l^p)] \\
\ddot{u}_1^{p+1} &= -\ddot{u}_1^p + \frac{2}{\Delta t} (\dot{u}_1^{p+1} - \dot{u}_1^p)
\end{aligned} \tag{8}
$$

For any node No.i in the finite element discrete model, the response quantities can be computed by equation (8), only if the node number '1' in equation (8) is replaced by 'i'.

Local Transmitting Boundary

The explicit formulas for computing the response quantities of the nodes inside the artificial boundaries (not including the nodes on the artificial boundary) in the finite element discrete model are introduced above. However, in order to bring about the step–by–step computation for the nodal response quantities, it is necessary to have the step–by–step formulas for computing the response quantities of nodes on the artificial boundaries. It is just the problem to be solved by any artificial boundary method.

Transmitting Boundary (Extrapolation Boundary) is an artificial boundary method widely used. In the method, the wave field is divided into the inward propagating wave and the outward propagating wave, and The method mainly simulates the outward propagating wave (simulates the motions of the mediums caused by the outward propagating wave on artificial boundaries). If assuming that u_{IJ} and u_{RJ} are the displacement quantities caused by the inward propagating wave and outward propagating wave at the node J on artificial boundaries, there is

$$u_J^{p+1} = u_{IJ}^{p+1} + u_{RJ}^{p+1} \tag{9}$$

where, u_J^{p+1} is the displacement quantity at the node J at the time p+1.

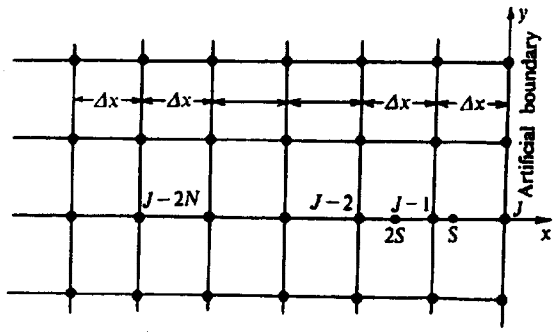

Figure 3. Transmitting Boundary in the discrete model in space

For the artificial boundary in the finite element discrete moel shown in Figure 4, following relationship is given by Generalized Transmitting Boundary (Li, 1993; Liao, et al., 1995)

$$u_{RJ}^{P+1} = \sum_{n=1}^{N}(-1)^{n+1}C_n^N u_{n,J}^{P+1} + \sum_{l=1}^{M}(-1)^{l+1}C_l^M\left(u_{RJ-l}^{P+1} - \sum_{n=1}^{N}(-1)^{n+1}C_n^N u_{n,J-l}^{P+1}\right) \tag{10}$$

in which,

$$u_{K,J'}^{P'} = \frac{1}{2}(1-S)(2-S)u_{K-1,J'}^{P'-1} + S(2-S)u_{K-1,J'-1}^{P'-1} + \frac{1}{2}S(S-1)u_{K-1,J'-2}^{P'-1}$$

$$u_{0,J'}^{P'} = u_{RJ'}^{P'}$$

$$P' = P+1,\cdots,P+1-N;\quad K = N,\cdots,1;\quad J' = J,\cdots,J-2N-M$$

in equation (10), M and N are the parameters controlling the simulating accuracy of Generalized Transmitting Boundary, in general case, $M=1$, $N=1$ or $M=1$, $N=2$; $S=C_a\Delta t/\Delta x$, C_a is artificial wave velocity. The response velocity quantity u is also given by Eq.10, only if the $\ddot{u}$ in Eq.9 is replaced by $\dot{u}$.

THE INWARD WAVE IN ARTIFICIAL BOUNDARY AREA

Because the Multi−transmitting Boundary mainly simulates the outward propagating wave, the inward propagating wave should be known if simulating the motions on artificial boundary by the the Multi−transmitting Boundary and the explicit finite element−finite difference method mentioned above. The following measures are suggested to determine the inward progagating wave field in artificial boundary area (see the Figure 3 and assume that the propagating direction of the incident wave is as shown in Figure 3):

a) considering that there is no inward propagating wave field in the right artificial boundary area;

b) considering the computing incident wave field as the inward propagating wave field in the bottom artificial boundary area ;

c) considering the computed free−field for the horizontally layered medium model as the inward propagating wave field in the left artificial boundary, the horizontally

THE HIGH–FREQUENCY INSTABILITY

The instability caused by the transmitting boundary is a major obstacle in wave motion simulating by means of the finite element technique, particularly when the required computing time is long. The high–frequency instability is a main form of the instability. Systematic studies are made by Liao et al. (1992) on the high–frequency instability. A measure is proposed for eliminating the high–frequency instability, that is, for the each time step p, the displacements u_j^p at the nodes in the artificial boundary area are replaced by $\tilde{u}_j^p$ which are computed by the following formula:

$$\tilde{u}_j^p = \beta u_j^p + \frac{1-\beta}{2}(u_{j-1}^p + u_{j+1}^p), \qquad j = J-1, \cdots, J-2N \tag{11}$$

When the value of β is small enough $(0<\beta<1)$, the measure above is effective for eliminating the high–frequency instability because of the strong filter effect for high frequency motions, but it may affect the low frequency motions in some degree.

Another measure for eliminating the high–frequency instability is recently proposed by author (Li, 1993), that is, suitably combining the step–by–step integration method which has strong dissipation with Multi–transmitting Boundary. Because of the strong dissipation (eliminating high frequency motion but affecting the low frequency motion very little), if the step–by–step integration method is used for solving the dynamic equation of the nodes inside the artificial boundary, the high frequency motions caused by the high–frequency instability of Multi–transmitting Boundary will be eliminated effectively. The explicit finite difference scheme proposed by author (Li et al., 1992) and used in the explicit finite element–finite difference method mentioned above is one of the step–by–step integration methods having the strong dissipation, and the degree of the dissipation is depended on the damping value (the value of β in equation (2)), little increase of the damping value will cause large increase of the degree of the dissipation, so the high–frequency instability of Multi–transmitting Boundary can be eliminated effectively by suitably controlling the computing damping value (in the case of not obviously going against the actual computing site condition).

THE DRIFT INSTABILITY

Besides the high–freqency instability (oscillation instability), there is another form of instability, drift instability, in the numerical integration method. In the Multi–transmitting Boundary, there is also the drift instability. This form of instability caused by Multi–transmitting Boundary was met in early time in application of the boundary method, but the phenomena do not appear or do not obviously appear for some special cases, so the study on the problem was not paid attation. Recently, author (Li, 1993) studied on the problem, and considering the feature of the instability phenomena as shown in Figure 4 (dotted lines), called this instability as ″Drift Instability of Multi–transmitting Boundary″.

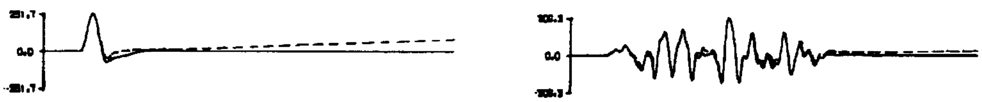

Fig. 4. The phenomena of the drift instability

The studies on the drift instability of Multi–transmitting Boundary show:

a) the origin of resulting in the drift instability is the computing error wave field, that is, the diference between the computed inward wave field and actual inward wave field in discrete model;

b) application of the one–order Transmitting Boundary does not cause the drift instability;

c) application of the two–order or higher Multi–transmitting Boundary may cause the drift instability, and if the drift instability appears, the degree of drift instability increases markedly with the increase of the transmitting order.

Based on the studies on the drift instability and the features of instability phenomena, a measure for eliminating the drift instability of Multi–transmitting Boundary is proposed. The basic idea is: in the computing processes, the computed nodal motions inside the artificial boundaries should be analyzed at each time step to judge whether the tendency of drift instability apears for each nodal motion in the artificial boundary area, if the tendency appears, the one–order Transmitting Boundary should be used to replace the Multi–transmitting Boundary originally used, and keep this case to do computation for several time steps, and then return to the original case (using the Multi–transmitting Boundary). The Figure 4 (soild lines) shows the computing results by the measure for eliminating the drift instability of Multi–transmitting Boundary.

CONCLUSION

The paper has introduced the basic idea about the high accuracy explicit finite element–finite difference method for analyzing the earthquake response of the viscoelastic inhomogenous local site, and has given the detail computing formulas. The basic theory of the method is also suitable for the problem of nonlinear local site, and for this case the explicit method will give full play to its superiority, that is, comparing with other numerical methods, the computing time decreases on a large scale. General discussions are made about the high–frequency instability and the drift instability of Multi–transmitting Boundary, and the effective measures for eliminating the instabilities are introduced. And Some suggestions are given on the determination of the inward propagating wave field in artificial boundary area.

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